Edmonton Junior High Mathematics Contest

May 3, 2000

Instructions

1. You are allowed up to 2 hours to write the contest.
2. Do not write your name on the contest paper.
3. The maximum score is 100.
4. Calculators are allowed.
5. Graph paper is allowed.

The Contest Consists of Two Parts.

Part I: Multiple Choice (10 questions)

A. Use the answer sheet.
B. Circle the correct answer
C. Each correct response is worth 5 marks.
D. Each unanswered question is worth 1 mark.
E. Each incorrect response is worth 0 marks.

Part II: Numeric Response (10 Questions)

A. Use the answer sheet to record the numerical value(s).
B. Each correct response is worth 5 marks
C. Each incorrect response is worth 0 marks.

EDMONTON JUNIOR HIGH MATHEMATICS CONTEST
MAY 3, 2000

PART 1: MULTIPLE CHOICE

1. What is the value of $\sqrt{2}-\sqrt{8}+\sqrt{18}$?

(a) 0 (b) $\sqrt{2}$ (c) $2\sqrt{2}$ (d) $3\sqrt{2}$

2. Donald mixed x kilograms of peanuts and y kilograms of raisins to make a mixture. The peanuts cost \$5 per kilogram while the raisins cost \$4 per kilogram. When the cost of the peanuts rose by 10% and the cost of the raisins dropped by 15%, Donald's total cost for making the mixture remained the same. What was the ratio $x : y$?

(a) 2:3 (b) 5:6 (c) 6:5 (d) 3:2

3. Alice is twice as old as Betty, and Betty is 7 years younger than Cecilia. The total of their ages is 67. How old is Betty?

(a) 10 (b) 12 (c) 15 (d) 20

4. What are the solution(s) of the equation $\sqrt{3x+10}=-x$?

(a) –2 only (b) 5 only (c) –2 and 5 only (d) no solution

5. Let x be any number, a = 2000x + 1999, b =2000x + 2000, c = 2000x + 2001. What is the value of $a^2+b^2+c^2-ab-bc-ca$?

(a) 3 (b) 4 (c) 6 (d) dependent on x

6. The numbers a and b satisfy $1 > -b > a > 0 > b$. Which of the following inequalities must be correct?

(a) $1-b>1+a>-b>a$ (b) $1+a>1-b>-b>a$

(c) $1+a>a>1-b>-b$ (d) $1-b>-b>1+a>a$

7. No two of the numbers a, b and c are equal, and none of the numbers x, y and z is equal to 0. Which of the following statements must be correct if $\frac{x}{b-c} = \frac{y}{c-a} = \frac{z}{a-b}$?

(a) $ax + by + cz = 0$ (b) $ax = by = cz$

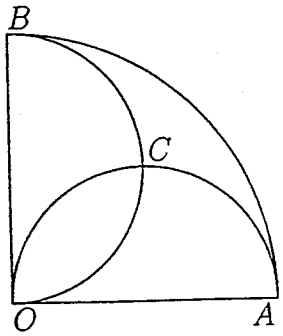
(c) $x + y + z = 0$ (d) $x = y = z$

8. In triangle ABC, $3\angle A > 5\angle B$ and $2\angle B \geq 3\angle C$. What kind of triangle is ABC?

(a) acute triangle (b) right triangle

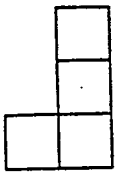
(c) obtuse triangle (d) inconclusive

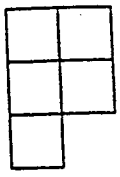
9. The diagram below shows a circular arc AB with centre O, such that OA is perpendicular to OB. Two semicircles with respective diameters OA and OB meet at C. Which has larger area, the football-shaped region OC or the scythe-shaped region ABC?



(a) OC (b) ABC (c) equal (d) inconclusive

10. The diagrams below show a L-shaped figure and a P-shaped figure. Leon wants to buid a similar larger P-shaped figure using several identical smaller copies of the L-shaped figure while Paul wants to build a similar larger L-shaped figure using several identical smaller copies of the P-shaped figure. Who may succeed?





(a) both (b) only Leon (c) only Paul (d) neither

PART 2: NUMERIC RESPONSE

1. If $2^8 + 4^4 + 16^2 + 256 = 2^x$, what is the value x?

2. In a contest, there are 80% more boys than girls, and the average of the girls is 20% higher than that of the boys. If the overall average is 75, what is the average of the girls?

3. The number of red balls is more than the number of green balls, but less than twice the number of green balls. Each green ball costs \$2 while each red ball costs \$3. If the total cost of the balls is \$60, what is the total number of balls?

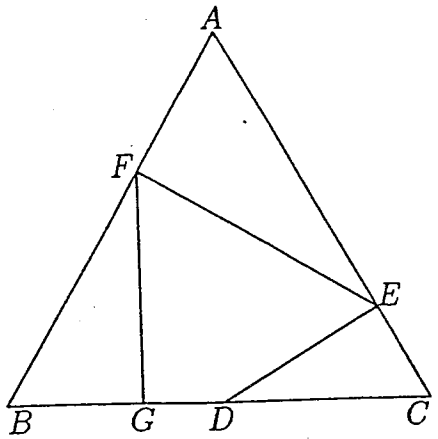
4. If both p and $p^5 + 5$ are prime numbers, what is the smallest value of $n > 5$ such that $p^n + n$ is also a prime number?

5. If $x + y = 5$, $y + z = 8$ and $z + x = 7$, what is the value of $2x + 3y + 5z$?

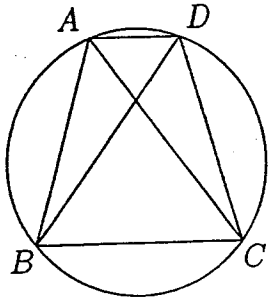
6. If $x + \frac{1}{x} = 3$, what is the value of $\frac{x^2}{x^4 + x^2 + 1}$?

7. How many integers x satisfy the inequality $\frac{4}{\sqrt{3}+\sqrt{2}} < x < \frac{60}{\sqrt{170}-\sqrt{2}}$?

8. The diagram below shows an equilateral triangle ABC of area 128. D is the midpoint of BC. E is the point on CA such that DE is perpendicular to CA. F is the point on AB such that EF is perpendicular to AB. G is the point on BC such that FG is perpendicular to BC. What is the area of the quadrilateral DEFG?



9. The diagram below is a rough sketch of a quadrilateral ABCD inscribed in a circle. The arcs AB, BC, and CD have the same length. The diagonals AC and BD meet at E, and $\angle AED = 70°$. What is the measure of $\angle ABD$?



10. The diagram below shows the first three of a sequence of towers. After the first, the next tower is obtained from the preceding one by adding a unit cube on top of each stack of cubes plus a row of unit cubes at the base in front. There are 1, 4, and 10 unit cubes in the first three towers respectively. How many unit cubes are needed to build the sixth tower?