2000 Edmonton Junior High Mathematics Contest Solutions

Part 2: Numeric Response

1. We have $2^8 + 4^4 + 16^2 + 256 = 2^8 + 2^8 + 2^8 + 2^8 = 2^2(2^8) = 2^{10}$. Answer = 10

2. Let the girls' average be 6n so that the boys' is 5n. Now the ratio of girls to boys is 5:9. Hence 5(6n) + 9(5n) = (5 + 9)(75) or n = 14. Answer = 84

3. Let there be x red balls and y green balls. Then y < x < 2y and 3x + 2y = 60. Note that x must be divisible by 2 and y by 3. Let x = 2u and y = 3v. Then 3v < 2u < 6v and u + v = 10. If u ≤ 6 then v ≥ 4 and 3v ≥ 2u. If u ≥ 8 then v ≤ 2 and 2u ≥ 6v. Hence u = 7, v = 3, x = 14 and y = 9. Answer = 23

4. Since $p^5 + 5$ is a prime number, p must be even. Since p is also a prime number, we must have p = 2 so that $p^5 + 5 = 37$. In order for $p^n + n$ to be a prime number also, n must be odd. When n = 7, we get 135 which is divisible by 5. When n = 9, we get 521 which is less than the square of 23. Since 521 is not divisible by any of the prime numbers 2, 3, 5, 6, 11, 13, 17,and 19, it is a prime number itself. Answer = 9

5. Adding the given equations, we have 2(x + y + z) = 20 or x + y + z = 10. Subtracting from this the given equations one at a time, we have z = 5, x = 2 and y = 3. Answer = 38

6. Squaring the given equation, we have $x^2 + 2 + \frac{1}{x^2} = 9$ so that $x^2 + 1 + \frac{1}{x^2} = 8$. Dividing the denominator of $\frac{x^2}{x^4+x^2+1}$ by the numerator, we obtain $x^2 + 1 + \frac{1}{x^2}$. Answer = $\frac{1}{8}$

7. We have $1 = \frac{4}{2+2} < \frac{4}{\sqrt{3}+\sqrt{2}} < \frac{4}{1+1} = 2 < 5 = \frac{60}{13-1} = \frac{60}{\sqrt{170}-\sqrt{2}} < \frac{60}{12-2} = 6$. Thus the acceptable values are 2, 3, 4 and 5. Answer = 4

8. Let CD = n and AB = BC = AC = 2n. This gives $128 = \frac{2n(\sqrt{3}n)}{2}$ $or\ n^2 = \frac{128}{\sqrt{3}}$ Knowing that ΔCED ~ ΔAFE ~ ΔBGF and that the ratio of the sides of a 30-60-90 triangle is 1-$\sqrt{3}$-2. We can find that CE = $\frac{n}{2}$, DE = $\frac{\sqrt{3}n}{2}$, AE = 1.5n, AF = $\frac{3n}{4}$, FE = $\frac{3\sqrt{3}}{4}n$, FG = $\frac{5\sqrt{3}}{8}n$, BF = 1.25n and BG = $\frac{5n}{8}$. It follows that the area of the quadrilateral [DEFG] = 128 – ([ΔCED]+[ΔAEF]+[ΔBGF]), where [m] denote the area of m.

$$128 - \left(\frac{1}{2} \times \frac{5}{8} \times \frac{5\sqrt{3}}{8} \times n^2 + \frac{n}{2} \times \frac{\sqrt{3}n}{4} + \frac{3n}{4} \times \frac{3\sqrt{3}n}{8}\right) = 128 - \left(\frac{25\sqrt{3}}{128}n^2 + \frac{\sqrt{3}}{8}n^2 + \frac{9\sqrt{3}n}{32}n^2\right)$$

$= 128 - \left(\frac{25\sqrt{3}+16\sqrt{3}+36\sqrt{3}}{128}\right)\left(\frac{128}{\sqrt{3}}\right)$ = 128 – 77 = 51. Answer = 51

9. From the equal arcs, we have ∠ACB = ∠CAB = ∠DBC. Since ∠CEB = ∠AED = 70° and ∠CEB + ∠ACB + ∠DBC =180°, each of those three equal angles has measure 55°. Now ∠ABD=∠AED – ∠CAB. Answer = 15°.

10. To go from the (n-1) tower to the nth, we add 1 + 2 + … + n = $\frac{n(n+1)}{2}$ unit cubes. Since the third tower requires 10 unit cubes, the number of unit cubes needed to build the sixth tower is 10 + 0.5(4 x 5 + 5 x6 + 6x7). Answer = 56.