Edmonton Junior High Mathematics Contest 2001

Answer Sheet

Multiple Choice	Numeric Response
1. a ● c d	1. $\frac{32}{3}$ or $10\frac{2}{3}$
2. a ● c d	2. 261
3. a b c ●	3. 6
4. a b c ●	4. 37
5. a b ● d	5. 1157
6. ● b c d	6. 36
7. a ● c d	7. 1
8. a b ● d	8. 8
9. a b ● d	9. 18900
10. a b c ●	10. 8

For Marker Only

Multiple Choice	
Numeric Response	
Total	

Place Student Label Below

Key

MULTIPLE CHOICE SOLUTIONS

1. Note that 49^1 ends in 49, 49^2 ends in 01, 49^3 ends in 49 and so on. Since 2001 is odd, 49^{2001} ends in 49. **Answer: (b)**

2. The sum is equal to $7 \times 7^7 = 7^8$. **Answer: (b)**

3. We have $3 \oplus (2 \otimes 4) = 3 \oplus (2^2 + 4^2 - 2 \times 4) = 3 \oplus 12 = 3 + 12 + 3 \times 12 - 1 = 50$. **Answer: (d)**

4. If the old price is 1, then the new one is $(1+\frac{r}{100})(1-\frac{r}{100}) = 1-\frac{r^2}{1000}$. **Answer: (d)**

5. Each day, the goat eats $\frac{1}{x}$ of the cabbage while the rabbit eats $\frac{1}{y}$. Hence the number of days the cabbage will last is $1 \div (\frac{1}{x}+\frac{1}{y}) = \frac{xy}{x+y}$. **Answer: (c)**

6. Let the present ages of Ace and Bea be x and y respectively. When Ace was y years old, Bea was y – (x – y) = 10 years old. When Bea is x years old, Ace will be x + (x – y) = 25 years old. Subtracting the first equation from the second equation, we have 3(x – y) = 15, so that Ace is 5 years older than Bea. **Answer: (a)**

7. Since the two sides differ by 5 and the perimeter is odd, the third side must be even and greater than 5. Hence the smallest possible value is 6. **Answer: (b)**

8. The sum of the angles of an n-sided polygon in (n – 2)180°. When 2001 is divided by 180, the quotient is 11 and the remainder is 21. Hence the largest possible value of n is 11 + 2 = 13. **Answer: (c)**

9. The circle with centre F and radius AF will pass through B since AF = FB, and also through E since ∠BEA = 90°. Hence BF = EF = Be so that BEF is an equilateral triangle. Now ∠CAB = 90° – ∠ABE = 30° and ∠BCA = 1/2 (180° – ∠CAB) = 75°. **Answer: (c)**

10. We have $(\frac{2}{x}+x)^2 = \frac{4}{x^2}+4+x^2 = (\frac{2}{x}-x)^2+8 = 9$ so that $\frac{2}{x}+x=3$. **Answer: (d)**

NUMERIC RESPONSE SOLUTIONS

1. $\dfrac{-2 \div \frac{1}{4} + -8}{-2 + \frac{1}{2}} = \dfrac{-16}{\frac{-3}{2}} = -16 \times \dfrac{-2}{3} = \dfrac{32}{3}$ **Answer:** $\dfrac{32}{3} or 10\dfrac{2}{3}$

2. Ace must receive at least $20 + (\frac{500-20}{2} + 1) = 261$ of the remaining 500 votes. **Answer: 261**

3. When Peter runs 5 laps, Steven runs 3 laps for a difference of 2 laps. For a difference of 4 laps, Peter must run 10 laps and Steven 6. **Answer: 6**

4. In order for n + 13 to be a multiple of 5, we must have n = 2, 7, 12, ... Now 2 – 13 is not divisible by 6, and while 7 – 13 is, it is not a positive multiple of 6. The least common multiple of 5 and 6 is 30, and 30 + 7 = 37 is the desired minimum. **Answer: 37**

5. We have $\frac{w}{z} = \frac{5^3}{8^3}$. Hence the smallest positive integers satisfying the given relation are $w = 5^3$, $x = 5^2 \times 8$, $y = 5 \times 8^2$ and $z = 8^3$, yielding w + x + y + z = 1157. **Answer: 1157**

6. Let E be the midpoint of AD. Then AE = 3. Since AC = CD, AD is perpendicular to CE. By Pythagoras' Theorem, $CE = \sqrt{AC^2 - AE^2} = 4$. The area of triangle CAD is $\frac{1}{2} AD \bullet CE = 12$. Since BD = 2CD, the area of triangle BAD is twice that of triangle CAD. It follows that the area of triangle ABC is 3 × 12 = 36. **Answer: 36**

7. The last digit of $1^3 - 2^3 + \ldots -10^3$ is the same as that of $11^3 - 12^3 + \ldots -20^3$, and so on. It follows that the last digit of $1^3 - 2^3 + \ldots - 2000^3$ must be 0, so that the last digit of $1^3 - 2^3 + 3^3 - \ldots + 2001^3$ is 1. **Answer: 1**

8. In order for the four-digit number to be divisible by 11, the sum of the first and the third digits must be 5. They may be either 1 and 4 or 2 and 3. Moreover, each pair can be permuted internally, yielding a total of 2 × 2 × 2 = 8 such numbers. **Answer: 8**

9. Length of each side of larger cube is 15 cm. Therefore the total surface area of painted surfaces on larger cube is 15 × 15 × 6 = 1350 cm^2. The total surface area of the individual 3375 individual 1 cm cubes is 3375 × 6 = 20 250 cm^2. Thus total unpainted surfaces is 20 250 – 1350 = 18 900.
Answer: 18 900

10. The area of triangle GAD is $\frac{1}{4} \times 24 = 6$. The area of triangle AGE is $\frac{1}{2} \times \frac{1}{4} \times 24 = 3$. The area of triangle BEF is $\frac{1}{2} \times \frac{3}{4} \times \frac{1}{3} \times 24 = 3$. The area of triangle CFG is $\frac{1}{2} \times \frac{1}{2} \times \frac{2}{3} \times 24 = 4$. Hence the area of triangle EFGis 24 – 6 – 3 – 3 – 4 = 8. **Answer: 8**