Edmonton Junior High Mathematics Contest
May 7, 2003

Part I: Multiple Choice

1. Half of $49\frac{1}{2}$ is equal to

a) $24\frac{1}{4}$ b) $24\frac{1}{2}$ c) $24\frac{3}{4}$ d) 99

(c) half of 99/2 is 99/4 = 24 ¾

2. After its price has been reduced by 20%, a tennis racket is still not selling. So the manager decides to restore the original price. This means a raise of

a) 16% b) 20% c) 24% d) 25%

(d) We may take the price of the racket to be $100. Then the 20% reduction is $20 drop in price. However, $20 is 25% of the new price $80.

3. A fair six-sided die has the numbers 1, 2, 3, 5, 6, 7 on its faces, while a fair eight-sided die has the numbers 1,2,4,6,8,10,12 and 13 on its faces. The two dice are rolled and the numbers on the top faces are added. The probability that this sum is even is

a) $\frac{5}{12}$ b) $\frac{1}{4}$ c) $\frac{1}{6}$ d) $\frac{4}{7}$

(a) There are 6 x 8 = 48 possible combinations. An even sum may result from a combination of two even numbers. There are 2 x 6 = 12 such ones. It may also result from a combination of two odd numbers. There are 4 x 2 = 8 such ones. Thus, the probability is (12 + 8)/48 = 5/12

4. Cranky became a bus driver at age 20. His company allowed retirement when his age and his years of service added up to at least 80. Cranky retired after 50 years of continuous service. His age at that time was

a) 30 b) 50 c) 60 d) 70

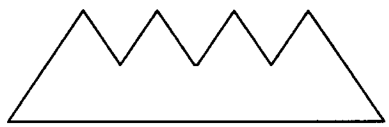
(d) Since Cranky started working at age 20 and retired after 50 years, he was 70 years old at that time.

5. Tom catches 5 mice and eats 2 rats per day, while Sylvester catches 3 rats and eats 1 mouse per day. As their retirement saving plan, they keep the uneaten mice and rat in a cage. There are no births or deaths in captivity. The difference in the numbers of mice and rats in the cage can never be

a) 12345 b) 12456 c) 23456 d) 123456

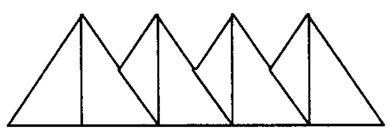
(c) Each day, 4 mice and 1 rat are put into the cage. Hence the total difference must be a multiple of 3. Using divisibility test for 3, 23456 is the only one not divisible by 3.

6. A, B,C ,D , E and F are points on a line such that AB = BC = CD = DE = EF = 1. On the same side of this line are four isosceles right triangles, with respective hypotenuses AC, BD, CE and DF. The total area covered by the resulting figure is



a) 3 b) $3\frac{1}{4}$ c) $3\frac{1}{2}$ d) $3\frac{3}{4}$

(b) Each of the large triangles in the diagram below has an area of half while each of the small triangles has an area of one-quarter. The total area is 5/2 + ¾ = 3 ¼



7. Let a and b be positive numbers such that (111+a)(111-b) = 12345. The relation between a and b is

a) a > b b) a = b c) a < b d) not constant

(a) 12321 + 111(a – b) – ab = 12345 or 111(a – b) = 24 + ab. Since the right side is positive, so is the left side. Hence a > b.

8. A cylinder has height 6 and base circumference 16. A bug is at a point B on the top edge of the cylinder, and wishes to crawl along its surface to the point on the bottom edge directly below the point diametrically opposite to B. The shortest crawling distance is

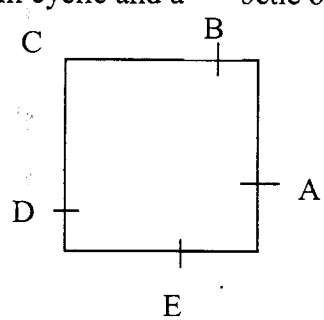
a) 6 b) 10 c) 14 d) 16

(b) Cut the hollow cylinder along the vertical line through B and unroll its curved surface into a 6 x 16 rectangle. The destination is the midpoint of the side of length 16 not ending at B. The shortest crawling distance is along a straight line from B to that point. By Pythagoras's Theorem, the distance is $\sqrt{6^2+8^2} = 10$

9. Dima lives at the north-west corner of a square city block, and Sunera lives in the south-east corner of the same block. Every morning, they start jogging around the block at the same time, Dima going clockwise and Sunera going counter-clockwise. Both jog at constant speeds, but Dima's is four times as fast as Sunera's. At their 2003rd meeting, they find themselves at a corner of the block. This corner is the

a) north-east b) north-west c) south-west d) south-east

(b) The five points A, B, C, D, and E in the diagram below divides the perimeter of the square block into five equal parts. Since Dima is four times as fast as Sunera, their meetings take place at these five points in cyclic and alphabetic order. It follows that their 2003rd meeting takes place at C.



10. When a barrel is 20% empty, it contains 20 litres more than when it is 20% full. How many litres does the barrel contain when full?

a) 80 b) $66\frac{1}{2}$ c) $66\frac{2}{3}$ d) $33\frac{1}{3}$

(d) Since 60% of the barrel's capacity is 20 litres, its full capacity is 26 x 5/3 = 33 1/3 litres.

Part II: Numeric Response

1. The shortest diagonal of a regular octagon has length $\sqrt{2}$. The longest diagonal of the same octagon has length _______.

Solution: 2. Let AC be a shortest diagonal and AE be a longest one. Then ACE is a right isosceles triangle. Since CE = AC = $\sqrt{2}$, AE = 2 by Pythogoras' Thm.

2. The fractions $\frac{1}{n}$ *and* $\frac{1}{n+3}$ can both be expressed as terminating decimals. The smallest positive integral value of such an n is 1, and the next smallest ones are 2 and 5. The next smallest is ________.

Solution: 125. The fraction 1/n can be expressed as a terminating decimal if and only if all prime divisors of n are 2s and 5s. Since one of n and n+3 is odd, it must be a power of 5. We already have the pairs (1,4), (2,5) and (5,8). None of 22, 28, and 122 are powers of 2. Hence the next pair is (125, 128).

3. There are ___ positive integers under 100 which can be expressed as products of two even numbers.

Solution: 24. A number can be expressed as products of two even numbers if and only if it is a multiple of 4. Not counting 100, there are 100/4 – 1 = 24 such numbers.

4. Jina wrote down a number and passed it to Rata. Rata doubled it and passed the product to Hema. Hema multipled it by 5 passed the product back to Jina. Jina subtracted her original number from it, and passed the difference to Niti. Niti divided it by 9, ignored any remainder and passed the quotient back to Jina. Jina again subtracted her original number from it. The maximum value of her final difference is ______.

Solution: 0. Regardless of the original number, the final difference will be 0.

5. Nine points are arranged uniformly in a 3 x 3 configuration. Among the distances between two of these points are _____ different values.

Solution: 5. If the two points are on the same row or the same column, the distance between them is 1 or 2. If not, then they are opposite corners of a rectangle which may be 1x1, 1x2, or 2x2. Hence there are 5 different values.

6. When the positive integer __ is added to 44 and to 100, both sums are squares of integers.

Solution: 125. Let the first square be a^2 and the second be b^2. Then (b-a)(b+a) = 100 – 44 = 56. Now b-a and b+a have the same parity. Since they cannot be both odd, we have either b-a=4 and b+a = 14, or b-a = 2 and b+a = 28. The first case leads to a = 5, but $a^2 = 25$ is less than 44. The second case leads to a = 13 so that the number added is $a^2 = 44 = 125$.

7. In the quadrilateral ABCD, AB = BC = CD = 5, AD = 11 and BC is parallel to AD. The area of ABCD is ______.

Solution: 32. Drop the perpendicular BE from B onto AD. Then AE = (AD – BC)/2 = 3 and by Pythagoras' Theorem, BE = $\sqrt{AB^2 - AE^2}$ = 4. Hence the area of ABCD is half of BE(BC + AD) = 32.

8. There are _____ two-digit prime numbers that form a different prime number when the order of the two digits is reversed.

Solution: 9. Both digits must come from 1, 3, 7, 9. If the two digits are identical, only 11 is prime. Of the six pairs of distinct digits, four yield pairs of primes, namely, 13, 31, 17, 71,37, 73, 79 and 97.

9. A book with 96 pages are printed on 24 sheets of paper. The first sheet contains pages 1 and 2 back to back, as well as pages 95 and 96 back to back. The second sheet contains pages 3, 4, 93 and 94, and so on. On the same sheet which contains page 37, the other odd-numbered page is page _____.

Solution: 59. The page printed on the same side of the sheet as page 37 is page 97 – 37 = 60. Hence the other odd-numbered page on this sheet is page 59.

10. In a circle with centre O, the perpendicular bisector of a radius cuts the circle at A and B. The measure of $\angle AOB$ is ______ degrees.

Solution: 120. Let the bisected radius be OC. Then AC=AO=OC, so that angle AOC = 60°. Similarly, angle COB = 60°, so that angle AOB = 120°