Edmonton Junior High Mathematics Contest 2006

Multiple-Choice Problems

Problem 1

Ace, Bea, Cec, Dee, Eve, Fth and Geo are 1, 2, 3, 4, 5, 6 and 7 years old, in some order. Dee is three times as old as Bea. Cec is four years older than Eve. Fth is older than Ace and Ace is older than Geo, but the combined age of Ace and Geo is greater than the age of Fth. The age of Ace is

(a) 2 (b) 3 (c) 4 (d) 5

Solution:

If Bea is 2, then Dee is 6. Now Eve and Cec are 1 and 5 respectively, or 3 and 7 respectively. In each case, the highest of the remaining three ages is equal to the sum of the other two. Hence Bea is 1. It follows that Dee is 3 while Eve and Cec are 2 and 6 respectively. Now Fth is 7, Geo is 4 and Ace is 5. The answer is (d).

Problem 2

When 10 girls joined the Math Club in January, the percentage of boys in the club dropped to 20%. Then 10 boys joined the club in February, and this percentage rose to 40%. The percentage of boys in the club originally was

(a) less than 30% (b) 30% (c) 40% (d) between 30% and 40%

Solution:

Suppose there were x boys and y girls in the club originally. Then $\frac{x}{x+y+10}=\frac{1}{5}$ and $\frac{x+10}{x+y+20}=\frac{2}{5}$. These equations simplify to 4x-y=10 and -3x+2y=10. Solving this system of equation, we have (x,y)=(6,14) and $\frac{6}{6+14}=\frac{3}{10}$.The answer is (b).

Problem 3

A $100 video game is not selling. The store gives an n% discount, but it is still not selling. So the store gives another n% discount on the reduced price, and the video game is finally sold for $64. The value of n is

(a) 6 (b) 8 (c) 18 (d) 20

Solution:

We have $100\left(1-\frac{n}{100}\right)^2=64$. Hence $1-\frac{n}{100}=\frac{8}{10}$ so that n=20. The answer is (d).

Problem 4

There are four elevators in a building. Each makes only three stops, which do not have to be on consecutive floors or include the main floor. For any two floors, there is at least one elevator which stops on both of them. The maximum number of floors in this building is

(a) 5 (b) 6 (c) 7 (d) 12

Solution:

There are twelve elevator stops altogether. Suppose there are six floors. Then some floor has at most two elevators stopping there. Each elevator connects this floor to two others, so that only four of the other five floors are connected to this floor. If there are seven or more floors, some floor has at most one elevator stopping there, and the situation is worse. Hence there are at most five floors. This is possible if the first elevator stops on floors 1, 4 and 5, the second on 2, 4 and 5, the third in 3, 4 and 5, and the fourth on 1, 2 and 3. The answer is(a).

Problem 5

A bookstore employed five part-time workers. Ace worked on Mondays, Tuesdays and Wednesdays. Bea worked on Wednesdays and Thursdays. Cec worked on Tuesdays and Thursdays. Dee worked on Mondays and Fridays. Eve worked on Thursdays and Fridays. During a certain week, they reported on the total numbers of books sold during the days in which they worked. The figures given by Ace, Bea, Cec, Dee and Eve were 115, 85, 90, 70 and 80 respectively. How many books were sold on Thursday of that week?

(a) 30 (b) 40 (c) 50 (d) 60

Solution:

According to Ace and Eve, the total number of books sold during that week was 115+80=195. According to Bea, Cec and Dee, the total number of books sold and the number of books sold on Thursday during that week was 85+90+70=245. Hence the number of books sold on Thursday of that week was 245-195=50.

Problem 6

The value of $\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}$ is

(a) $\sqrt{6}$ (b) $2\sqrt{2}$ (c) $2\sqrt{3}$ (d) 6

Solution:

Squaring the expression, we have $2+\sqrt{3}+2\sqrt{(2+\sqrt{3})(2-\sqrt{3})}+2-\sqrt{3}=6$. The answer is (a).

Problem 7

If a and b satisfy a+b-ab=1 and a is not an integer, then

(a) b cannot be an integer. (b) b must be a positive integer.

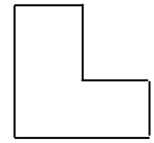
(c) b = 0. (d) b must be a negative integer.

Solution:

Solving for b, we have b(1-a)=1-a. Since a is not an integer,1-a ≠ 0 and can be cancelled. Hence b=1, and is a positive integer. The answer is (b)

Problem 8

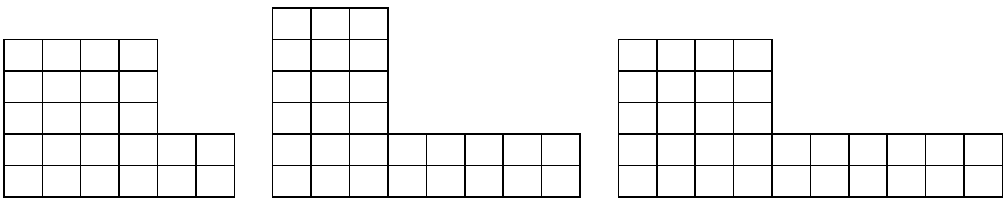
Every two adjacent sides of a hexagon are perpendicular to each other. See diagram on the right for one such example. Five of the sides have lengths 6, 5, 4, 3 and 2, in some order. The area of this hexagon cannot be



(a) 24 (b) 28 (c) 32 (d) all are possible

Solution:

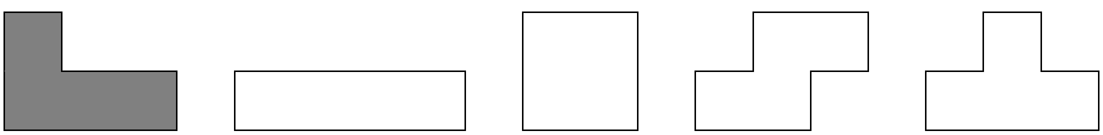
The diagrams below shows that the answer is (d).



Problem 9

There are five Tetris pieces, each consisting of four unit squares joined edge to edge. Lily's favorite is the piece shaped like the letter L (shaded below). She tries to use it and one of the remaining four pieces to form a shape with one line of symmetry. The number of cases for which this is possible is

(a) 1 (b) 2 (c) 3 (d) 4



Solution:

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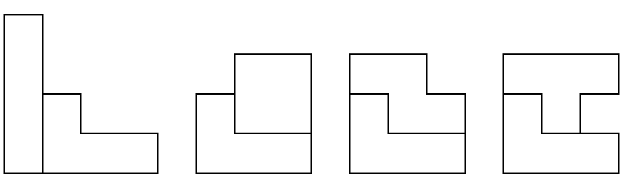
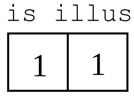


Problem 10

A domino consists of two unit squares joined edge to edge, each with a number on it. The domino numbered 11 is illustrated below.



Fifteen dominoes, numbered 11, 12, 13, 14, 15, 22, 23, 24, 25, 33, 34, 35, 44, 45 and 55, are assembled into the 5 by 6 rectangular grid shown in the diagram below. A domino can be placed vertically or horizontally on the grid. The square 4 in the second row and second column (in boldface) forms a domino with the square numbered

(a) 1 above it (d) 4 below it (c) 1 to the left of it (d) 3 to the right of it

1	1	3	5	2	3
1	**4**	3	1	5	2
2	4	5	5	3	2
3	3	1	1	2	4
2	5	4	5	4	4

Solution:

First, note that the 55 domino must be at the centre while the 22 domino can only be on the right edge. Then the 3 at the top right corner must form a domino with the 2 to the left. This means that the 2 at the bottom left corner must form a domino with the 5 to the right, and the 3 above this 2 must form a domino with the 3 to the right. We can also draw two lines separating a 33 pair and a 23 pair. Now the 3 in the third row forms a domino with the 5 above, arriving at the diagram below on the left. The 5 in the first row cannot form a domino with the 3 to the left, and must therefore form a domino with the 1 below. This forces the 3 on the second row to form a domino with the 4 to the left. The answer is (d). The complete construction is shown in the diagram below on the right.

Answers-Only Problems

Answers with fractions must be written in lowest terms.

Problem 1

Evan had no money. So Ray gave one third of his own money to Evan, Chauncey gave one quarter of his own money to Evan, and Sean gave one fifth of his own money to Evan. Evan received the same amount from each of the three. What fraction of the group's money did Evan have now?

Solution:

Let the money be put into envelopes, with the same amount in each. We may assume that Sven got one envelope from each friend. Then Ray had two envelopes left, Chauncey had three left and Sean four. The total number of envelopes was twelve. Hence Sven had 3/12 = 1/4 of the group's money.

Problem 2

On the left pan of a balance scale are 9 oranges, and on the right pan are 2 apples. The weight of each apple is $1\frac{1}{3}$ times that of an orange. If only whole fruits can be added, what is the smallest number of fruit which must be added to the right pan in order to achieve equilibrium?

Solution:

Since an apple weighs more than an orange, we should add apples as far as we can. Now 4 oranges have the same total weight as 3 apples. To balance 8 of the oranges on the left pan, we need 6 apples. Since we already have 2 on the right pan, we must add 4 apples. Finally, we must add 1 orange to the right pan to balance the ninth one on the left pan. Thus the smallest number of fruit we must add is 5.

Problem 3

Each of the digits from 0 to 9 are placed in one of the squares in a 2X5 table. The digit 0 goes into the square in the first row and the first column. The sum of the two numbers in each column except the first is constant. How many different digits could have gone into the square in the second row and the first column?

	C1	C2	C3	C4	C5
R1	0				
R2					

Solution:

The sum of the nine non-zero digits is 45, 1 more than a multiple of 4. The sum of the eight digits not in the first column is a multiple of 4. Hence the digit which goes below 0 is 1 more than a multiple of 4, and it may be 1, 5 or 9. Hence there are three different digits that could have gone there, and the diagrams below show that all three are possible.

0	2	3	4	5
1	9	8	7	6

0	1	2	3	4
5	9	8	7	6

0	1	2	3	4
9	8	7	6	5

Problem 4

A nine-digit number consists of the nine non-zero digits in some order. The first digit is smaller than the last digit. The sum of the digits 1 and 2 and all the digits between them is 9. The sum of the digits 2 and 3 and all the digits between them is 19. The sum of the digits 3 and 4 and all the digits between them is 45. The sum of the digits 4 and 5 and all the digits between them is 18. What is this number?

Solution:

Since the sum of the nine non-zero digits is 45, the digits 3 and 4 are at either end of the number, and since the first digit is smaller than the last digit, the first digit is 3 and the last digit is 4. Since 19+18<45, 2 comes before 5, and the sum of the digits between them is 8. It may be a single digit 8, or they may be the digits 1 and 7. However, in the latter case, the sum of the digits 1 and 2 and all the digits between them will either be 3 or 10. Hence the digit 8 is between 2 and 5. Now the sum of the digits 4 and 5 and all the digits between them is 18. We must have the digit 9 between them. Also, the sum of the digits 1 and 2 and all the digits between them is 9. We must have the digit 6 between them. This leaves the digit 7 between the digits 3 and 1, so that the number is 371628594.

Problem 5

Consider a fraction between 0 and 1. The sum of the numerator and the denominator of the fraction is 33. How many fractions are there such that the numerator and the denominator have no common divisors greater than 1?

Solution:

There are 16 fractions less than 1 in which the sum of the numerator and the denominator is 33, namely, 1/32, 2/31, 3/30, …, 16/17. In 5 of them, namely, 3/30, 6/27, 9/24, 12/21 and 15/18, the numerator and the denominator have 3 as a common divisor. In the fraction 11/22, the numerator and the denominator have 11 as a common divisor. There are no others since 3 and 11 are the only prime divisors of 33. It follows that the number of fractions we seek is 16-5-1=10.

Problem 6

The sum of seven consecutive positive integers is the cube of an integer. The sum of the middle three of the seven numbers is the square of an integer. What is the smallest possible value of the middle one of the seven numbers?

Solution:

Let the seven numbers be x-3, x-2, x-1, x, x+1, x+2 and x+3. Then 3x is a square and 7x is a cube. Hence x must contain a factor of 3 in order for 3x to be a square. Then x must contain two more factors of 3 in order for 7x to be a cube, and 3x is still a square. Similarly, x must contain two factors of 7 in order for 7x to be a cube, and 3x is still a square. It follows that the smallest possible value of x is $3^3 7^2$=1323.

Problem 7

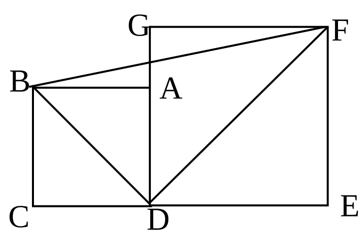
A Pizza Joint cuts circular pizzas through the center into six equal slices. Each slice may be ham and pineapple, or salami and mushroom. How many different pizzas can be made up this way, if two pizzas which become the same after a rotation is counted as one?

Solution:

There is 1 pizza with all ham and pineapple and 1 pizza with all salami and mushroom. There is 1 pizza with five slices of ham and pineapple and one slice of salami and mushroom, and 1 pizza with five slices of salami and mushroom and one slice of ham and pineapple. There are 3 pizzas with four slices of ham and pineapple and two slices of salami and mushroom, because the two slices of salami and mushroom may be adjacent, one slice apart or diametrically opposite. Similarly, there are 3 pizzas with four slices of salami and mushroom and two slices of ham and pineapple. Finally, there are 4 pizzas with three slices of each kind. One of them has the two kinds in alternate slices. A second one has the two kinds each forming a semicircle. The third one has two slices of ham and pineapple followed by two slices of salami and mushroom, one slice of ham and pineapple and one slice of salami and mushroom, in clockwise order. The fourth one has the same combination but in counter-clockwise order. The total number of different pizzas is 14.

Problem 8

ABCD and DEFG are squares such that C, D and E lie on a straight line and A lies on DG. If AB=3 and FG=4, what is the area of triangle BDF?

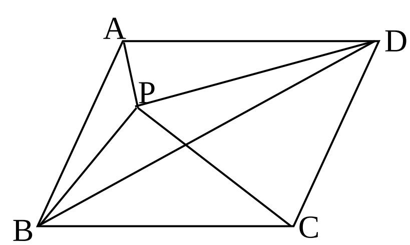


Solution:

We have BD=$3\sqrt{2}$ and DF=$4\sqrt{2}$. Moreover, BD and DF are perpendicular to each other. Hence the area of triangle BDF is $\frac{1}{2}(3\sqrt{2})(4\sqrt{2}) = 12.$

Problem 9

ABCD is a parallelogram and P is a point inside triangle BAD. If the area of triangle PAB is 2 and the area of triangle PCB is 5, what is the area of triangle PBD?

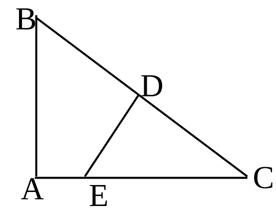


Solution:

Triangles PCB and PAD have bases equal to that of ABCD, and their combined height is equal to that of ABCD. Hence their combined area is half that of ABCD. The combined area of triangles PAD, PAB and PBD is equal to that of BAD, which is in turn equal to half the area of ABCD. It follows that the area of PBD is the difference between the areas of PCB and PAB, which is 5-3=2.

Problem 10

Triangle ABC is cut out of a piece of paper, where AB=24, AC=32 and $\angle$CAB=90°. The paper triangle is folded so that B and C coincide. What is the length of the crease?



Solution:

By Pythagoras' Theorem, BC=$\sqrt{24^2+32^2}=40$. Let D be the midpoint of BC. Then DC=20. Let E be the point on AC such that DE is perpendicular to BC. Then triangles CDE and CAB are similar. Hence $\frac{DE}{DC}=\frac{AB}{AC}$ so that DE=15.