Print ID # __________ /90

School Name ____________________ Student Name

(Print First, Last)

2009 Edmonton Junior High Math Contest - Solution

Part I: Multiple Choice (PRINT neatly, use CAPITAL letters, 4 points each)

1. B	6. C	11. A
2. B	7. B	12. D
3. D	8. A	13. C
4. D	9. A	14. B
5. C	10. B	15. C

Part II: Short Answers (PRINT small but legible, 6 points each)

16. **13, 95 or 669** 17. **15** 18. **90** 19. **3** 20. **27**

Part I: ________ x 4 + _________ x 2 = ________ (be sure blanks ≤ 3)		MARKER ONLY
Correct Blank		
Part II: ______ x 6 = ________		
Correct		
Total = ________ (enter total score on top)		

Instruction:

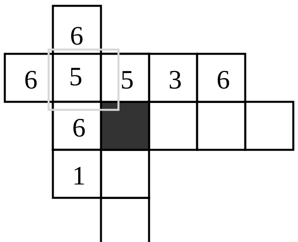
1. Calculator, grid paper and scrap paper are permitted. You may write on the booklet.
2. Programmable calculator and Cell phones are not allowed to be brought into class.
3. Don't write your answers too LARGE to avoid others seeing your answers. COVER your answers at all time.
4. All fractions must be proper and reduced to lowest terms.
5. Each correct answer is worth 4 points for multiple choices and 6 points for short answers.
6. Each incorrect answer is worth 0 point.
7. Each unanswered question in Part I is worth 2 points up to a maximum of 6 points.
8. Unanswered questions in Part II is worth 0 point.
9. You have 90 minutes of writing time.
10. When done, carefully REMOVE and HAND IN only page 1.

1. Each of the eight squares in the figure at the right must contain one digit. The sum of the eight digits is 38. Reading from top to bottom, the four digits in the vertical column form a number that is a power with 3 as its base. Reading from left to right, the five digits in the horizontal row form a number that is a power with 4 as its base.

What digit will be in the square of the shaded box?

- a. 6
- * b. 5
- c. 3
- d. 1

Solution:



Powers of 3: 3, 9, 27, 81, 243, 729, 2187, **6561**, 19683, …

Powers of 4: 4, 16, 64, 256, 1024, 4096, 16 384, **65 5<u>3</u>6**, 262 144, …

2. How many whole numbers from 10 to 60, inclusive, are divisible by both its units digit as well as its tens digit?

(E.g., Although 39 is divisible by 3, it is not divisible by 9. Therefore, 39 is not one of the numbers in the solution.)

- a. ten numbers
- *b. eleven numbers
- c. seventeen numbers
- d. eighteen numbers

Solution:

Solve by testing. The eleven numbers are circled below:

10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31
32	33	34	35	36	37	38	39	40	41	42
43	44	45	46	47	48	49	50	51	52	53
54	55	56	57	58	59	60				

3. Twenty people at Snow Lodge came either to ski or to snowboard. (*Nobody does both activities*.) The ratio of skiers to snowboarders is 3 to 2. The ratio of male skiers to female skiers is 5 to 1. The ratio of children snowboarders to adult snowboarders is 1 to 3. What is the sum of the number of male skiers and child snowboarders at Snow Lodge?

a. 6
b. 8
c. 10
* d. 12

Solution: 10 male skiers + 2 child snowboarders = 12 people

Test possibilities where the ratio of skiers to snowboarders = 3 to 2, until the number of people is 20.

Skiers	3	6	9	12
Snowboarders	2	4	6	8
Total people	5	10	15	20

There must be 12 skiers and 8 snowboarders.

Test possibilities where the ratio of male skiers to female skiers = 5 to 1, until the number of skiers is 12.

Male skiers	5	10
Female skiers	1	2
Total skiers	6	12

There must be 10 male skiers and 2 female skiers.

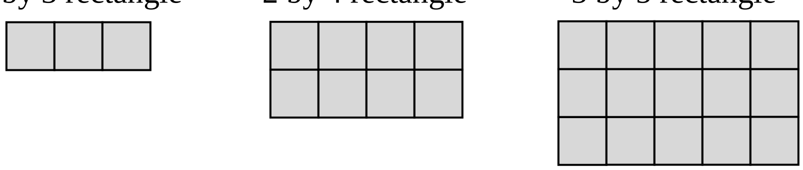
Test possibilities where the ratio of children to adult snowboarders is 1 to 3, until the number of snowboarders is 8.

child snowboarders	1	2
adult snowboarders	3	6
Total snowboarders	4	8

There must be 2 child snowboarders and 6 adult snowboarders.

4. Rectangles can be formed by joining unit squares. Some of the line segments are part of their perimeters and some are not. For the three examples shown below, their perimeters are, respectively, 8 units, 12 units, and 16 units. The number of line segments that are not part of their perimeters are, respectively, 2, 10, and 22.

1 by 3 rectangle 2 by 4 rectangle 3 by 5 rectangle



If a 50 by 100 rectangle is formed with unit squares, how many line segments would **not** be part of the perimeter?

a. 4900
b. 4950

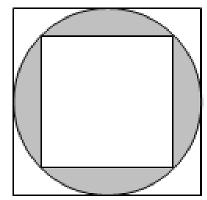
c. 9050
*d. 9850

Solution: Number of vertical segments = width × (length – 1)
Number of horizontal segments = (width – 1) × length

For a 50 by 100 rectangle:

- the number of vertical segments = 50 × 99 = 4950, and
- the number of horizontal segments = 49 × 100 = 4900, so
- the total number of unit segments = 4950 + 4900 = 9850.

5. In the figure at the right, the smaller square is inscribed in a circle, which is inscribed in a larger square with a side of 10 cm.
Approximately what area of the entire figure is **<u>not</u>** shaded?



a. 21.5 cm^2
b. 50 cm^2
*c. 71.5 cm^2
d. 78.5 cm^2

Solution: Unshaded area = Area of small square + (Area of large square – Area of circle)

Diameter of circle = side of large square = diagonal of small square = 10 cm

If side of small square = s, then $2s^2$ = 10 cm × 10 cm = 100 cm^2

Area of small square = s^2 = 100 cm^2 ÷ 2 = 50 cm^2

Area of large square = 10 cm × 10 cm = 100 cm^2

If diameter of circle = 10 cm, then its radius = 5 cm

Area of circle = 25 $\pi \approx 25 \times 3.14 \approx 78.5$ cm^2

Area of large square – Area of circle ≈ 100 cm^2 – 25 π
≈ 100 cm^2 – 78.5 cm^2
≈ 21.5 cm^2

Unshaded area ≈ 50 cm^2 + 21. 5 cm^2 ≈ 71.5 cm^2

6. The front wheel of a tricycle has a diameter of 25 cm; the rear wheels have a diameter of 15 cm. If the tricycle travels 100 m, about how many more rotations will the rear wheels make than the front wheel?

a. 212
b. 127
* c. 85
d. 31

Solution: Diameter of front wheel = 25 cm × π ≈ 78.5 cm
Diameter of rear wheel = 15 cm × π ≈ 47.1 cm
100 m = 10 000 cm

10 000 ÷ 78.5 ≈ 127 and 10 000 ÷ 47.1 ≈ 212

212 rotations – 127 rotations = 85 rotations

7. The year 1961 has 180˚ rotational symmetry because it reads the same when it is turned upside down. How many years since 1000 have 180˚ rotational symmetry?

a. 3
*b. 5
c. 8
d. 10

Solution: Numbers that have 180˚ rotational symmetry are 0, 1 and 8; also 6 becomes 9 and 9 becomes 6.

The first and last digits must be 1:
1001, 1111, 1691, 1881, 1961

8. Which of the numbers in the set { -50 , -1.5 , -1 , -0.2 , 0 , $\sqrt{2}$, $\frac{4}{5}$, π , 40 } are greater than their reciprocals?

* a. -0.2 , $\sqrt{2}$, π , 40
b. -50 , -1.5 , -1 , 0 , $\frac{4}{5}$
c. -0.2 , 0 , $\sqrt{2}$, π , 40
d. $\sqrt{2}$, $\frac{4}{5}$, π , 40

Solution:

Number (x)	-50	-1.5	-1	-0.2	0	$\sqrt{2}$	$\frac{4}{5}$	π	40
Reciprocal ($\frac{1}{x}$)	$-\frac{1}{50}$	$-\frac{2}{3}$	-1	-5	Undefined	≈ 0.7	$\frac{5}{4}$	≈ 0.3	$\frac{1}{40}$
Is $x > (\frac{1}{x})$?	no	no	no	yes	no	yes	no	yes	yes

9. On her last test, Crystal scored 92%, and in doing so, raised her average by 2%, to 82%. What percent must she get on her next test to raise her average one more percent, to 83%?

* a. 89%
b. 86%
c. 83%
d. 82%

Solution: If Josh raised his average by 2 points to 82%, then his previous average was 80%. If he previously wrote n tests, then the sum of his scores = 80n + 92.

$$\frac{(80n + 92)}{n + 1} = 82$$

$80n + 92 = 82 \times (n + 1)$
$80n + 92 = 82n + 82$
$80n + 10 = 82n$
$10 = 2n$
$5 = n$
His total scores on the 5 tests = $5 \times 80 = 400$

To raise his average to 83%, $(400 + 92 + m) \div 7 = 83$

$$\frac{492 + m}{7} = 83$$
$$492 + m = 7 \times 83$$
$$492 + m = 581$$
$$m = 89$$

10. Erhan has a pair of special dice. The six faces of each die are labelled 1, 4, 5, 7, 11, and 14. He rolls the pair of dice and finds the sum of the two numbers. What is the probability that the sum will be both even and less than 20?

a. $\frac{5}{12}$

* b. $\frac{1}{2}$

c. $\frac{5}{9}$

d. $\frac{5}{6}$

Solution: There are 36 combinations (possible outcomes). Of those sums, 18 are both even and less than 20.

+	1	4	5	7	11	14
1	E & <20		E & <20	E & <20	E & <20	
4		E & <20				E <20
5	E & <20		E & <20	E & <20	E & <20	
7	E & <20		E & <20	E & <20	E & <20	
11	E & <20		E & <20	E & <20		
14		E & <20				

Therefore, the probability = $\frac{18}{36} = \frac{1}{2}$

11. The tens digit of the square of an integer is 5. The ones digit

*a. must be 6

b. must be 4

c. can be 4 or 6

d. none of the above

Solution: The square of an integer can only end in 0, 1, 4, 5, 6 or 9. If the tens digit is 5 and the units digit is 1, 5 or 9, this would leave a remainder of 3 when divided by 4. If the tens digit is 5 and the units digit is 0 or 4, this would leave a remainder of 2 when divided by 4. This leaves the unit digit to be a 6, the square of 34 is one such number, 1156.

12. In the quadrilateral ABCD, $\angle ABC = \angle BCD = 90°$, $\angle CAB = 45°$ and $\angle CBD = 60°$. The diagonals AC and BD intersect at point E. The ratio of the areas of the triangles CDE and ABE is

a. $\sqrt{2}:1$

b. $\sqrt{3}:1$

c. 2:1

* d. 3:1

Solution:
Let AB = 1. Then BC = 1 and CD = $\sqrt{3}$. Since triangles CDE and ABE are similar, the desired ratio is $(\sqrt{3})^2 \; to \; (1)^2$ or 3:1.

13. A small box of chocolate costs $21 while a large box costs $41. How many different combinations of small and large boxes cost exactly $2009?

a. 1

b. 2

* c. 3

d. 4

Solution:

Let n be the number of small boxes and m be the number of large boxes. We know that m, n are non negative integers and

$$21n + 41m = 2009$$

Since 7 divides both 21 and 2009, 7 must divide 41m. Since 41 is prime, we get that 7 divides m. Thus m = 7a for some nonnegative integer a.

Since 41 divides both 41 and 2009, 41 must divide 21n. Since 41 is prime, we get that 41 divides n. Thus n = 41b for some nonnegative integer b. Replacing in the equation we get:

$$21(41b) + 41(7a) = 2009$$

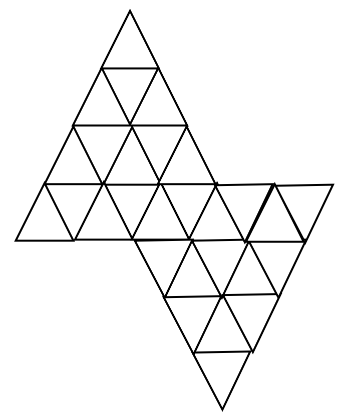
Dividing by 287 = 41(7) we get:

$$3b + a = 7$$

Since a, b ≥ 0 and are integers, we get three solutions

a = 7, b = 0 implies n = 0 and m = 49
a = 4, b = 1 implies n = 41 and m = 28
a = 1, b = 2 implies n = 82 and m = 7.

14. What is the total number of downward-pointing triangles in the diagram below? (Consider all sizes of triangles.)



a. 14

* b. 25

c. 28

d. 32

Solution:

Triangles with 1-unit sides: 14
Triangles with 2-unit sides: 7
Triangles with 3-unit sides: 3
Triangles with 4-unit sides: 1
Total = 14 + 7 + 3 + 1 = 25

15. A square has its vertices at (1, 1), (1, 3), (3, 1) and (3, 3). It is reflected by a mirror line that runs through (3, 1) and (3, 3). Then, anchored at vertex (3, 3), the first image is magnified to form a second image whose area is quadruple that of the original. The second image is then rotated 90˚ clockwise using (1, 3) as the turn centre. What are the coordinates of the final image?

a. (1, 1), (-1, 3), (1, 7) and (-3, -1)

b. (1, 5), (5, 5), (1,9) and (5,9)

* c. (1, 1), (-3, -3), (1, -3), and (-3,1)

d. (1,1), (1, -7), (-7,1) and (-7, -7)

Solution:

When the square is first reflected, its new coordinates are (3, 1), (3, 3), (5, 1) and (5, 3).

When the new square is quadrupled, its coordinates are (3, 3), (3, -1), (7, 3) and (7, -1).

When the enlarged square is rotated, its coordinates are (1, 1), (-3,-3), (1, -3) and (-3,-1).

Short Answers:

16. Find a positive integer n so that $\frac{6n+2013}{3n+2}$ is an integer.

Answers: 13, 95 or 669

Solution:

$$\frac{6n+2013}{3n+2} = \frac{6n+4+2009}{3n+2} = 2 + \frac{2009}{3n+2}$$

$\frac{6n+2013}{3n+2}$ is an integer if and only if $\frac{2009}{3n+2}$ is an integer. Thus 3n+2 must divide 2009. The divisors of 2009 are 1, 7, 41, 49, 287 and 2009. Only 41, 287 and 2009 are of type 3n+2. Thus the values of n are 13, 95 and 669.

17. Emily earned some spending money by running a lemonade stand. She paid her mother 5% of what she earned for the supplies that she used. She spent $\frac{3}{5}$ of what was left on entertainment and then saved the remaining \$114. How much money did Emily pay her mom for supplies?

Answer: \$15.00

Solution:

100% – 5% = 95%
$\frac{3}{5}$ of 95% = 95% × $\frac{3}{5}$ = 57%
95% – 57% = 38%

c = cost of supplies (paid to her mom)

$$\frac{\$114}{38\%} = \frac{\$c}{5\%}$$

$c = (114)(5) \div 38$
$c = \$15$

Alternate Solution:

e = amount she earned
Cost of Supplies = 0.05e
What was left = e – 0.05e = 0.95e
Spent on entertainment = (0.6)(0.95)e = 0.57e
Remaining profits (amount saved) = \$114

$e = 0.05e + 0.57e + 114$
$0.38e = 114$
$e = 114 \div 0.38 = \$300$
Cost of Supplies = 0.05e = (0.05)(300) = \$15

18. Laura, Svitlana and Robert deliver flyers. In one week, they delivered a total of 270 flyers. If Laura delivered 50% less flyers than Svitlana, and if Robert delivered 50% more flyers than Laura, how many flyers did Robert deliver?

Answer: Robert delivered 90 flyers.

Solution: Let number of flyers delivered by Laura = n
Then Robert delivered $1.5n$ flyers, and Svitlana delivered $2n$ flyers

$n + 1.5n + 2n = 270$
$4.5n = 270$
$n = 270 \div 4.5 = 60$
Therefore, 1.5n = (1.5)(60) = 90
Robert delivered 90 flyers.

19. The table at the right shows input and output values for a function machine. Find the missing output value.

Input (x)	Output (y)
-1	0
0	1.5
1	?
5	9
10	16.5
15	24

Answer: 3

Solution:

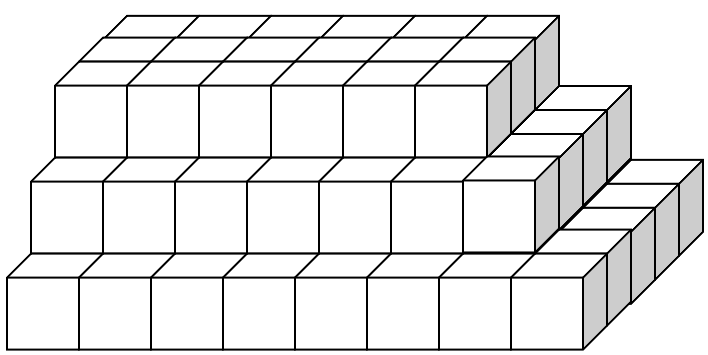
Input (x)	Output (y – 1.5)
-1	-1.5
0	0
1	?
5	7.5
10	15
15	22.5

Subtract 1.5 from each output value, and by inspection see that each new output value is 1.5 times the input value.

The equation is $y = 1.5\,x + 1.5$, or $y = (1.5)\,(x + 1)$

- So, the missing value (y) = $1.5 \times 1 + 1.5 = 1.5 + 1.5 = 3$
  (or $y = (1.5)\,(x + 1) = (1.5)\,(1 + 1) = (1.5)\,(2) = 3$

20. The solid 3-D object shown below is composed of layers of unit cubes. The top layer has 18 cubes, the middle layer has 28 cubes, and the bottom layer has 40 cubes. If the object is completely dipped in paint, how many unit cubes will have exactly two faces painted?



Answer: 27

Solution:

Keeping in mind that the entire base of the 3-D object is painted:

- Cubes on bottom level with 2 painted faces: 9
- Cubes on middle level with 2 painted faces: 8
- Cubes on top level with 2 painted faces: 10
- Total = 9 + 8 + 10 = 27