Edmonton Junior High Mathematics Contest 2007

Multiple-Choice Problems

Problem 1

A sequence is simply a list of numbers in order. The sequence of odd integers is $1,3,5,7,9,\ldots$. If we add any number of consecutive odd numbers, always starting at 1, then the result will always be

A. an even number B. an odd number
C. a perfect square D. a perfect cube

Solution C

Solution
The n[th] odd number is given by $t_n = 2n-1, n \in N$.

Using $S_n = \frac{n}{2}(a+t_n)$, we have

$$S_n = \frac{n}{2}(1+2n-1) = n^2$$

Problem 2

You have an unlimited number of nickels (5 cents) and dimes (10 cents) from which you can use only nickles, only dimes or a combination of both to make a sum of 55 cents. The number of different combinations of coins that can be used is

A. 11 B. 6
C. 5 D. 4

Solution B

n	*d*
11	0
9	1
7	2
5	3
3	4
1	5

Problem 3

In the real number system, if the sum of $a + 3.464\ 466\ 444\ 666...$ is to be an integer, then a could have a value of

A. 0.535 533 555 333...

B. 1.646 644 666 444...

C. 2.202 200 222 000...

D. 3.313 311 333 111...

Solution A

Solution:

$$\begin{array}{r} 3.464466444666... \\ 0.535533555333.. \\ \hline 3.999999999999... \end{array} = 3.\bar{9} = 4$$

Problem 4

Consider any three consecutive integers *a*, *b* and *c*, where $a < b < c$ and $a > 1$. The expression that gives a correct relationship among *a*, *b* and *c* is

A. $ac = b^2 - 1$

B. $ac = b^2$

C. $2b = ac$

D. $c = a + b$

Solution A

Solution

If a, b and c are consecutive, then they can be expressed (respectively) as $n-1, n$ and $n+1$.

Since $(n-1)(n+1) = n^2 - 1$, we have

$ac = b^2 - 1$

Problem 5

If $2x^2 + x + 3 + 2y = -4x^2 + 4x - 6$, then $y =$

A. $-12x^2 + 6x - 18$ (multiplication by 2)

B. $-4x^2 + x - 6$ (divides incorrectly by 2)

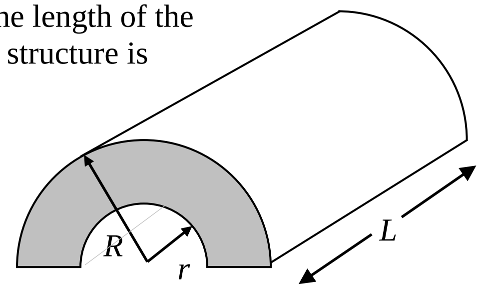
C. $-x^2 + 2.5x - 1.5$ (errors in transposition)

D. $-3x^2 + 1.5x - 4.5$

Solution D

Problem 6

An engineer designs a hollow reinforced concrete support structure in the shape of a semi-cylinder. If the inner radius is r, the outer radius is R and the length of the structure is L, then an expression for the volume of concrete in the structure is



A. $V = \frac{\pi}{2}\left(\frac{R}{r} - L\right)^2$

B. $V = \frac{\pi L}{2}(R - r)^2$

C. $V = \frac{\pi}{2}(LR - r)^2$

D. $V = \frac{\pi L}{2}(R - r)(R + r)$

Solution D

Problem 7

A rectangle can be made longer and narrower without changing its area. For example, if the lengths of one pair of its sides are increased by 60%, then the lengths of its other pair of sides must be decreased by

A. 62.5%

B. 60.0%

C. 40.0%

D. 37.5%

Solution D

Solution:

$$A = lw$$

$$A = 1.60l\left(\frac{w}{1.60}\right)$$

$$A = 1.60l(0.625w)$$

Since w must become $0.625w$, we have a 37.5% decrease on w.

Problem 8

The value of $\sqrt{3+\sqrt{8}}+\sqrt{3-\sqrt{8}}$ is equal to

A. 3

B. $2\sqrt{2}$

C. $2\sqrt{3}$

D. $\sqrt{6}$

Solution B

Problem 9

Each student in a class of 25 students wrote 2 different tests. It is known that

- 18 students passed the first test.
- 22 students passed the second test.
- No students failed both tests.

The number of students who passed both tests is

A. 15 students

B. 10 students

C. 20 students

D. 40 students

Solution A

Solution: if n is the number who passed both tests, then 18-n passed test 1 only, and 22 - n passed test 2 only, $18 - n + n + 22 - n = 25$ gives $n = 15$

Problem 10

If n is a whole number then the number of different values of n where $7n + 1$ is a multiple of $3n + 5$ is

A. 0

B. 1

C. 2

D. infinite

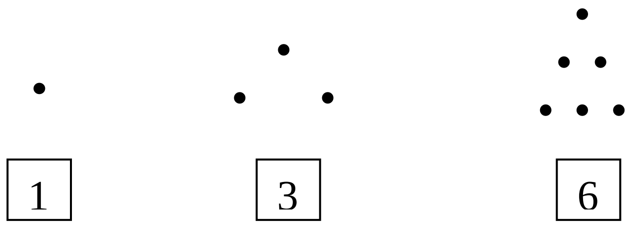
Solution C

If $7n + 1 = 3n + 5$, we have $n = 1$. If $7n + 1 = 2(3n + 5)$, we have $n = 9$. We cannot have $7n + 1 = k(3n + 5)$ for any $k \geq 3$, as the right side is clearly larger than the left side.

Answers-Only Problems

Problem 1

Numbers such as 1, 3 and 6 are sometimes referred to as triangular numbers, because the value of the number can be represented by a triangular shape as shown below.



The sum of the first 10 triangular numbers is **220** .

(Solution: 1 + 3 + 6 + (6 + 4) + (10 + 5) + (15+6) + (21+7) + (28+8) + (36+9) + (45+10) = 220

Problem 2

John and Sam both leave point A at the same time, heading in exactly opposite directions. If John walks at 4 km/h and Sam walks at 3.5 km/h, then the number of minutes it takes for them to be 2.5 km apart is **20**.

Solution: 4.0t + 3.5t = 2.5 gives t = 2.5/7.5 = 1/3 hours or 20

Problem 3

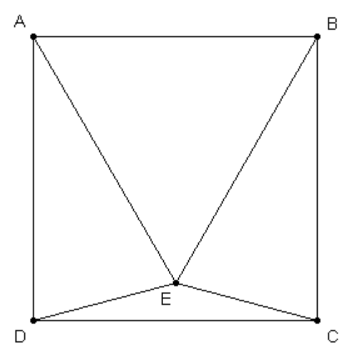
A set of 5 numbers has an average of 13. If a 6th number is included, then the average is 23. The value of the 6th number is **73** .

Solution:

$$\frac{65 + x_6}{6} = 23 \rightarrow x_6 = 73$$

Problem 4

In the diagram to the right, quadrilateral ABCD is a square, and ΔABE is an equilateral triangle. The measure in degrees of $\angle$ECD is **15°** .



$AB = BC = AD$ *and* $AB = AE = EB$

$\angle BAE$ *and* $\angle ABE = 60°$

ΔEBC is isosceles $\therefore BE = BC$

$< EBC = 30°$

$< BCE = 75°$

$< ECD = 90° - 75°$

$< ECD = 15°$

Problem 5

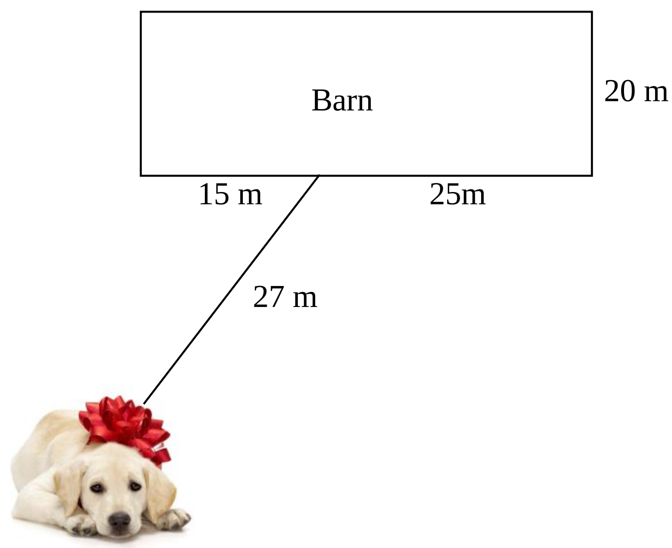
Each integer from 1 to 100 inclusive is written on an identical card, one number per card. The cards are placed into a box and mixed thoroughly. If a single card is drawn at random, then the probability that the number on the card is divisible by either 3 or 5, expressed as a decimal to the nearest hundredth, is **0.47** .

Solution: The sequence 3, 6, 9, …, 99 contains 33 terms each of which is divisible by 3
The sequence 5, 10, 15, …,100 contains 20 terms, each of which is divisible by 5
Note that the numbers 15, 30, 45, 60, 75 and 90 are contained in both sequence.

$$\therefore P(\text{divisible by 3 or 5}) = \frac{33+20-6}{100} = \frac{47}{100} = 0.47$$

Problem 6

A farmer fastens the end of his dog leash to the edge of his barn at a point that is 15 m from one corner and 25 m from another corner of the barn, as llustrated in the diagram below. The barn is 20 m wide and the leash is 27 m long. The area of the region where the dog is able to reach while leashed to the wall, to the nearest whole square metre, is **1261** .



Solution:

$$A = \frac{\pi}{2}27^2 + \frac{\pi}{4}(2)^2 + \frac{\pi}{4}(12)^2$$

$$= \frac{\pi}{4}\left[2(27)^2 + (2)^2 + (12)^2\right]$$

$$= \frac{\pi}{4}(1606)$$

$$= 1261.34\,\text{m}^2$$

Problem 7

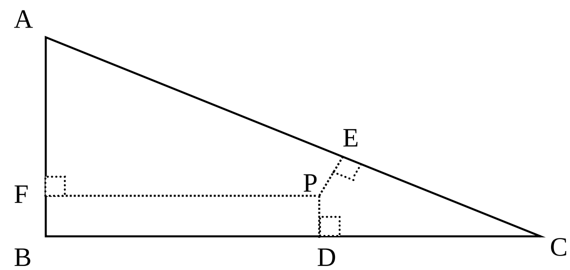
Consider the irrational number 0.454 554 555 455 554.... The total number of 5s that occur in the number before the digit 4 appears for the 100th time is **4950**.

Between 1st and 2nd 4 – 1 occurrence
Between 2nd and 3rd – 2 occurrences
3rd and 4th – 3 occurrences
nth and n+1th – n occurrences
Therefore there are 99 5s between the 99th and the 100th 4

$$S_n = \frac{99(1+99)}{2}$$
$$= 4950$$

Problem 8

A point *P*, is inside a triangle *ABC*, where *AB* = 7, *BC* = 24 and *CA* = *25*.
If *PD* = 2, and *PE* = 2, then the length of *PF* to the nearest whole number is **10** .



Solution: Since $AB^2 + BC^2 = 49 + 576 = 625 = CA^2$, *AB* is perpendicular to *BC*. Hence the area of triangle *ABC* is equal to ½ (7)(24) = 84. The respective areas of triangles *PCB* and *PAC* are ½ (2)(24) = 24 and ½ (2)(25) = 25. Hence the area of triangle *PBA* is equal to 84 − 24 − 25 = 35 = ½ (7)*PF*. Hence *PF* = 10.

Tan angle C = 7/24, which gives angle C = 16.26 degrees.
Tringles PCD and PCE are congruent. Therefore, angle PCD is 8.13 degrees.
Then using tangent of angle PCD, side DC can be found to be 14.0007. Thus FP = 24 – 14 = 10

Problem 9

When 80, 97 and 158 are divided by a certain even positive integer, the sum of the three remainders is 39. This even positive integer is **74** .

The sum of the given numbers is 335. When 39 is subtracted from this sum, the difference 296 must be a multiple of the unknown even number. Now the prime factorization of 296 is $2^3 \times 37$. Hence the even number is one of 74, 148 or 296, but the last two are too large. Thus the answer is **74**.

Problem 10

Let *a*, *b*, and *c* be non-zero numbers such that $a+b+c=0$. The value of $a\left(\frac{1}{b}+\frac{1}{c}\right)+b\left(\frac{1}{c}+\frac{1}{a}\right)+c\left(\frac{1}{a}+\frac{1}{b}\right)$ is $-\mathbf{3}$.

The given expression is equal to $\frac{b+c}{a}+\frac{c+a}{b}+\frac{a+b}{c}$. Now $b+c=-a$, $c+a=-b$ and $a+b=-c$. Hence the answer is $-\mathbf{3}$.