Edmonton Junior High Mathematics Contest 2005

Solutions to Questions with Multiple Choices

1. If there are 5 female employees among 8, then they account for 62.5% of the number of employees overall. Since $\frac{1}{3}<\frac{1}{2}=\frac{2}{4}=\frac{3}{6}<\frac{4}{7}<\frac{3}{5}=60\%<65\%<\frac{2}{3}=\frac{4}{6}<\frac{5}{7}<\frac{3}{4}<\frac{4}{5}<1$, the number of employees overall cannot be less than 8. **Answer: (a)**

2. The number of students who study French is 14, and the number of students who study German but not French is 8-3=5 . Since every student studies either French or German, the total number of students is 14+5=19. **Answer: (b)**

3. There are 6 x 8=48 patrol spots. The students who are on duty only once fill up 3 x 6=18 spots. Of the remaining 48-18=30 spots, each remaining student who is on duty takes up at least 2, so that there are at most 30/ 2=15 other students on duty. Thus the largest possible number of students on duty is 18+15=33. **Answers: (c)**

4. If a, d and f are positive while b, c and e are negative, then only -ac is positive. Hence the number of positive number can be as low as 1. If none of them is positive, then a and c have the same sign, as do b and e , and likewise c and f . Now two of these three pairs must have the same sign. So one of ab, cd and ef is positive, and we have a contradiction. Hence at least one of the numbers is positive. **Answer: (b)**

5. When two pebbles go their separate ways, they say goodbye to each other. Thus the numbers written down are the numbers of goodbye exchanges said after each move. Since there are 15 pebbles in all, each says goodbye 14 times. Hence the total number of goodbye exchanges is $\frac{15x14}{2}$=105. **Answer: (a)**

6. The minimum number of pieces is 4. We may have a piece 9 centimeters long which can form the top and the bottom of the cube linked by a vertical edge. Then we need 3 more pieces each 1 centimeter long to form the other three vertical edges. Note that 3 edges come together at each of the 8 vertices of the cube. Thus each vertex must either be the starting point or the end point of one of the pieces. Since each piece has 1 starting point and 1 end point, we need 4 pieces to cover the 8 vertices. **Answer: (d)**

7. Draw a line FC parallel to BA . Then ∠ BCF=∠ ABC=50° and ∠DCF=∠ CDE=50° . Hence ∠BCD=∠BCF+∠DCF =100° . **Answer: (c)**

8. Each hexagon has area 54/3=18 square centimeters. Since it consists of six equilateral triangles, each triangle has area 18/ 6=3 square centimeters. Hence the area of the irregular hexagon is 54 + 4x3=66 square centimeters. **Answer: (b)**

9. Since the thickness of each CD set is a multiple, at most 99 centimeters of each rack may be used, yielding a total display space of 594 centimeters. If all 450 CD sets are singles, we would require 450 centimeters of display space. Thus we have a spare of 144 centimeters. The conversion of a single into an album uses up an additional 3 centimeters. Hence the maximum number of albums is 48. **Answer: (a)**

10. The total of the four remaining numbers is 9+10+12+13=44. Initially, all four are 0s. Each move increases exactly two of them, each by 1. Hence the total number of moves made is 44/2=22. Now the number in the central square is increased by 1 after each move. Since it starts as 0, it ends as 22. **Answer: (a)**

Solutions to Problems requiring Answers Only

1. Since Ace plays at least once every two games and he has played exactly 10 games, the total number of games is at most 21. Since Bea has played 21 games, she plays every game. It follows that Cec has played exactly 21-10=11 games. **Answer: 11**

2. Let the lower and upper pieces be of lengths y and z meters respectively. Then z+y=27. By Pythagoras' Theorem, $z^2-y^2=9^2$ or (z+y)(z-y)=81 . Division yields z-y=3. Hence y=(27-3)/ 2=12 so that the lower piece has length 12 meters. **Answer: 12**

3. Triangles CEN, CNW, CWS and CSF are all similar. Since $\frac{CE}{CN}=\frac{192}{256}=\frac{3}{4}$, we have $CW=\frac{3}{4}CN=144, CS=\frac{3}{4}CW=108, CF=\frac{3}{4}CS=81$. Hence the distance between E and F is 256-81=175 meters. **Answer: 175**

4. The total number of participants must be odd in order for Sven to place exactly in the middle. Since Sean places 16th, the number of participants is at least 17. Since Ray places 10th and lower than Sven, Sven places no lower than 9th. Hence the number of participants is at most 17. It is therefore exactly 17. **Answer: 17**

5. Since 17 and 15 are relatively prime, the numbers of male and female voters are 17k and 15k respectively for some positive integer k. Hence $\frac{17k-90}{15k-80}=\frac{8}{7}$. Cross multiplication yields 119k-630=120k-640. Hence k=10 and the total number of voters is 17k+15k=320. **Answer: 320**

6. Let m be the larger number, n the smaller and d their difference. In order for d<10000, the first digit of m must exceed the first digit of n by 1. In order for d<1000, the second digit of m must be 0 and the second digit of n must be 9. We cannot make d<100, so that d will be a three-digit number. To minimize its first digit, the third digit of m should be 1 and the third digit of n should be 8. The same reasoning shows that the last two digits of m should be 2 and 3, and the last two digits of n should

be 7 and 6, respectively. This leaves 5 as the first digit of m and 4 as the first digit of n, and we have d=m-n=50123-49876=247. **Answer: 247**

7. In any multiple of 9, the sum of the digits is also a multiple of 9. In order for our multiple to be as small as possible, the digits, starting from the second, should be 0, 1, 2 and so on for as long as we can keep the digit-sum a multiple of 9. Hence the smallest multiple is 8012349. **Answer: 8012349**

8. Let the pair of two digit numbers be n=10a + b and m = 10b + a. The sum is 11(a + b). In order for this to be a square, we must have a+b=11. Possible choices are (2,9), (3,8) (4,7)(5,6). since n>m we have 92, 83, 74 and 65. **Answers: 92, 83, 74, and 65**

9. The prime factorization of 3193 is 31 x 103. Since 31 < 50< 103, the bookshop had 103 copies of the book and sold each for $31. Hence the reduction was $(50-31). **Answer: 19**

10. Let n be the initial number. We have $\frac{n}{7}+72=7n-72.$ Hence 144 = $\left(7-\frac{1}{7}\right)n$ so that $n=144x\frac{7}{48}=21.$ the correct answer is 7n-72=75.
Answer:75