Factoring Trinomials: A Student's Perspective

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It is when we least expect it that a student will make an observation about a conventional method that we virtually take for granted. In this case, it was factoring the trinomial form $ax^2 + bx + c$, where the factors of 'ac' add up to 'b' by decomposition. In particular, I presented $3x^2 - 2x - 8$ for consideration. The question posed was, "How can we factor this if the 3 is not a perfect square?" Pega Alerasool, an industrious student with a different view of this procedure, did not understand the conventional algorithm as it had been presented to her. It was clear that my perception of this method and hers were very different. So after explaining that the 3 or any other coefficient of the x^2 term didn't have to be a perfect square, she asked, "Why can't it be a perfect square?" That is, why couldn't the coefficient of x^2 always be a perfect square? Now I was curious and asked her to explain her approach. In essence, let $A = 3x^2 - 2x - 8$ and then multiply both sides by 3 in order to make the coefficient of x^2 a perfect square. The advantage here is that when we apply decomposition, we do not have to worry about the positioning of the constant terms. The illustrated format looks like this:

Factor	$3x^2 - 2x - 8$	Given
Let	$A = 3x^2 - 2x - 8$	Substitution
then	$3A = 9x^2 - 2(3)x - 24$	Multiplication
	$3A = (3x)^2 - 2(3x) - 24$	
	$3A = (3x - 6)(3x + 4)$	$9x^2 = 3x \bullet 3x$ and the factors of -24 whose sum is −2 are - 6 and 4

As with previous methods of factoring, the position of the variable and constant terms is crucial. Here, however, we no longer have this problem. Since the numerical coefficient of the x term is the same in both sets of parentheses, it does not matter where we place the constant terms, factors, in this case −6 and 4. Continuing this process we have:

$3A = (3x - 6)(3x + 4)$	from above
$3A = 3(x - 2)(3x + 4)$	factor out the GCF
$A = (x - 2)(3x + 4)$	division

In general, the sequence looks like this:

Factor	$ax^2 + bx + c$	where the factors of ac add up to be factorable
Let	$A = ax^2 + bx + c$	by substitution
Then	$Aa = a^2x^2 + abx + ac$	by multiplication to make the coefficient of x^2 a perfect square
Now	$Aa = (ax + a)(ax + c)$	factors of ac are a and c regardless of order
	$Aa = a(x + 1)(ax + c)$	factor out the GCF
	$A = (x + 1)(ax + c)$	division by a

The only real question remaining is what to do when the coefficient of x^2 in a given question is already a perfect square. As tempting as it is to factor "as is," the procedure doesn't work as shown below (Example 1). We must still multiply the coefficient of x^2 by itself so that we can get the constant factors to work properly, as in (Example 2).

Example 1

$4x^2 - 5x + 1$

does not work as there are no values a and b for which the form $(2x-a)(2x-b)$ works.

Example 2

Let	$A = 4x^2 - 3x - 1$	substitution
	$4A = 16x^2 - 3(4)x - 4$	multiplication by 4
	$4A = (4x - 4)(4x + 1)$	$16x^2 = 4x \bullet 4x$ and $-4 = -4 - 1$ and $-3 = -4 + 1$
	$4A = 4(x - 1)(4x + 1)$	factor out the GCF
	$A = (x - 1)(4x + 1)$	division

The above was Pega's take on factoring by decomposition. This is how it made sense to her. If it appeals to other students like Pega, then it may become an alternative method of factoring. Try it, you'll like it!

Duncan McDougall has been teaching for 27 years, including 13 years of teaching in the public school systems of Quebec, Alberta and British Columbia. During the past 15 years, he has taught mathematics to high school and university students and to elementary school teachers. He owns and operates TutorFind Learning Centre in Victoria, British Columbia. Pega Alerasool is in her third year as a coop student in mechanical engineering. The work presented in this article occurred when she was a member of a high-energy Grade 11 honours math class that savoured all the material Duncan McDougall presented to them. She was a keen student who liked to tinker with methods and techniques demonstrated in math and science classrooms.