Reducing Fractions and Its Application to Rational Expressions

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There are various reasons a given student doesn't learn or master a presented method or technique. Teachers are aware of diverse learning styles and conditions in the classroom. But sometimes despite hard work and willingness to learn on the part of the student, the objective is not met. What do we do? Alternative approaches to a problem are often sought, and this is where thinking outside the box may come in handy, especially when a novel idea works and appeals to others.

Imagine the plight of high school math students whose factoring skills are less than adequate. They may find factoring a chore because of a lack of success with a conventional method. Students might need another approach to help them succeed.

Consider reducing the fraction 39/65 to lowest terms. If we know that 13 is a factor of both 39 and 65, then we can write it as $\frac{39}{65}=\frac{13\times 3}{13\times 5}=\frac{3}{5}$. The educator knows that 13 is the greatest common factor, but the student may not. Similarly, how would it be apparent to a student in an expression like $\frac{x^2-2x-3}{x^2-7x+12}$?

Previously, I have demonstrated the premise that the only possible factors available to reduce a fraction to lowest terms come from the difference between the numerator and the denominator (McDougall 1990). Numerically, it looks like this:

Reduce $\frac{39}{65}$

Steps:

(1) $65-39=26$ demonstrates the difference between the numerator and the denominator

(2) Factors of 26 1, 2, 13, 26

(3) Disregard 1. Disregard 2 and 26 because they are even

(4) Try 13. If 13 doesn't work, then nothing else will.

(5) $\frac{39}{65}\div\frac{13}{13}=\frac{3}{5}$

When transferring this concept to rational expressions, examine the following two examples:

Example 1

Reduce $\frac{x^2-2x-3}{x^2-7x+12}$

Steps:

(1) $(x^2-2x-3)-(x^2-7x+12)$

$=x^2-2x-3-x^2+7x-12$

$-5x-15$

$=5(x-3)$

(2) Disregard 5 and consider $(x-3)$ because 5 doesn't divide evenly into the numerator or the denominator but $(x-3)$ might.

(3) $x^2-2x-3=(x-3)(x+1)$ and $x^2-7x+12=(x-3)(x-4)$

At this stage, we can factor more easily because we know that $(x-3)$ is one of the desired factors; failing that, use long division

(4) $\frac{x^2-2x-3}{x^2-7x+12}=\frac{x+1}{x-4}$

Example 2

Reduce $\frac{x^3-1}{x-1}$

Steps:

(1) $(x^3-1)-(x-1)$

$=x^3-1-x+1$

$=x^3-x$

$=x(x^2-1)$

$=x(x-1)(x+1)$

(2) Disregard x and $(x+1)$ but consider $(x-1)$ because neither x nor $(x+1)$ divide evenly into neither the numerator nor the denominator; the only remaining factor to consider is $(x-1)$.

(3) $x^3 - 1 = (x-1)(x^2+x+1)$

(4) $\frac{x^3-1}{x-1} = \frac{(x-1)(x^2+x+1)}{x-1}$

$= x^2 + x + 1$

What I also like about this method is that we can discover which factors will not work in a given situation:

Consider $\frac{x^2-x-12}{x^2-x-6}$ **Steps:**

(1) $(x^2-x-12)-(x^2-x-6)$

$= x^2 - x - 12 - x^2 + x + 6$

$= -6$

Immediately we can see that there cannot be a common factor of the form $(x+a)$ (assuming that the algebra is done correctly of course).

As we can see, there is no point in factoring and looking for a common term if indeed none exists to begin with. Now it's no secret that a method for finding the GCF of two polynomials does exist, but it does involve long division, and therefore, it would look something like this:

Find the GCF for x^2-2x-3 and $x^2-7x+12$, or GCF $(x^2-2x-3, x^2-7x+12)$.

Divide one into the other, and keep track of the remainder. Now divide the remainder into the previous divisor, and again, keep track of the remainder. Continue this last step until the remainder is zero. The divisor, which gives zero as a remainder, is our GCF. This means we would have:

$$\begin{array}{r} 1 \\ x^2-2x-3\,\overline{)\,x^2-7x+12} \\ \underline{x^2-2x-3} \\ -5x+15 = -5(x-3) \end{array}$$

Take only $(x-3)$ because -5 is not a factor of the form $(x+a)$ and because -5 doesn't divide evenly into either the numerator or the denominator.

$$\begin{array}{r} x+1 \\ x-3\,\overline{)\,x^2-2x-3} \\ \underline{x^2-3x} \\ x-3 \\ \underline{x-3} \\ 0 \end{array}$$

Actually, finding the GCF this way is part of the reason the above method of subtraction works. The teacher now has more than one way of presenting this material to various types of learners and can provide alternatives for the reluctant student. A welcome application of this approach is the calculation of limits for the calculus student. In general, we have:

$$\lim_{x\to -a} \frac{x^2+x(a+b)+ab}{x+a}$$

Instead of evaluating directly, and giving the indeterminate form $0/0$, we can subtract the two polynomials, factor this difference, and then try to reduce it to its lowest terms. This would create the following:

$$\begin{aligned}(x^2+x(a+b)+ab)-(x+a) &= x^2+ax+bx+ab-x-a \\ &= x(x+a)+b(x+a)-1(x+a) \\ &= (x+a)(x+b-1)\end{aligned}$$

This reveals that (1) the expression can be reduced, and (2) $(x+a)$ is the common factor. A numerical example would look like:

$$\lim_{x\to -2} \frac{x^2+3x+2}{x+2}$$

$$\begin{aligned}&(x^2+3x+2)-(x+2) \\ &= x^2+3x+2-x-2 \\ &= x^2+2x \\ &= x(x+2)\end{aligned}$$

$$= \lim_{x\to -2} \frac{(x+1)(x+2)}{x+2}$$

Disregard x and consider $(x+2)$ because x doesn't divide evenly into the numerator or the denominator.

$$= \lim_{x\to -2} x+1$$

$= -2 + 1$

$= -1$

In summary, some students see math as a necessary evil. However, now they can get into it a bit more because someone has found a method that makes sense to them.

Reference

McDougall, D E. 1991. "Reducing Fractions." *International Journal of Mathematics Education and Science Technology* 22, no 4: 683–93.

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