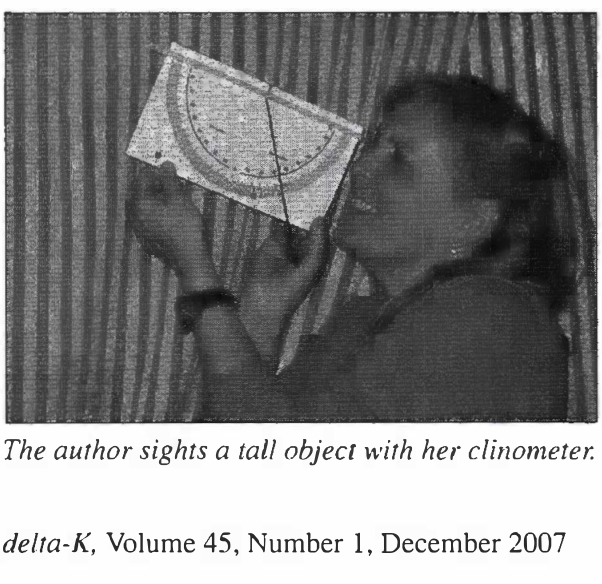
Taking Measurement Mathematics to New Heights with Student-Made Clinometers

Sherry Talbot

The use of tools in teaching and learning measurement allows students to experience mathematics in a direct and personal way. Tools help students connect diagrams and problems to the world around them and to their own experience. By the time they reach secondary school, students have a wealth of experience with rulers and measuring sticks, and they are beginning to look at how to use degrees to measure angles and construct polygons. We can build on these experiences by having students construct triangles to solve problems and use a clinometer, an instrument for determining angles of inclination or slope, to determine the heights of objects too tall to be measured with a measuring stick.

Building and using a clinometer to measure angles of elevation and depression helps students develop their understanding of length, height, degrees, right triangles, ratio and proportion, estimation, accuracy, trigonometry, and elevation and depression. The problems that students develop have solutions that help them determine whether their calculations are correct and accurate. Because they have constructed the problems themselves, students often gain a sense of efficacy and trust in their ability to work with mathematics.



The author sights a tall object with her clinometer.

Constructing and Using a Clinometer

My first experience with having students make clinometers was decades ago, when I was a student teacher. We used a copy of a protractor to make the clinometers. It worked, but the students had to add their readings to or subtract them from 90°.

More recently, I used Google Image Search and found several images that were much better. The one I chose, which works beautifully, can be found at www.learner.org/channel/workshops/lala/clinometer.html.

Students will have to work in pairs to use the clinometers, but they will benefit from each making a clinometer of their own.

To make the clinometer, each student will need a copy of the clinometer image (a half-circle with degrees marked on it), a piece of cardboard, a piece of string and a weight (a coin, pebble or washer will do) to make a plumb line, a straw for a sighting device, and tape and glue.

First, students will glue the half-circle to the cardboard.

Next, they will tape the string and the weight to the top centre (where the dot is). The plumb line should fall right across the 0 mark when the clinometer is held perpendicular to the ground.

Finally, the students will tape or glue the straw (the sighting device) right on the diameter of the clinometer.

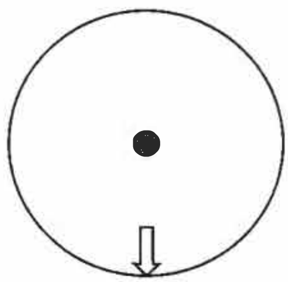
Voila! The clinometer is ready to be used for sighting angles.

Using a clinometer is a collaborative endeavour. One person sights the object through the straw, and the other reads the degrees of elevation or depression indicated by the plumb line. Suddenly, mathematical problems spring off the page and into students' lived experience.

Constructing and Using a Trundle Wheel

A complementary tool students can construct to help them measure triangles is a trundle wheel (see Figure 1). After measuring a variety of circles and determining π (circumference divided by diameter), students can construct a cardboard circle with a circumference of exactly 1 m. A pencil through the centre of the circle acts as an axle, and an arrow drawn on the circle indicates the starting point. The students then have a trundle wheel to measure distances along the ground.

Figure 1

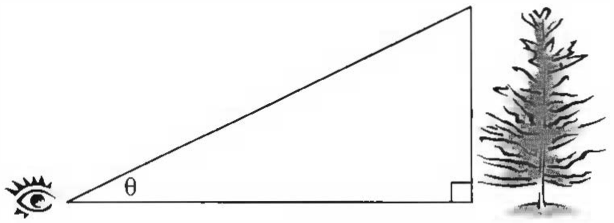


Now the students are equipped with tools to find and measure all kinds of triangles in their surroundings.

Finding the Heights of Objects Too Tall to Measure

Students who have not yet encountered trigonometry can use similar triangles and proportions to determine the heights of tall objects, such as flagpoles, tall buildings, trees and monuments (see Figure 2).

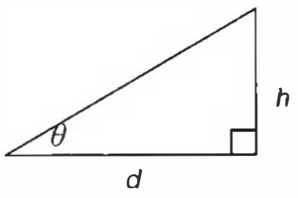
Figure 2



To use similar triangles, the students need to have a right triangle for which they know the measurements. In the classroom, gym or corridor, mark a spot on the wall at a specified height, h (3 m or higher is best). Put a measuring tape, or masking tape with metres marked out, on the floor from directly below the mark on the wall.

In pairs, students then choose a distance, d, and measure their angle, θ, by sighting the mark on the wall with the clinometer. Now they have a triangle for which they know the length of the two arms adjacent to the right angle, as well as the angle of inclination. Although they could determine all six angles and arm measurements, they need only d, h and θ (see Figure 3). This becomes the triangle that will be similar to all the triangles the student will measure outside the classroom.

Figure 3



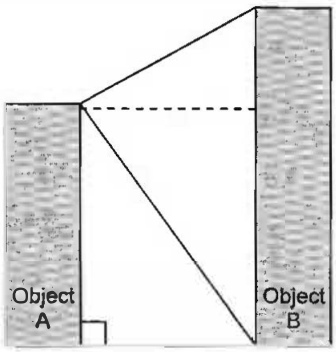
To make a large triangle similar to this reference triangle, students must sight the top of a tall object through the straw on the clinometer and then move back and forth until the angle of inclination is exactly θ. From that spot, they must run the trundle wheel to the base of the object to determine d. Once the students draw the triangle for the tall object beside the drawing of their reference triangle, they can see that their triangles are similar and that they can use ratios and proportions to calculate the height of the tall object.

When students find the length of the vertical arm of the right triangle, they are finding the height of the object from eye level. Therefore, the calculation of the object's total height is not complete. The height of the clinometer at the time of the sighting must be added to the calculated height of the object. Once students realize this, they must determine a way to do it. Some students measure the height of their eyes, and others lie on the ground to take their readings. One of the cleverest solutions I have seen was to use a metre stick as a stand for the clinometer; then, the students only had to add 1 m to the calculated measurement to find the total height.

As students move on to trigonometry, they no longer need to use a reference triangle. Once students sight the top of an object to get θ and measure d from the sighting spot, they need only to apply the tangent ratio to find the height. The challenge, then, is to determine the heights of objects using angles of depression, or a combination of elevation and depression. To do this, students can stand on a balcony or in front of a window on a floor above ground level. A building with an atrium also offers an opportunity to make measurements with elevation and depression.

Let's look at Figure 4. If students were able to measure the distance between Object A and Object B, and to take readings of elevation and depression with a clinometer from the top of Object A, many interesting problems involving angles and heights would arise. They would be able to work with the sum of angles in triangles and quadrilaterals, parallel lines, distances, the Pythagorean theorem and trigonometric ratios.

Figure 4



Experiences with Measurement Mathematics

Using a clinometer involves students in mathematics by situating them in measurement with their movements. Students not only become physically involved but also estimate, use mental math, communicate with their peers, make mathematics connections and use tools they have constructed themselves. In my experience, a wide range of students have done much mathematics with curiosity and a positive attitude while using these tools in problem-solving projects.

Although the mathematics that students use when working with clinometers and trundle wheels can be quite sophisticated, the tools themselves and the context of the problems are concrete, allowing students to get a personal feel for the mathematics.

The clinometer gives students direct experience with moving their heads to make different angles, connecting that movement with the number of degrees representing the angle measurement. When students draw the angle in a diagram, they can see how the problem looks on paper and relate it to what they did with the clinometer.

While using a trundle wheel to measure distances, students begin to make sense of large measures of length. These actions help them to visualize problems that they encounter, making connections between words, numbers and experiences.

These moving and feeling experiences give variety to students' connections to mathematics. For students who learn better through direct experience, this involvement can provide an entry point for learning mathematics. And diversity of experience can help all students remember mathematics more effectively.

Because working with clinometers requires students to work together to take measurements, students talk about what they are doing. They discuss whether they need to be more precise or if they may be doing something wrong. They use terminology that helps them work with and understand measurement and geometry problems. If one person makes a mistake, the other may be able to recognize and help rectify it.

Once the students have taken measurements, they must draw diagrams and solve the measurement problems. In doing so, they are writing about their experiences, describing them through mathematics. They are expressing through symbols what they have discussed and felt. While working together to draw their diagrams, discuss the process of solving the problems and do the calculations, students have opportunities to interact in a constructive mathematical fashion. At this stage, if students have made mistakes, they can recognize that there must be something wrong with their measurements or processes. If their result contradicts their sense of size (such as a school roof that is 2 m high or a flagpole that is 200 m high), they are able to see for themselves that they need to remeasure or recalculate.

Using clinometers gives students an opportunity to get out of the classroom to work with mathematics. It is a mathematical occasion when students can put their minds and bodies—and some simple tools—to work in producing and solving interesting measurement and geometry problems.

Sherry Talbot has taught mathematics to students in Grades 6–12 in Edmonton and at international schools in Munich and London. She is completing her master's degree in secondary mathematics education at the University of Alberta. Her research interests lie in looking at how students collaborate to learn mathematics. Over her many years of teaching, she has developed several projects that allow students to work together to build their mathematical understanding.