delta-k

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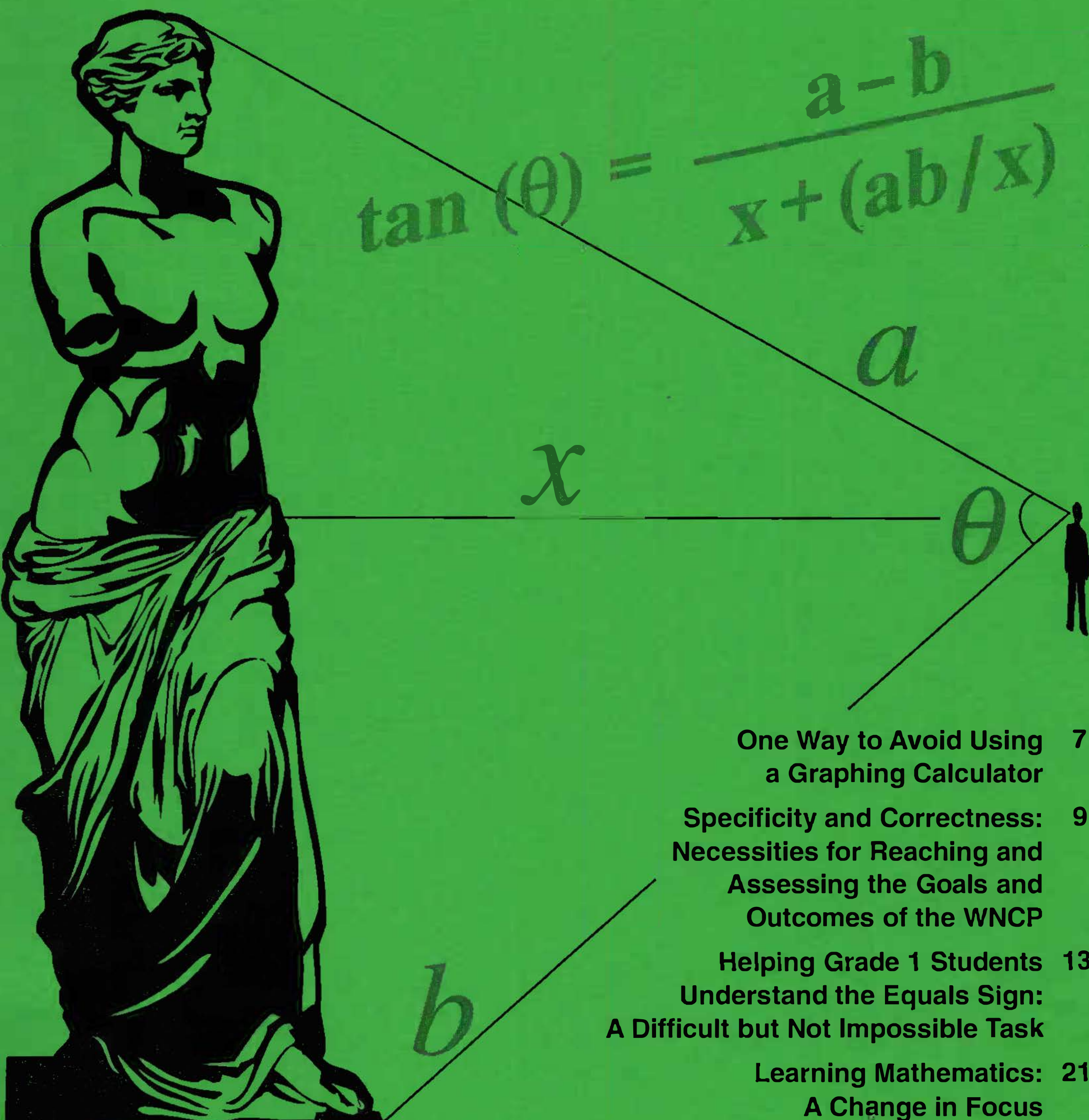

GUIDELINES FOR MANUSCRIPTS

delta-K is a professional journal for mathematics teachers in Alberta. It is published twice a year to

- promote the professional development of mathematics educators, and
- stimulate thinking, explore new ideas and offer various viewpoints.

Submissions are requested that have a classroom as well as a scholarly focus. They may include

- personal explorations of significant classroom experiences;
- descriptions of innovative classroom and school practices;
- reviews or evaluations of instructional and curricular methods, programs or materials;
- discussions of trends, issues or policies;
- a specific focus on technology in the classroom; or
- a focus on the curriculum, professional and assessment standards of the NCTM.

Suggestions for Writers

1. *delta-K* is a refereed journal. Manuscripts submitted to *delta-K* should be original material. Articles currently under consideration by other journals will not be reviewed.
2. All manuscripts should be typewritten, double-spaced and properly referenced. All pages should be numbered.
3. The author's name and full address should be provided on a separate page. If an article has more than one author, the contact author must be clearly identified. Authors should avoid all other references that may reveal their identities to the reviewers.
4. All manuscripts should be submitted electronically, using Microsoft Word format.
5. Pictures or illustrations should be clearly labelled and placed where you want them to appear in the article. Please also include all graphics as separate files (JPEG, GIF, TIF). A caption and photo credit should accompany each photograph.
6. References should be formatted using *The Chicago Manual of Style*'s author-date system.
7. If any student work is included, please provide a release letter from the student's parent/guardian allowing publication in the journal.
8. Limit your manuscript to no more than eight pages double-spaced.
9. Letters to the editor and reviews of curriculum materials are welcome.
10. Send manuscripts and inquiries to the editor: Gladys Sterenberg, 195 Sheep River Cove, Okotoks, AB T1S 2L4; e-mail gladyss@ualberta.ca.

MCATA Mission Statement

Providing leadership to encourage the continuing enhancement of teaching, learning and understanding mathematics.

delta-K

Volume 45, Number 1 December 2007

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EDITORIAL

During the summer, I had an opportunity to work with elementary teachers in Bhutan. Led by Marian Small, this project involved writing mathematics textbook materials for a new curriculum and providing professional development in the form of a writers' workshop.

During a meeting with the secretary of education, one teacher shared the following:

> I've heard of a species of vulture that has a lifespan of about 40 years. When these vultures get older, their upper beaks grow around their lower ones, fusing them together. The vultures then cannot eat, and they die of starvation. About 5 per cent of the elder vultures fly to the rock cliffs close to their nesting grounds. They proceed to smash their beaks on the rocks, effectively breaking them. This is a very painful process, but over time their beaks regenerate. Their lifespan is then extended by another 30 years. This is what this new curriculum seems like. We are painfully smashing old ideas about teaching and learning so that we can bring new life into Bhutan.

Throughout this fall, I have been reminded of how painful, yet exciting, new curriculum implementation can be. The articles in this issue of *delta-K* demonstrate the innovative ways teachers are changing mathematics learning for their students.

The issue opens with two opinion pieces. Indy Lagu presents reasons for rethinking how we use graphing calculators. Werner Liedtke challenges us to carefully consider the goals of the new Western and Northern Canadian Protocol (WNCP) document.

Helping students understand mathematical concepts remains a focus, and Laurent Theis leads us through his own research with Grade 1 students. Janelle McFeetors emphasizes the importance of metalearning and reminds us that mathematical conversations and listening can enhance mathematical learning. Sherry Talbot shows how teachers are making a difference in their students' learning and presents an inventive measurement task.

Each year, University of Alberta professor Andy Liu is involved with the Alberta High School Mathematics Competition. Included here are lists of the 2006/07 winners, as well as the test questions and solutions. Congratulations to all teachers and students who took part! If you are interested in having your students participate in this competition, contact Andy at (780) 492-3527 or aliu@math.ualberta.ca.

The issue concludes with a page of mathematical problems by Craig Loewen. These problems are intended to provide rich experiences in mathematical learning for students at all grade levels.

To give voice to mathematics teachers in this province, I invite letters to the editor on the topic of the new curriculum. May we be inspired by the unconventional story of the vultures as we enter this new phase of mathematics education in Alberta.

Gladys Sterenberg

From the President's Pen

As Christmas approached, we were all busy with school and family activities. Whether your life was filled with preparing for concerts, trying to finish concepts before the break, preparing students for diploma exams or just trying to survive until the holidays, I hope you were able to reflect on the spirit of the season. School breaks are meant to be a time of rejuvenation, a time to reflect on why we do what we do and how we can make student learning meaningful.

The past year has been a busy one, especially for those who are delivering the new curriculum in kindergarten and Grades 1, 4 and 7. Although a new curriculum usually means a lot of extra planning, it also gives us a sense of renewal and opportunities for networking. The program of studies for K–9 mathematics has been adopted and is now available on the Alberta Education website (http://education.alberta.ca). If you haven't had a chance to look at it yet, I encourage you to do so.

The Mathematics Council of the Alberta Teachers' Association (MCATA) has sent out a needs assessment for K–6 and 7–9 to help in preparing for future conferences. I thank those of you who responded over the summer to our survey on the proposed new senior high curriculum. The responses gave us a sense of the future needs for senior high, and we have shared the results with Alberta Education representatives. The curriculum for senior high has not yet been finalized, but it is getting closer. Please take any opportunity you can to provide feedback now.

The MCATA executive enjoyed meeting many of you at our conference in October. With the new automatic specialist council membership offered by the ATA, we are hoping to see more teachers become involved with MCATA. If you have colleagues teaching math who are not MCATA members, please encourage them to join us. We all know that math teachers have the most fun! If you are already a member and would like to become more active, please contact any of the executive members (see the inside back cover for contact information). We have lots of opportunities for people to become involved.

From now until 2012, when the new curriculum is fully implemented, math teachers will be forming strong professional learning communities. We will be helping each other reach the necessary depths of knowledge that our students will require. Never forget that Alberta students are among the best math students in the world because of the expertise of Alberta teachers. You have a lot to share with each other, so please consider presenting at conferences, helping to organize miniconferences or forming regional learning communities.

As MCATA president this year, I look forward to working with and getting to know more of my mathematical colleagues. I hope you had a wonderful holiday and took some time to recharge!

Sharon Gach

Conference 2007: “Mathematical Tapestries: Weaving the Connections”

Message from the Conference Director

Conference 2007: “Mathematical Tapestries: Weaving the Connections,” held October 18–20 at the Fantasyland Hotel in Edmonton, highlighted the interconnectedness of mathematics and the lives of students. The connections can be easy to lose sight of as we struggle to help our students cover curriculum and prepare for tests. With the upcoming implementation of the new K–12 curricula, designed to provide space for deeper consideration of fewer topics, we are again invited to reconsider how we might weave richer connections within and between mathematical topics and students’ own experiences.

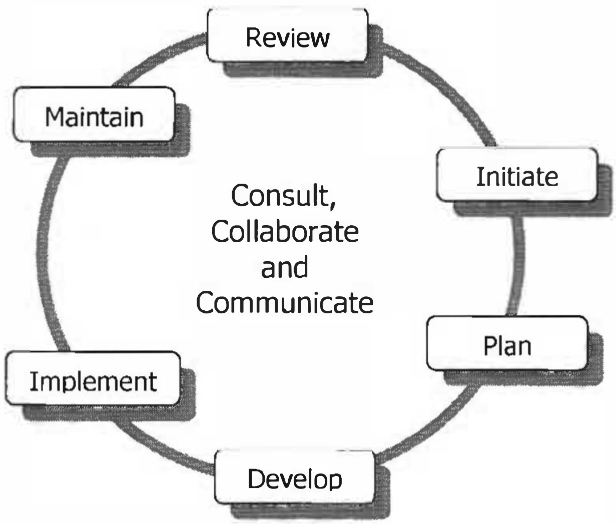
Our opening keynote speaker, David Jardine, has written extensively on the fragmentation of curricula, particularly in mathematics. He works closely with teachers and students to explore more richly connected alternatives. His talk “Time Is Always Running Out” offered a fresh look at mathematics as a living discipline rather than a collection of disparate and predetermined objectives.

We were also pleased to welcome Dave Mitchell, an award-winning math teacher and popular speaker, whose closing keynote address stressed the importance of enjoying mathematics.

Additional descriptions and photographs of the conference will be included in the next issue of *delta-K*. Please continue to check the MCATA website (www.mathteachers.ab.ca) for information on upcoming conferences.

Martina Metz

The Right Angle: Report from Alberta Education



In the last instalment of "The Right Angle," Jennifer Dolecki examined the curriculum development cycle.

Currently, Mathematics 31 and Mathematics Preparation 10 are in the maintenance phase. The Alberta program of studies for K–9 mathematics was completed and posted on the Alberta Education website (http://education.alberta.ca) in the spring of 2007 and entered the implementation phase in September. The implementation schedule for the K–9 program is shown below.

	September 2007	September 2008	September 2009	September 2010
Optional Implementation	K, 1, 4, 7	2, 5, 8	3, 6, 9	—
Provincial Implementation	—	K, 1, 4, 7	2, 5, 8	3, 6, 9

Implementation of the revised program has been supported through workshops offered by the Alberta Regional Professional Development Consortia (ARPDC), as well as through a summer institute offered by Alberta Education. The summer institute saw over 300 teachers engaged in learning about and discussing the revised program of studies, its philosophy and pedagogy to support its implementation. Also, the online guide to implementation has been launched, and new content will be added on an ongoing basis.

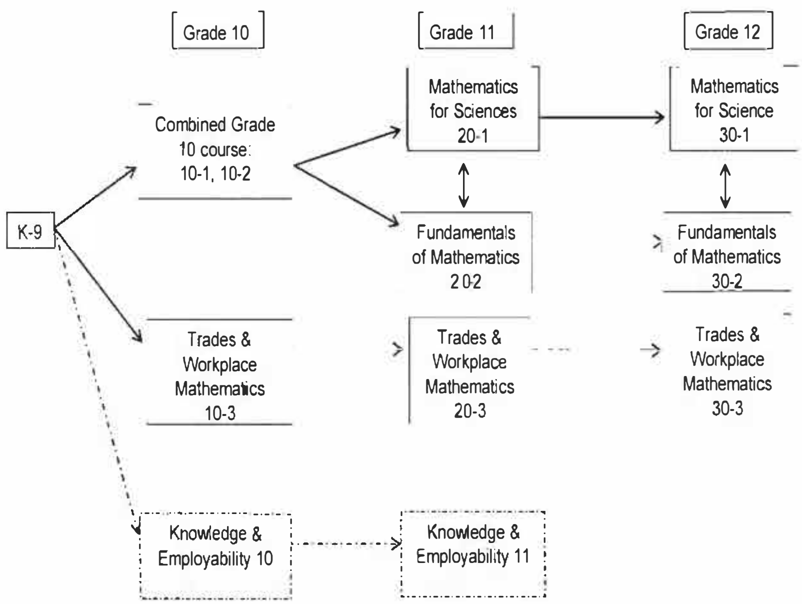
The high school mathematics program is currently in the development phase. Along with our Western and Northern Canadian Protocol (WNCP) partners, we have revised the consultation draft of the Common Curriculum Framework (CCF) based on feedback from our stakeholders. This feedback was gathered through many face-to-face consultations and an online survey. The revised CCF was provided to publishers in the fall of 2007. Final revisions are being made to the CCF, and it is scheduled to be signed off in the spring of 2008.

Though Alberta remains a strong partner in the WNCP, feedback from our stakeholders indicated the need for a different fundamentals of mathematics pathway from our partner jurisdictions. The WNCP need for a distinct pathway from the mathematics for sciences pathway was contrary to the transferability between pathways that Alberta stakeholders desired. Because of these differing needs, Alberta Education (with input from teachers and postsecondary stakeholders) will develop unique fundamentals of mathematics courses for Grades 11 and 12.

The WNCP CCF is scheduled for completion in June 2008. This will be followed by the development of the Alberta high school program of studies in 2009.

The structure of the high school math programs follows.

Structure for Senior High School Mathematics



Provincial implementation of the Grade 10 programs of study is scheduled for September 2010, followed by Grade 11 in 2011 and Grade 12 in 2012.

Alberta Education thanks all the stakeholders who have provided input on the programs of study. Your input ensures strong programs that will meet the needs of our students.

Kathy McCabe

One Way to Avoid Using a Graphing Calculator

Indy Lagu

I was recently looking through a calculus text, and a question caught my eye. The question is quite common; I suspect that it appears in most calculus books. It concerns the optimal distance for viewing a statue. Variations of the problem abound: where to sit in a movie theatre and from where to attempt an extra point in rugby come to mind.

Imagine that you are standing and looking at a statue. Let b be the distance from your eyes to the base of the statue, and let a be the distance from your eyes to the top of the statue. If the distance from you to the statue is x, then the angular size of the statue is given by

$$\theta = \tan^{-1}\left(\frac{a}{x}\right) - \tan^{-1}\left(\frac{b}{x}\right).$$

The task is to determine x to maximize θ.

The approach taken in a calculus class is to take a derivative:[1]

$$\frac{d\theta}{dx} = \frac{-a/x^2}{1+\left(a^2/x^2\right)} - \frac{-b/x^2}{1+\left(b^2/x^2\right)}.$$

A bit (all right, a lot) of algebra reveals

$$\frac{d\theta}{dx} = \frac{-a}{x^2+a^2} + \frac{b}{x^2+b^2},$$

which is 0 when $x = \sqrt{ab}$. Note that $\theta \to 0$ as $x \to 0$ and as $x \to \infty$, so that this x gives a maximum.

A lower-tech approach (mathematically and intellectually) is to plug the equation into a graphing calculator and eyeball the maximum.

I do not mind the first approach, but I abhor the graphing calculator approach. Now, I think my resistance is not to the use of technology but to the *misuse* of technology. One objection is, of course, that eyeballing a maximum is not mathematically correct—indeed, the power of mathematics is that we can find the exact answer here and not resort to eyeballing. However, my objection is deeper: the act of finding an answer to this question misses the point entirely. Surely the point of solving a problem such as this is not merely to get an answer but to *learn* something.

To that end, I offer a mathematically correct approach that provides an opportunity to use some facts about trigonometric functions.

Recall that we wish to find x to maximize θ, where

$$\theta = \tan^{-1}\left(\frac{a}{x}\right) - \tan^{-1}\left(\frac{b}{x}\right).$$

Taking the tangent of both sides yields

$$\tan(\theta) = \tan\left[\tan^{-1}\left(\frac{a}{x}\right) - \tan^{-1}\left(\frac{b}{x}\right)\right]$$

$$= \frac{a/x - b/x}{1+\left(ab/x^2\right)}$$

$$= \frac{a-b}{x+\left(ab/x\right)},$$

where we have used the identity

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}.$$

Now, since $\tan(\theta)$ increases with θ, and $a - b$ is a constant, it is enough to maximize the function

$$\frac{1}{x+\left(ab/x\right)},$$

and therefore it is enough to minimize the function

$$f(x) := x + \frac{ab}{x}.$$

Even now, the temptation to stop thinking and use a graphing calculator can be avoided. Since x is increasing and ab/x is decreasing, a good guess is that $f(x)$ is minimized when $x = ab/x$. That is, at $x = \sqrt{ab}$.

Since $f(\sqrt{ab}) = 2\sqrt{ab}$, the question now is to try to show that

$$x + \frac{ab}{x} \geq 2\sqrt{ab},$$

and even this can be done without resorting to calculus or calculators. I will leave this as an exercise for the reader. Your hint is to start with the fact that

$$\left(\sqrt{x} - \sqrt{\frac{ab}{x}}\right)^2 \geq 0.$$

Incidentally, the quantity $\sqrt{ab}$ is called the *geometric mean of a and b,*[2] and it has an interesting geometric interpretation. Imagine a rectangle with sides *a* and *b*. Its area is *ab*. Now imagine a square with the same area. The sides of the square must be $\sqrt{ab}$.

I guess that the approach you favour in solving this problem depends on your goal. If your goal is to estimate the answer, and avoid any depth of thought, then using a graphing calculator is all right, I suppose. If your goal is to obtain the correct answer and to flex your calculus muscles, then differentiation is the way to go.

My goal would be to see the utility of trigonometry; hence, I would avoid using a graphing calculator, or even calculus, and use my head. What makes a problem good is not the realism or practicality in it but, rather, where it takes you.

Notes

1. If you have never taken a calculus class, skip this part. The headache is not worth it.

2. The arithmetic mean—$(a + b)/2$—is probably familiar to most readers and is often called *the average*.

Indy Lagu completed his PhD in mathematics at the University of Calgary in 1996. His interests moved from numerical analysis to the state of mathematics education in Alberta. He has been a member of the MCATA executive for seven years and has been working with teachers and students in the province for much longer. Currently, Indy is chair of the Department of Mathematics, Physics and Engineering at Mount Royal College in Calgary. He spends his free time learning Spanish, playing golf and wondering when the Montreal Canadiens will win another Stanley Cup.

Specificity and Correctness: Necessities for Reaching and Assessing the Goals and Outcomes of the WNCP

Werner Liedtke

The content of this article is based on the assumption that without the use of specific, nonsubjective language and correct mathematical terminology, many of our students could experience difficulty as they try to reach the goals and outcomes of the Western and Northern Canadian Protocol (WNCP). The examples used here have been selected from comments on assignments made by teachers and teachers-to-be, from comments overheard at conferences and meetings I have attended, and from statements found not only in newspapers but also (I am sad to say) in many professional journals.

Statements of a general nature—and I am going beyond those that end in *etcetera* or *blah blah blah*—tend to be part of our everyday conversations. There are many times when I ask (or am tempted to ask), "What do you mean?" or "What options or what examples might you have in mind?" For example, a degree of specificity would be advantageous in statements that include phrases such as *will be there shortly, won't be long* and *will be right there,* especially when one is waiting for a taxi or assistance of some sort.

Statements Related to Assessment

A lack of specificity and a degree of subjectivity are present in many statements that involve aspects of assessment in the mathematics classroom. Students' comments are sometimes referred to as being "very clear" or indicating "good understanding," "logical estimation skills" or "deep knowledge." These examples begin to illustrate my concern. There is no doubt in my mind that these descriptions can sound impressive; however, interpretation of intent is not possible. The levels of clarity, understanding, estimation ability and knowledge that those making the statements have in mind are not known. Even if a list of categories were included, any interpretation would involve a high degree of subjectivity and would likely differ from one person to another.

Many checklists and assessment rubrics suggested for use in the mathematics classroom include descriptors of the type *rarely, sometimes* and *often.* If those who are trying to use scales of this sort are not told what, for a given grade, might be considered *rarely, sometimes* and *often,* a range of differences for classifying responses is sure to exist. Some of those differences could be related to such things as years of teaching experience and background knowledge in the subject area. Familiarity with the curriculum and the specific learning outcomes for a grade (as well as for the previous grades) should allow teachers to be specific and make numerical suggestions, or at least a range of numerical suggestions, for categories of this type.

Since conceptual understanding and number sense are two important WNCP components of mathematics learning, parents should be informed about their child's progress in those areas of numeracy. Reports should be specific, and any degree of subjectivity should be avoided. It may be tempting to generate impressive-sounding statements such as "Abby has good conceptual understanding" and "Ryan is developing number sense." But these statements do not tell a parent anything about learning outcomes that have been met or any indicators of the child's conceptual understanding or number sense that have been observed and collected.

I believe that many proposed assessment criteria and many current examples of assessment statements are much too general and therefore meaningless—and that includes some statements that have been published and recommended as effective and appropriate.

A colleague (who teaches measurement and evaluation courses) and I have on occasion examined teachers' report card remarks. Close scrutiny has revealed that little, if any, specific or valuable information about students' performance and abilities is provided. My colleague shared his insight on the following three comments, each intended to tell parents something different about their child's adjustment:

- "Your child is adjusting to the routines and procedures of the full-day kindergarten program."

- “Your child has adjusted to the routines and procedures of the full-day kindergarten program.”
- “Your child is having difficulty adjusting to the routines and procedures of the full-day kindergarten program.”

He suggested that a parent who received one of these comments without seeing the others would not know whether anything was wrong—or right, for that matter. I agree. Without indicators that describe aspects of adjustment, the statements are empty and meaningless.

The following have been recommended as effective statements, but as my questions show, they are too general:

- *“The child is developing a better attitude toward ...”* Better than what? How was this change in attitude assessed? What were some of the indicators?
- *“The child is learning to be a better listener.”* Better than what? How was this conclusion reached? What are some areas of difficulty right now?
- *“The child is learning to be careful, cooperative and fair.”* Why list all these important characteristics in one statement? Are all three currently lacking? How was the learning observed?
- *“The following suggestions might improve the child's ...”* Why the caution? What might the possible improvement depend on? If the teacher is not sure about a suggestion for improvement, how can it be of help to a parent—or to most parents?
- *“The child needs to apply skills to all written work.”* What are the skills? What is included in *all written work?*

It would be interesting to see readers' reactions and the questions they would pose about suggestions such as “Megan needs to work democratically with others,” “Brandon needs to be urged,” “Jessica is maturing” and “Please try to encourage Matthew to do things on his own.” Teachers-to-be have shared some interesting and sometimes even amusing reactions with me.

Perhaps these and other general statements are a result of some of the vagueness and subjectivity found in the assessment strategies and rubrics recommended for use. The following information comes from rubrics provided by various authors in the Spring 2006 issue of *Vector*, the journal of the British Columbia Association of Mathematics Teachers (pp 16 and 37):

- *The category Meets Expectations is described as follows: “The work satisfies the most basic requirements of the task, but is flawed in some way. The student may need some help.”* How is *most basic* defined? If something is flawed, why say that the student *may* need help?
- *A description of the category Fully Meets Expectations includes the expressions “minor errors” and “simple extensions.”* What are some examples of minor errors and simple extensions? How are *minor* and *simple* defined? Without examples or definitions, is it possible to reach agreement about these terms?
- *In the category Exceeds Expectations, reference is made to being efficient.* How might we assess efficiency, or is it even possible?

In a math writing rubric, the category labelled Meets Expectations (Minimal Level) contains this statement: “May include some illogical and irrelevant mathematical ideas.” How are *illogical* and *irrelevant* defined? It would be interesting to examine examples of illogical and irrelevant mathematical ideas and the level of agreement reached by people with different backgrounds. The use of the word *minimal* reminds me of a report in the local newspaper about the sinking of the Queen of the North. The fuel leak on the ferry was described as being “fairly minimal.” Such a description raises questions about the meaning of *minimal.* Without any kind of reference point, the description is meaningless.

The category Fully Meets Expectations includes this description: “Usually uses mathematically sound, relevant arguments to support the main idea.” How can something that occurs only “usually” be classified as fully meeting expectations? How would *sound, relevant,* and *sound, relevant* be defined, and what would be some examples of such arguments? Should all three of these phrases be in the same descriptor?

Not too long ago, we were informed by the then British Columbia minister of education that eduspeak and sugar-coated report cards for students in BC public schools were on the way out. To illustrate the intent, the minister used the following example:

> A student struggling with fractions would receive a report card stating, “Johnny needs help with fractions” rather than putting a positive spin on Johnny's learning by saying he “understands addition and subtraction and is working hard in other areas.”

Aside from wondering why it is always a Johnny when a student experiencing difficulty is discussed, I think that both statements lack the necessary specificity and are therefore meaningless. It would be impossible to plan and prepare a student's IEP (individual education plan) based on an observation that the student simply “needs help.” Unless the minister was misquoted (which I unsuccessfully tried to determine), the conclusion has to be that eduspeak is present in both statements. Parents receiving either of these comments would not learn anything specific about their child.

Another example of a lack of specificity is a newspaper advertisement for a math program, which informs us of the following program features:

- *"Materials correspond exactly with the learner's level of ability."* We are not told what abilities this might refer to (recall, reflective problem solving, computation, employing personal strategies).
- *"Rate of progress is determined by that student's achievement, not by the teacher."* Wow! This sounds impressive indeed, but what is the ad trying to tell us? What achievement, or achievement of what? Who determines and interprets the achievement?
- *"Material is skilfully organized into natural, coherent and logical progression, so students stay focused and are able to enjoy meaningful results."* It is tempting to request an example of materials that are unskilfully organized into unnatural, incoherent and illogical progression. The claim about resulting focus and enjoyment of meaningful results requires a lot of elaboration.

No doubt many will be impressed by these claims and tempted to buy the goods being sold.

Correctness: Expressions and Terms

In a letter to the *Vector* editors,[1] Barnes presents and discusses a common error made in the mathematics classroom: teachers' saying or writing "Consider the function *f(x)*" rather than "Consider the function *f*." Barnes asks, "Can we blame our students for getting confused and acquiring a distaste for mathematics?" A lack of specificity, along with common errors, can contribute to confusion about mathematics in students, parents and teachers.

Another error encountered frequently—during conversations, in newspapers, in mathematics references—is the confusion between *amount* (continuous quantity) and *number* (discontinuous or discrete quantity). Whenever I see the phrase *amount of people,* I try to visualize an attempt to determine mass. To foster the development of number sense, which includes the visualization of number, the manipulative materials children use must clearly illustrate discreteness. Some young children find it difficult to deal with and interpret anything presented in a row-and-column matrix. (Since it is impossible for young children to develop aspects of number sense by looking at a matrix of ordinals, the WNCP does not make reference to calendar math.)

Failing to distinguish between number (numerousness, cardinal number) and a name for numbers (numerals) can be confusing for young children and detrimental to the development of number sense, which is a key goal of the WNCP. Adults can use context to determine whether the word *number* refers to a discrete quantity or to a symbol assigned to the quantity. Young children are not able to do that. Teachers need to know what young children are thinking when they are asked to think of a number, and children need to know what to do when they are requested to use a drawing to show a number. For children who know the difference between number and name for a number, a request like "Print a number" would not make sense.

Relating numbers and names for numbers is an important aspect of the development of number sense. The language of comparisons is often used incorrectly and carelessly, and as a result many of our students (as well as many adults) are confused. They need to learn when it is appropriate to use the phrases *more, fewer, as many as, greater than* and *less than.*

Many people, including teachers, fail to distinguish between *guessing* and *estimating.* Sometimes subjective descriptors such as *intellectual, logical* or *very good* are used for these terms. There are some who combine the terms and talk about *guesstimates.* How confusing it must be for students to attempt to sort through these terms and descriptors! There is a need for clarity and correctness, and future students should benefit from the related specific learning outcomes and the achievement indicators in the WNCP.

A failure to distinguish between *figure* and *shape* also leads to a lot of confusion, as can be observed in many adults who have gone through our school system. Names of shapes are assigned to three-dimensional objects (for example, a triangular-shaped pattern block is referred to as a triangle). In references for teachers, students are requested to perform an impossible task: "Select and pick up the different triangles." If no differentiation is made between two- and three-dimensional figures (objects) and their characteristics (shape), confusion will remain throughout school and beyond.

When everyday usage of terminology differs from usage in the mathematics classroom, confusion can result, especially if students are not reminded about the differences. Use of the term *fraction* is an example. When fractions are introduced, equal regions are used—where *equal* refers to the same size and shape. At home or outside the classroom, a piece of any size or shape is often referred to as a fraction. If students are not reminded of these different usages, it is not all that surprising that a published test result showed that most students were unable to correctly select a diagram representing one-third. Only 24 per cent were able to do so by Grade 8; 43 per cent selected a diagram showing three pieces that were not equal (Lesh and Zawojewski 1988, 63–64).

Our students also need to know the difference between *design, pattern* and *design with a pattern.* Many times people say, "That is an interesting pattern," when in fact the term *design* would be correct. If no distinction is made between *design* and *pattern,* students will be confused. Students also need to know that patterns can be repeating and growing and that it is easy to change one type into the other. Patterns allow us to correctly describe a member or members of a pattern that are hidden.

Flexible thinking about numbers is an important aspect of number sense; numbers can be shown and named in different ways. This aspect allows us to avoid such a nonmathematical expression as *borrowing* (which is used differently in the mathematics classroom than it is outside of the classroom—unless, of course, one has neighbours who "borrow" something and never give it back). Some student and teacher references would identify, for example, *20 + 4* and *2 tens and 4 ones* as different names for *24,* which is not the case. They are the same. A different name for *24* would be *1 ten and 14 ones.*

Does a lack of agreement on definitions, or the inability to define terminology frequently used in mathematics and mathematics education, contribute to confusion? I believe it does. Teachers and parents talk about *the basics* without ever attempting to find out if they are thinking about the same ideas and skills. (For some surprising responses, ask several people what they think the basics are). *The basic facts* (that is, *the basic addition facts*) are an important part of the curriculum, yet few people are able to define what they are. People—including teachers—also have different definitions for *times table* (for example, some will refer to "the 6 times table"). This lack of agreement on definitions contributes to students' confusion.

Confusion can be avoided if the terminology we use is accompanied by a list of criteria or examples. It is easy to utter such important-sounding phrases as *mathematical reasoning* and *computational fluency,* but if we do not inform the reader or listener of the intent or parameters of such utterances, too many different meanings will be attached to them. Even a group of mathematics educators, who might be tempted to declare that there is no need to define those terms for themselves, would come up with definitions that differ from one person to the next—which is a somewhat sobering thought.

Conclusion

A new curriculum (WNCP) provides us with a golden opportunity to focus on specificity and objectivity and to rid ourselves of possible causes of confusion. As a result, our students and future students should and will benefit.

Allow me to pose the question, Is it possible to be too specific? As far as learning outcomes for students are concerned, no. As a speaker at a conference I attended stated, "Without specific outcomes, nothing specific will come out!"

However, two examples come to mind when I think about something being too specific.

The first example is the question, "Do you have the correct time?" The range of response options is reduced by this very specific question—even if one is tempted to utter a few somewhat facetious remarks as part of the response. What are people who ask a question like this thinking? What are they worried about?

Another example is a pamphlet on Air Canada flights that informs passengers of the ranges of the planes in the airline's fleet. The range, believe it or not, is given to the nearest kilometre, which is difficult to fathom. For example, the range of an Airbus A319 is given as 4,442 km, at a cruising speed of 837 km/h. Amazing accuracy! One cannot help but think about the number of test pilots who might have been lost trying to arrive at this result, while smiling at the lack of number sense evident when someone converts miles to kilometres with a greater degree of precision than possessed by the original measurement.

Note

1. M Barnes, letter to the editors, *Vector* 47, no 1 (2006): 7.

Reference

Lesh, R, and J Zawojewski. 1988. "Problem Solving." In *Teaching Mathematics in Grades K–8: Research-Based Methods*, ed T R Post, 40–77. Boston: Allyn & Bacon.

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Helping Grade 1 Students Understand the Equals Sign: A Difficult but Not Impossible Task

Laurent Theis

Context

Difficulty in developing a proper understanding of the equals sign is a widespread, though often unrecognized, phenomenon among lower-level elementary school students. When I discussed my research on Grade 1 students' understanding of the equals sign with elementary school teachers, most of them were surprised that the equals sign could pose such difficulty for their students. I encouraged those teachers to ask several of their students about the meaning of the equals sign. The teachers were surprised to find that most students saw the equals sign not as an indicator of a relationship but, rather, as an indication to perform an operation or write an answer.

The aim of my research was to describe the development of Grade 1 students' understanding of the equals sign. To observe this development, I chose to involve students in a constructivist teaching experiment in which they worked on the meaning of the equals sign in number sentences involving addition.

At the beginning of my research, I conducted a pretest involving 11 Grade 1 students in an urban district. In six to nine half-hour individual sessions, I then taught to students the equals sign as an indicator of a relationship. Each session was videotaped and then transcribed, allowing me to analyze the students' reasoning. Approximately 10 days after the last lesson, I conducted a posttest, which required the students to answer questions similar to those in the pretest.

In this article, I will describe how Melissa, who was initially assessed by her teacher as being an average performer in mathematics, managed to develop an accurate understanding of the equals sign. I will also highlight the difficulties of two other participants, Mathieu and Caroline, in acquiring this new understanding.

Misunderstanding of the Equals Sign as an Operator

During the pretest, the students were asked to assess whether various number sentences were correct and to complete some equations. They were asked to justify their answers. When they stated that a number sentence was not correct, I asked them to change it to make it correct. They also had to explain the meaning of the equals sign. At this stage, my aim was to gain access to the children's understanding; I did not yet try to make the children think differently about the equals sign.

From the beginning of the pretest interview, Melissa showed an ambiguous understanding of the equals sign. To gain access to her initial understanding, I asked her to tell me whether the number sentence $4 + 5 = 9$ was correct or incorrect. Her answer led me to believe that Melissa saw the equals sign as an indicator of a relationship:

TEACHER [*showing $4 + 5 = 9$*]. Can you tell me whether this number sentence is correct or incorrect?
MELISSA. It is correct.
TEACHER. Why do you think it is correct?
MELISSA. Because we do lots of these additions in school.
TEACHER [*pointing to the equals sign*]. Can you tell me what this sign means?
MELISSA. It says that when it is equal, it also means "the same thing."

However, when I used the number sentence 7 = 3 + 4, Melissa's answers showed that she accepted only number sentences in which the equals sign preceded the last number.

TEACHER [*showing 7 = 3 + 4*]. Can you tell me whether this number sentence is correct or incorrect?

MELISSA. It is the wrong way round. In the last example, the plus sign was here [*indicating placement after the first number*], and now it is here [*indicating placement before the last number*].

TEACHER. Do you think that we can write a number sentence like this?

MELISSA. I'm not sure we can. I don't think so.

TEACHER. How should this number sentence be so that it is correct?

MELISSA. We should put the plus sign here [*indicating placement after the first number*] and the equals sign here [*indicating placement before the sum*]. It would be 4 plus 3 equals 7.

Melissa had used a strategy of reading backward.

Her answers when she had to assess other number sentences showed that she accepted only $a + b = c$ number sentences.

TEACHER [*showing 3 + 4 = 6 + 1*]. Can you tell me whether this number sentence is correct or incorrect?

MELISSA. It is wrong.

TEACHER. Why do you think it is wrong?

MELISSA. Because it is not equal here [*showing the position of the equals sign*].

TEACHER. Why can't you put the equals sign here?

MELISSA. Because, when you add up, you have to .... There is something wrong here, because 4 plus 3 equals 7, not 1.

This example shows that it was important to Melissa that the equals sign was followed immediately by the sum of the numbers preceding it; she would not accept that the equals sign was followed by another sum.

Similar difficulties appeared when Melissa was asked to complete a number sentence. When I asked her to add the correct number to the number sentence 6 + 2 = __ + 3, Melissa stated that 8 must be the missing number.

TEACHER. Why do you think that 8 is the missing number?

MELISSA: Because 6 plus 2 equals 8. It is 6 plus 1, and 1 again, which adds up to 8.

This understanding of the equals sign as an operator (which must be followed by the answer to a question—the operation that precedes it) was not isolated to Melissa. All the students I interviewed displayed, in various degrees, a similar understanding of the equals sign.

The research literature confirms that this understanding of the equals sign is very common (Carpenter and Levi 2000; Sáenz-Ludlow and Walgamuth 1998). While this conception of the equals sign allows students to solve number sentences like 2 + 5 = __, significant difficulties arise when they have to assess number sentences like 8 = 7 + 1 or complete equations like __ = 2 + 3. Students are often unable to find the correct answer, or they choose to read the number sentence backward. That strategy, while efficient for number sentences involving addition, will cause significant problems with number sentences containing subtraction. Because subtraction is not commutative, backward reading will lead to a wrong answer.

Difficulties also arise when students are asked to complete equations like 3 + 5 = __ + 2 (Falkner, Levi and Carpenter 1999; Sáenz-Ludlow and Walgamuth 1998; Shoecraft 1989). A common error of students who view the equals sign as an operator would be to designate 8 (the sum of the numbers preceding the equals sign) as the unknown number. This type of error is not only common with Grade 1 students but also frequently found with older students. In a research study involving 752 elementary school students, the success rate for both Grade 1 and Grade 6 students who were asked to complete 8 + 4 = __ + 5 was below 10 per cent (Falkner, Levi and Carpenter 1999).

The conception of the equals sign as an operator also plays an important role in learning algebra. As Bodin and Capponi (1996) point out, this conception has been clearly identified as a main obstacle in the transition from arithmetic to algebraic thinking.

Which Classroom Strategies Allow Children to Develop a More Accurate Understanding of the Equals Sign?

In the past, several researchers have tried to find strategies that would allow students to develop a better understanding of the equals sign. Their results have been mixed. For instance, 30 years ago, Denmark, Barco and Voran (1976) proposed a balance model that Grade 1 students could use to illustrate various number sentences. However, their research was inconclusive, and they decided that Grade 1 students are simply too young to conceive of the equals sign as an indicator of a relationship.

However, the failure of Denmark, Barco and Voran's (1976) strategy can be explained by the type of number sentence a balance model can illustrate. Generally, a number sentence can be represented in at least two ways. I will use the number sentence $2 + 5 = 7$ to explain the differences between the types of representation.

In one situation, someone has two marbles in the left hand and five marbles in the right hand—seven marbles altogether. In this situation, the seven marbles do not exist independently from the two marbles and the five marbles; they represent the sum of the two subgroups. Later, I will refer to this situation as an inclusive representation.

In another situation, someone has two green marbles and five red marbles, and another person has seven marbles. Both people have the same number of marbles. I will refer to this situation as a comparative representation, because in the solid representation, the seven marbles are not physically the same as the two marbles and the five marbles.

A balance model can be used to illustrate only a comparative representation, because the seven marbles are distinct from the two marbles and the five marbles when the two sides of the balance are stable. However, students' first experiences with addition and number sentences refer much more often to an inclusive situation, where they are attempting to find the sum of the two addends. Denmark, Barco and Voran's (1976) use of a comparative representation, which does not correspond to students' previous experiences with addition, could explain, at least partly, why the exclusive use of a balance model did not help the Grade 1 children in their study to understand the equals sign as an indicator of a relationship.

More recently, Carpenter, Franke and Levi (2003) experimented with different methods of challenging students' conceptions of the equals sign. They found that using true-false number sentences was an effective way to change students' misconceptions. In their classroom throughout the year, they repeatedly engaged Grade 1 students in discussions about true-false number sentences by presenting a list of number sentences, some true and some false. Subgroups of students were organized for discussion purposes. The classroom discussions focused on the children's justifications for considering a number sentence to be true or false. The researchers also encouraged students to use words that expressed the equality relation more directly. For instance, the statement "8 is the same amount as 5 plus 3" gives a clearer description of the underlying relationship than does "8 equals 5 plus 3." This approach allowed most Grade 1 students to use the equals sign appropriately by the end of the year (Carpenter and Levi 2000). However, the researchers point out that the students' understanding was fragile; therefore, it is important to work on understanding of the equals sign regularly.

A Sequence of Activities to Help Children Develop a Better Understanding of the Equals Sign

Several principles guided the development of a sequence of activities presented to the children in my teaching experiment.

First, my interventions during the sequence consisted mainly of questioning the children's strategies. The only information I explicitly gave them was an explanation, during the first activity of the sequence, of the equals sign as a symbol that says there is the same amount on both sides.[1] I referred to this explanation several times during the sequence, but I intentionally did not show the students specific strategies that could influence the way they resolved the situations.

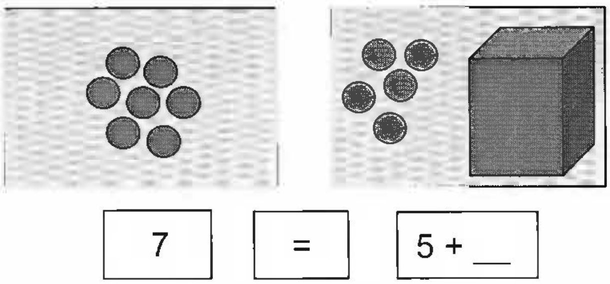
Second, the children were asked to work on two types of tasks throughout the teaching experiment. The first type of task asked them to determine whether a given number sentence was true or false. When they detected an error in a number sentence, students were asked to modify the sentence in such a way as to make it correct. In the second type of task, the children were asked to complete various equations.

Third, I used various types of equations and number sentences. At the beginning of my sequence of activities, we worked exclusively on $a + b = c$ and $a = b + c$ number sentences. Later, I introduced the $a + b = c + d$ structure, which is more difficult for children to understand.

Fourth, to help establish a link between mathematical symbolization and concrete representation, I introduced each number sentence or equation coupled with solid objects at the beginning of the work on each type of question. Those concrete representations were gradually withdrawn later in the sequence, with the aim of facilitating students' ability to work on number sentences and equations solely with mathematical symbols.

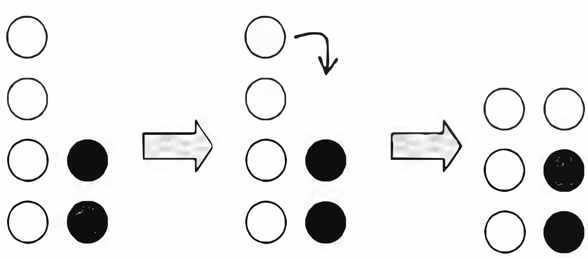
Fifth, the unknown numbers in the various equations were represented in two ways. In one situation, children had to add as many objects as necessary to a transparent plastic bag to make both collections equal. In another situation, the unknown number was represented by a nontransparent box that contained the correct number of objects. The children were told that the same number of objects were present on both sides and then asked to calculate how many objects were in the box (see Figure 1). This situation was more difficult for the students to work on, because they could not see or manipulate the solid objects that represented the unknown.

Figure 1



Finally, throughout the sequence, I alternated comparative and inclusive representations. Previously, I had mapped out the differences between the types of representation in $a + b = c$ number sentences. The same differences could apply to $a + b = c + d$ number sentences. For instance, a comparative representation of $4 + 2 = 3 + 3$ implies the presence of four red and two black marbles in one hand, and three black and three red marbles in the other hand. An inclusive representation of $4 + 2 = 3 + 3$ implies the transfer of one marble from the collection of four marbles to the collection of two marbles, as illustrated in Figure 2.

Figure 2



Study Results

Accurate Understanding for Some Students

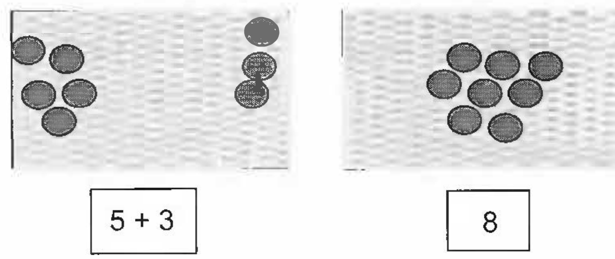
Melissa, like many other participants, made significant progress in understanding the equals sign during the teaching experiment. At the end of the study, she was able to correctly complete $a + b = \_\_ + d$ number sentences. Nevertheless, understanding the equals sign as an indicator of a relationship was a challenge for her. In this section, I will explore Melissa's difficulty accepting a new meaning of the equals sign at the beginning of the study. I will then describe my perceptions of her understanding of the equals sign during the posttest, which shows that she made significant progress during the teaching experiment.

Accepting a New Meaning of the Equals Sign

The aim of the first activity of the sequence was to introduce the children to the meaning of the equals sign as an indication of the same quantity on both sides.

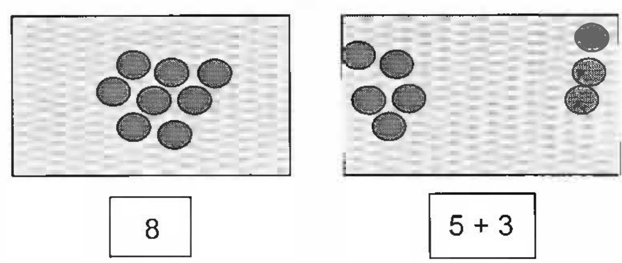
I first presented to the students a comparative representation of the number sentence $5 + 3 = 8$ (see Figure 3).

Figure 3



The students then had to discover whether the same quantities were represented in both situations, and whether the equals sign could be used between $5 + 3$ and 8. The situations were then inverted, to get the number sentence $8 = 5 + 3$, as shown in Figure 4.

Figure 4



The children were then asked whether there was still the same number of counters on both sides and whether it was appropriate to use the equals sign between 8 and $5 + 3$. At first, Melissa was not sure if she should use the equals sign in this situation.

TEACHER. Do you think that you can use the equals sign now?

MELISSA. I think so.

TEACHER. Can you read the number sentence aloud?

MELISSA. Can I read this way? [*She indicates a progression from the right to the left.*]

TEACHER. You can do it the way you think it should be done.

MELISSA. You have to turn this around, put the 5 plus 3 here [*indicating placement before the equals sign*] and the 8 here [*indicating placement after the equals sign*].

At that point, I explained to Melissa that the equals sign means that there is the same quantity on both sides of the sign, but Melissa was unwilling to adopt this new meaning: "I don't think that it is the way you just explained it. If you have 8 here [before the equals sign], it doesn't work. We haven't learned the equals sign, but I think it means ... I don't remember what it means." Her strong resistance to changing her conception of the equals sign was a first indicator of her difficulty accepting the equals sign as an indicator of a relationship.

Accurate Understanding at the End of the Sequence

After seven half-hour sessions of working with Melissa on the described tasks related to the meaning of the equals sign, I conducted a posttest interview aimed at illustrating her understanding. During the posttest, Melissa's answers clearly showed that she now understood the equals sign as an indicator of the same quantity on both sides of the sign. For instance, when I asked her to assess whether the number sentence 4 + 2 = 6 + 1 was correct, her answer revealed that she had made significant progress in her understanding of the equals sign:

TEACHER [*showing 4 + 2 = 6 + 1*]. Can you tell me whether this number sentence is right or wrong?
MELISSA. Do I have to say whether it is the same thing?
TEACHER. I want to know whether the number sentence is right or wrong.[2]
MELISSA. It is wrong, because 4 plus 2 equals 6, and there is an equals sign that tells us that it is the same thing, but after the equals sign, it is 6 plus 1, which equals 7, not 6.
TEACHER. How could you modify this number sentence to make it a correct one?
MELISSA. [*She replaces the 1 with a 0.*]

During the posttest, Melissa was also able to complete number sentences correctly, without having to use a concrete representation.

TEACHER [*showing 7 + 1 = __ + 2*]. Can you tell me what number you have to write in the box to make this a correct number sentence?
MELISSA [*after thinking awhile*]. It must be 6.
TEACHER. Why do you think it should be 6?
MELISSA. Because I thought 8 minus 2, which equals 6.
TEACHER. Can you read me the number sentence now?
MELISSA. 7 plus 1 equals 6 plus 2.

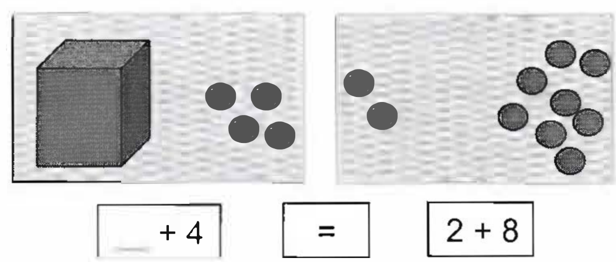
More Difficult Progress for Other Students

Although it seems that Melissa was able to develop a more coherent understanding of the equals sign as an indicator of a relationship, progress was more difficult for other children. This section describes some of the difficulties Mathieu and Caroline experienced during our research.

Mathieu was initially perceived by the teacher as one of the strongest students in the class. During the pretest, his understanding of the equals sign was similar to Melissa's. By the end of the study he was, like Melissa, able to correctly assess $a + b = c + d$ number sentences and to complete $a + b =$ __ $+ d$ number sentences. However, as the following examples illustrate, Mathieu's conception of the equals sign was much more fragile than Melissa's.

Throughout the teaching experiment, Mathieu showed a tendency to return to his conception of the equals sign as an operator, even if in other situations he considered it an indicator of a relationship. For example, during the last session Mathieu had to complete the number sentence __ + 4 = 2 + 8. This number sentence is illustrated by Figure 5, in which the nontransparent box contains six objects.

Figure 5



Mathieu had to determine how many objects were in the box if the use of the equals sign between __ + 4 and 2 + 8 was to be possible.

TEACHER. If we can use the equals sign here, how many objects should be in the box?
MATHIEU. There must be 4, because 4 plus 4 equals 8.
TEACHER. Can you read the number sentence?
MATHIEU. 4 plus 4 equals 2 plus 8.

Mathieu had transformed the number sentence into an $a + b = c$ structure, which allowed him to think, once again, in terms of a question–answer pattern.

Mathieu also showed a tendency, especially during the early sessions, to read certain number sentences backward. For instance, when asked to complete the number sentence 7 = 2 + __, he first thought that 9 was the missing number. Then, reading backward seemed to be an adequate strategy for him. "Can I read in the other direction [from right to left], too?" When I asked him about the reasons for this change of direction, he referred explicitly to an understanding of the equals sign as an operator: "You always put the operation first and the result after."

During the posttest, even if Mathieu was generally able to use the equals sign as an indicator of a relationship, he referred to the equals sign as an operator on one occasion, at the beginning of the posttest, when he insisted on reading the number sentence 8 = 4 + 4 backward.

TEACHER [*showing 8 = 4 + 4*]. Can you tell me whether this number sentence is right or wrong?

MATHIEU. It is correct, because 4 plus 4 equals 8.

TEACHER. Can you read the number sentence?

MATHIEU. 8 plus … no, this doesn't work. But you always have to begin on the side where the window is. 4 plus 4 equals 8.

TEACHER. Why do you read this way?

MATHIEU. You always read this way. You always start on the side where the window is.

In the room where I conducted the posttest, the window was to Mathieu's right, whereas the window was to his left in the classroom. This change was sufficient to encourage Mathieu to read backward, indicating the fragility of his conception of the equals sign as an indicator of a relationship.

Caroline, another student on whom I conducted an in-depth analysis, was perceived by her teacher as having major difficulties in school, particularly in mathematics. During the pretest interview, she displayed an understanding of the equals sign similar to that of Melissa and Mathieu. However, unlike the other two students, Caroline had major difficulties during the teaching experiment in developing a coherent understanding of the equals sign as an indicator of a relationship. She returned to her understanding of the equals sign as an operator on numerous occasions and was often unable to manipulate the concrete representation appropriately. At the beginning of the posttest interview, she seemed convinced that the equals sign is used as an operator.

TEACHER [*showing 8 = 4 + 4*]. Can you tell me whether this number sentence is right or wrong?

CAROLINE. It is wrong.

TEACHER. Why do you think it is wrong?

CAROLINE. It is the wrong way around.

TEACHER. How do you think it should be?

CAROLINE. These [*indicating 4 + 4*] should be at the beginning, then the equals sign, and finally the answer.

However, later in the interview she seemed to remember that the equals sign is an indicator of a relationship.

TEACHER [*showing 4 + 2 = 6 + 1*]. Do you think that this number sentence is right or wrong?

CAROLINE. It is correct.

TEACHER. Why do you think it is correct?

CAROLINE. No, it is wrong, because it is not the same. The additions are not the same. In the first one, it is 4 plus 2, and 6 plus 1 equals 7.

TEACHER. So, do you think that this number sentence is right or wrong?

CAROLINE. It is wrong, because there is not the same amount on both sides.

From this moment on in the posttest, Caroline's answers were coherent with an understanding of the equals sign as an indicator of a relationship. However, the fact that she considered the equals sign as an operator at the beginning of the posttest interview clearly indicates that her understanding of the equals sign was fragile.

Discussion

Several conclusions may be drawn from my research.

First, if the equals sign is not taught explicitly, children will likely develop a conception of the equals sign as an operator. In the class in which I conducted my research, the equals sign had not been explicitly investigated by the teacher, and all the students initially believed that they had to write an answer after the equals sign. This finding is consistent with other research on the understanding of the equals sign: several other researchers have confirmed children's common conception of the equals sign as an operator. Furthermore, this conception is held not only by Grade 1 students but also by much older students. Carpenter, Franke and Levi (2003) support the necessity of explicitly teaching the equals sign as an indicator of a relationship, because developing an understanding of the equals sign is not simply a process of maturation but, rather, must be addressed more directly.

Also, it seems realistic to allow students to change their conception of the equals sign under certain conditions. This idea is consistent with recent research (Carpenter and Levi 2000; Falkner, Levi and Carpenter 1999), which suggests that even Grade 1 students can develop a flexible understanding of the equals sign.

Even if it is possible to influence a change in students' conceptions of the equals sign, students are often reluctant to change and will try to stick with their initial conception. I have described strategies the participants in my research used, which can also be found in the literature. For example, Sáenz-Ludlow and Walgamuth (1998, 185) found that it is difficult to make children understand that the equals sign is an indicator of a relationship: "The dialogues and the arithmetical tasks on equality indicate these children's intellectual commitment, logical coherence and persistence to defend their thinking unless they were convinced otherwise." Carpenter, Franke and Levi (2003, 12) also point out the difficulty of changing students' conceptions of the equals sign:

> Children may cling tenaciously to the conceptions they have formed about how the equals sign should be used, and simply explaining the correct use of the symbol is not sufficient to convince most children to abandon their prior conceptions and adopt the accepted use of the equals sign.

Carpenter and Levi (2000) also observed that students' new understanding of the equals sign was not stable. After the researchers had investigated the equals sign with Grade 1 students, many of the children returned to their initial conceptions of the sign several months after the end of the teaching sessions. Therefore, Carpenter, Franke and Levi (2003) recommend a continuation of the use of nonconventional number sentences throughout the year. Sáenz-Ludlow and Walgamuth (1998) emphasize that developing an adequate understanding of the equals sign is a long-term process. It is therefore important to adopt a long-term approach in the classroom and to have children work repeatedly on the meaning of the equals sign.

Considering Grade 1 students' difficulties with the equals sign, one might wonder whether the introduction of the equals sign should be delayed. Several arguments support this proposition. As Dougherty (2004, 29) mentions, an early introduction of the equals sign means that teachers will need to undo childrens' misconceptions later: "In order for older children to solve equations with meaning, we have to first undo their idea about the equals sign before any approach to solving an equation makes sense."

However, is postponing the introduction of the equals sign a viable solution? Sáenz-Ludlow and Walgamuth's (1998) research seems to indicate that even when children are confronted later with the equals sign, they have difficulties understanding the symbol as an indicator of a relationship. In their research, the equals sign was taught to Grade 3 students who had not had to use the equals sign in previous tasks. During Grades 1 and 2, they had seen additions and subtractions written horizontally, without the equals sign. However, even those students tended to conceive of the equals sign as an operator. The learning of this symbol as an indicator of a relationship is therefore also a significant cognitive obstacle, and postponing the introduction of the equals sign is not sufficient to help students better understand its meaning. In this context, Dougherty (2004, 29) recommends helping children work on the equality relationship at a concrete level in measurement situations before switching to a numerical level:

> Showing that students can solve equations with different methods at an earlier age is encouraging. If they are capable of using these methods, even after coming from a strictly numerical perspective in their early beginnings in mathematics, what would be possible if students started with a focus on the structure of mathematics within a measurement context?

Furthermore, it is important to remember that children have probably already developed a conception of the equals sign, even before starting school. Many children's books, especially those that deal with counting, use the equals sign, so most children have already encountered the symbol.

There seems to be no easy solution in addressing young children's misunderstanding of the equals sign. However, the important thing to remember is that if the equals sign is not taught explicitly, children will develop a conception of this symbol as an operator—a conception that will be difficult to deconstruct later. On the other hand, the constant use of appropriate classroom activities that promote an accurate understanding of the equals sign seems to help even young children develop a better understanding of the symbol and the underlying relationships.

Notes

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1. Because the definition of the equals sign is a convention, it is impossible for children to simply discover its meaning, especially because they are already convinced that the equals sign is always followed by an answer to a question preceding it.

2. Here, I chose to repeat the question I had asked Melissa rather than answer her question. I did not want to influence her answer or give her hints about the meaning of the equals sign.

References

Bodin, A, and B Capponi. 1996. "Junior Secondary School Practices." In *International Handbook of Mathematics Education,* ed A J Bishop, K Clements, C Keitel, J Kilpatrick and C Laborde, 565–614. Dordrecht, Netherlands: Kluwer.

Carpenter, T P, M L Franke and L Levi. 2003. *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School.* Portsmouth, NH: Heinemann.

Carpenter, T P, and L Levi. 2000. *Developing Conceptions of Algebraic Reasoning in the Primary Grades.* Research Report 00-2. Madison, Wisc: National Center for Improving Student Learning and Achievement in Mathematics and Science. Also available at www.wcer.wisc.edu/NCISLA/Publications/reports/RR-002.PDF (accessed December 27, 2007).

Denmark, T, E Barco and J Voran. 1976. *Final Report: A Teaching Experiment on Equality.* Project for the Mathematical Development of Children (PMDC) Technical Report No 6. Tallahassee, Fla: Florida State University.

Dougherty, B. 2004. "Early Algebra: Perspectives and Assumptions." *For the Learning of Mathematics* 24, no 3: 28–30.

Falkner, K P, L Levi and T P Carpenter. 1999. "Children's Understanding of Equality: A Foundation for Algebra." *Teaching Children Mathematics* 6, no 4 (December): 232–36.

Sáenz-Ludlow, A, and C Walgamuth. 1998. "Third Graders' Interpretations of Equality and the Equal Symbol." *Educational Studies in Mathematics* 35, no 2 (February): 153–87.

Shoecraft, P. 1989. "'Equals' Means 'the Same As.'" *The Arithmetic Teacher* 36, no 8: 36–40.

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Learning Mathematics: A Change in Focus

P Janelle McFeetors

Over the last several decades, reforms made to classroom practice have significantly shifted teaching methods from teacher-centred to student-centred instruction (Chazan 2000; National Council of Teachers of Mathematics 2000). These reforms have brought about changes from direct instruction to inquiry models of teaching (Boldt and Levine 1999; Borasi 1992). Teachers have focused on transforming their teaching practice, noticing the shift in their role from delivering content to guiding students' exploration (Brown and Smith 1997; Verkaik and Ritsema 2006). Teachers take note of their effectiveness in the classroom by recognizing their students' decreased dependence on them during the learning process.

The shift in teaching focus has kept the content of mathematics in direct view. In exploring better teaching practice, reforms in mathematics education have focused on different methods of delivering or exploring mathematical content (Simon 1995; Ward 2001). Many mathematics courses, especially at the high school level, are overflowing with content; there seems to be an enormous challenge to teach all the outcomes in a course. Although teachers have shifted their teaching practice, they remain focused on getting through heavily weighted courses by addressing the content.

What if the reforms to mathematics education were intended to support more than just instructional change? What other types of change could improve mathematics education?

Mathematics Instruction: Making Space for Conversation

Three scenarios familiar to junior high and high school mathematics teachers follow. While reading the scenarios, consider the commonalities between the teachers' orientations toward the learning of mathematics.

In the first scenario, a mathematics student approaches a teacher at lunchtime for some extra help. The teacher willingly agrees, and sits down with the student. He asks the student what topic is troublesome and asks to see some of the student's individual practice. The teacher looks through the student's steps, asking probing questions. Carefully and thoughtfully, the teacher determines which step the student has completed incorrectly in each question, diagnosing the problem. Patiently, the teacher explains (perhaps in a different way from his explanation that day in class) how to perform the step, and he demonstrates the correct method for the student. The teacher encourages the student to perform the step several times, scaffolding each attempt, until the student begins to feel confident and the teacher feels that he has addressed the problem. The student leaves knowing how to correctly complete the step and finishes the individual practice that evening for homework.

In the second scenario, a mathematics teacher sits down to plan the next unit of instruction. She carefully reads the curriculum document and considers each specific learning outcome, breaking down each individual skill and concept the students are expected to learn. Considering the time allocated for the unit, she begins to sequence the learning outcomes. She notices that each day the students will be required to learn a new skill or concept. The pace seems fast, with so much content to cover, so she designs each lesson to include a large segment of direct instruction and some individual practice time in order to reach all the specific learning outcomes in a short amount of time. She feels confident that with the ordering of the learning outcomes and her ability to explain the connections between each lesson, in addition to the students' practice of the skills, the students will be able to perform the necessary skills by the end of the unit.

In the third scenario, the time for a reporting period has arrived. A mathematics teacher is preparing for parent–teacher interviews after providing marks and comments on each student's report card. The comments have been selected from a collection of prewritten phrases. For each student, the teacher considers thoroughly each assessment task that has been collected. He notes the mathematics skills each student has developed during the term, as well as skills that need improvement. The comment "Needs to improve basic fraction calculations" seems appropriate for one student. In approaching the parent–teacher interview, the teacher creates a plan to help the student get better at using fractions that includes a weekly tutorial in the mathematics lab and extra practice. The interview is successful, and the student and her parents appreciate the teacher's suggestions and are willing to implement the plan to see improvement.

What do these three scenarios have in common? Perhaps you noted the positive tone of each situation, with a resolution that gives the teacher and the student confidence in the experiences in the mathematics classroom. Perhaps you noted that each student attends a mathematics class in which every day offers opportunities to learn new mathematical skills and concepts with a teacher who is thoughtful about her or his practice. Perhaps you noted that each teacher is concerned with the student as an individual mathematics student. Finally, perhaps you noted that scenarios like these often occur in the mathematics classroom.

However, there is an additional commonality between these three scenarios that, when explored, could lead to further reform in the mathematics classroom. Perhaps asking the question in a different way will uncover this similarity. In each scenario, what is the *focus* of the teacher's efforts? In the first scenario, the teacher focuses on the mathematics content with which the student is having difficulty; the extra help session focuses on the individual step the student missed and corrects only that step. In the second scenario, the teacher focuses on the mathematical concepts in the unit and the mathematical skills the students should acquire during that unit. In the third scenario, the comment the teacher selects for the report card focuses on a mathematical skill the student needs, and the conversation at the parent–teacher interview is concerned with that mathematical content.

In each scenario, the teacher is focusing on the mathematical content that is in view. However, could each student's experiences in mathematics class be enhanced through a change in focus—a focus on the student's acts of learning? During the extra help session, how could the student have benefited from a conversation about how he was learning during the teacher's explanation in class, or how he was learning from his individual practice? Within the unit of instruction, how could a student's learning be improved by conversations through assessment tasks that focus on how the student learned a particular skill? How could the discourse at the parent–teacher interview have been more effective for the student if it had included a conversation about how to learn to perform arithmetic operations on fractions, or the general learning processes best for that student?

Incorporating conversations about how students are learning, including discourse about how to improve learning, is important in supporting students' successful mathematical learning. These learning-based conversations require a shift from a focus solely on mathematics content to a focus on learning that uses the mathematical content as a vehicle for the learning-based conversations.

This article will introduce the concept of metalearning as a focus in the secondary mathematics classroom. Metalearning moves beyond a teacher's focus on mathematics (learning) and promotes student engagement in conversations about their own learning, with the intent of improving their learning (of mathematics). After describing the classroom ethos that encourages teacher–student conversations about learning, I will discuss how assessment practices can incorporate metalearning as an additional purpose for such tasks.

Metalearning: Learning as a Focus of Conversation

Metalearning is a term that describes students' thinking about their learning processes (Jackson 2004). *Learning processes* refers to ways in which students come to know and understand, as well as their positioning with authority, their beliefs about the reception or construction of knowledge, and other related actions (Baxter Magolda 1992; Belenky et al 1986; Chickering and Reisser 1993). Engaging in thinking about learning is a higher level of cognition that invites those in a classroom to analyze and make meaning of the way in which they are students and learners in the classroom. This higher level of cognition would be situated within a higher order of thinking in the classroom, such as Marzano and Pickering's (1997) "habits of mind" or fifth dimension of learning. When students engage in metalearning, they move beyond learning content to critically view their learning.

Metalearning occurs both while students are learning and, more commonly, as they look back on the learning they have done. When teachers invite their students to engage in metalearning, they begin a conversation (Gordon Calvert 2001) with students about what they are learning and how they are learning. This gives teachers opportunities to assess what new concepts and skills students have learned and the way in which they have learned them. An important element of metalearning is the feedback students receive from teachers. Feedback, often in written form, is given to students when they have completed a metalearning activity. Teachers model, through feedback, how to think in a metalearning way, in an effort to extend students' learning about learning (Norton, Owens and Clark 2004). Thus, the act of metalearning is both assessment *of* learning and assessment *for* learning.

Six purposes can be identified for inviting students to engage in metalearning. First, metalearning brings learning into focus in the classroom setting, rather than focusing on specific content. This shifts the valuation of knowledge and understanding to encompass goals broader than merely learning specific mathematical outcomes.

Second, because metalearning seeks to help teachers and students understand how and what students learn, it addresses the multiple forms of assessment (assessment *of* learning, assessment *for* learning and assessment *as* learning) recently incorporated into provincial assessment frameworks (Alberta Assessment Consortium 2005; Manitoba Education, Citizenship and Youth 2006).

Third, when individual students become analytic about their learning, they can learn how to get better at their learning processes. They are encouraged not only to describe their learning processes but also to consider how they might make those processes more effective. The development of effective learning processes is critical to lifelong learning.

Fourth, metalearning focuses on the learning processes of the individual, allowing for differentiation of thinking and learning that is appropriate and effective for each student in the classroom (Manitoba Education and Youth 1996; Tomlinson 1999).

Fifth, if students become aware of how they learn, especially in different settings and with different focuses (for example, conceptual understanding or skill development), they can improve their learning of specific mathematical content.

Sixth, metalearning changes the didactic contract (Herbst and Kilpatrick 1999) and the asymmetrical power relationship inherent in many classrooms. The shift in power relations occurs within the context of metalearning because the learners become experts on their own learning, and the teacher acts as a guide to prompt metalearning awareness and the growth of learning processes. The asymmetrical power relationship between students and the teacher is minimized because the teacher is learning about the students' metalearning alongside the students (Freire 2000). Together, in mutuality, they are engaging in discourse about each student's learning.

Some similarities can be seen between metacognition and metalearning. Both occur on a higher level of cognition within the classroom, and both invite individuals to be aware of and analytical about the processes in which they engage daily in the classroom. As they engage in both metacognitive and metalearning thought, they have opportunities to get better at related processes. Both also aim at achieving learning and thinking goals that are broader than specific subject content outcomes. However, an important distinction must be drawn between metacognition and metalearning. Metacognition focuses solely on the *thinking* of students in the classroom (Schoenfeld 1987). Metalearning cannot be a subelement of metacognition, because the purposes and stances addressed in metacognition and metalearning are quite different. Rather than considering only cognitive processes, metalearning also takes into account the individual's relationship to others and to school as an institution (McLaren 1994).

Listening: The Foundation for Conversation

Conversations about student learning and general learning processes can take place in any mathematics classroom, but a certain classroom dynamic must be established. In a student-centred classroom, students and their teacher have already developed power relationships that encourage mutual exploration of mathematical content. In this classroom, learning about mathematics alongside one another is a common experience; space for conversation about mathematical ideas already exists. Mathematical experiences of this type can be extended to create similar metalearning experiences.

Engaging students in metalearning takes place within a community that has cultivated authentic discourse between students and the teacher. The community of learners in the classroom is built on respect and mutuality. Metalearning situates itself within a pedagogical relationship characterized as teacher-with-learner and learner-with-teacher. The teacher is "as much a participant as a person who leads the students, yet retains the responsibility for the learning, the teaching, and the environment in the classroom" (Romano 2000, 59). The community of learners forms because of the members' willingness to learn something together, affecting each other's sense making in a particular context (Craig 1995). The community is forged through caring relationships (Noddings 1984) and through directed interactions between the teacher and each learner that encourage dialogue about metacognition, learning and self-awareness.

Forging a community of learners is challenging. Although it is not dependent on methodology or strategy, listening to students is central. Listening can be enacted in several ways in the classroom, depending on the roles and the relationship of the students to the teacher and the school system. At times, students are listened to as students; the listening focuses on how students interact with the schooling system, but they are not seen as particular individuals and learners. At times, students are listened to as cognizers; the listening focuses on the way in which students are thinking about mathematical content. However, the listening central to metalearning can be referred to as *authentic listening*, where each student is listened to as a particular learner in the classroom. The teacher forges a pedagogical relationship with each student; listening to the whole individual, the teacher comes to know each as a student, a learner and a human being. Van Manen (1986, 17) describes this listening as hearing the student "as a whole human being involved in self-formative growth."

Authentic listening is critical in conversations about learning. These conversations can take place in a variety of contexts in a mathematics classroom—through classroom observations, instructional moments, classroom interactions and specific assessment tasks. An informed conversation begins with the teacher's classroom observations (one method of listening) of each student's learning processes. The conversations can be supported, and often prompted, by instructional moments—where the teacher takes time in class to discuss a particular learning strategy and why it is effective. In initiating a learning-based conversation, the teacher signifies to the students a change from a focus on mathematical content to a focus that encompasses learning processes. Whole-class discussions can lead to conversations with individual students, and the strategy can be differentiated. Through one-on-one conversations in class or in assessment items collected from each student, the teacher can authentically listen to each student.

The following section explores how teachers can incorporate metalearning in existing assessment tasks.

Assessment: The Opportunities for Conversation

Listening to students not only fosters the success of all learners but also is an effective way to assess student learning. A change in focus allows teachers to attend to the learning students have done and the ways in which they learn. Teachers can gain a much richer picture of student learning and can assess what has been learned, what areas need improvement and how to support improvement. Through thinking and writing about their learning, students engage in self-assessment of learning and learning processes (not just self-assessment of content). In conversations with students, teachers can model how to assess learning and how to build on previous learning.

Assessment tasks that incorporate a written element can be effective for engaging students in conversations about their learning. The writing occurs in a conversational style between the teacher and the student. This can be considered a conversation because sustained interactions are encouraged through the assessment tasks. The focus is on how the students are learning and what the students are learning; students respond to prompts that support their thinking and writing about their learning. Metalearning is part of assessment in a classroom because both the students and the teacher are observing what the students have learned. Additionally, incorporating metalearning elements into existing assessment items, instead of increasing the number of assignments taken in by a teacher, makes the task less onerous for the teacher.

Metalearning developed in my practice through my use of learning-based prompts in students' assessment pieces in a variety of secondary mathematics courses (including pure math, applied math, and Math 14 and 24). I conducted a research study (McFeetors 2003) in a Grade 10 consumer mathematics course (for course information, see Manitoba Education, Training and Youth 2002), a course similar to Math 14. I found that written assessment tasks helped me learn more about my students' learning processes, and my students became increasingly adept at writing about their learning. I used a variety of assessment forms to encourage a focus on learning; however, I found that a simple adaptation of journal writing became a foundational element for engaging in metalearning with my students.

The role of journal writing can be expanded from merely explaining mathematical skills and concepts to prompting students to write about what they have learned and their learning processes. This shift signifies a change in focus and intention for student writing, bringing each student's learning into direct view. Student writing about learning, with responses from the teacher, is interpersonal in nature because it allows the learner and the teacher to come to understand each other's views—listening and relating. The student and the teacher can have a dialogue about the student's progress, creating opportunities to recognize and celebrate successful learning. Further, student writing about learning helps the teacher recognize what can be done to promote each student's success in the mathematics classroom. Recognizing each learner as a person builds an environment that invites each to become the best learner, student and human being he or she can be.

Engaging in conversations about learning is a new focus for many students in the mathematics classroom. Because this is an emergent process that students will not initially engage in independently, teachers should scaffold opportunities for students to think about their learning and their learning processes. Let's further explore journal writing as an example. When students submit their journals, the teacher can write back to each student, instead of marking each journal with a numerical score. In the written response, the teacher can interact with the student's ideas, model metacognitive and metalearning thinking, and encourage the student to think more deeply about her or his learning. This is called interactive writing (Mason and McFeetors 2002). Interactive writing allows the teacher to have a conversation with each student, at the most appropriate place in that student's learning and thinking. It also allows the teacher to come to understand what and how each student is learning, supporting the change in focus from mathematics to learning. See Figure 1 for suggested prompts focusing on student learning.

Interactive writing is only one of a variety of activities and assessment tasks that can be used to engage students in metalearning in the mathematics classroom. Consider assessment tasks that you already use

Figure 1

Learning-Focused Prompts

General Prompts Focused on Learning

- What do you want to learn in this course? Describe two things you will do to be successful in your learning goals.
- Think back to math class last year. What were two things you did that supported your learning?
- Again, think back to math class last year. What were two things you did that did not support your learning? How are you going to improve this year?
- How did you study for the math test yesterday? (Be specific.)
- Describe how you do your math homework every evening (how often, what you do and so on).
- Pick two items that can be shared with your parents at parent–teacher interviews. Describe why you picked those particular items. (In other words, what do they show about your learning?)
- Describe your day-to-day study habits for math during this last unit (include actions in class and at home). Can you see a connection between your study habits and your learning and test mark? What can you do to improve?
- Tell me about the hardest test question that you did well on. What made the question challenging? How did you get it in the end?
- Look back at your goals and strategies. Then take a look at your report card mark. Tell me about your progress. Talk about whether you are using your strategies. Do you need to set a new goal or use different strategies?
- Check off what you have completed from the list of assignments. How does the completion match up to how you did on the test? Which topic(s) do you need to work on? How will you improve?
- Create two report card comments for yourself. Explain why they are appropriate.

Homework Prompts Focused on Learning

- One thing I now know how to do because I did my homework is …
- One thing I learned from doing my homework was …
- I got stuck on question __. To get unstuck, I …
- One thing I still do not understand how to do is … To improve, I will …
- A question I now understand after homework question time is … What I now understand is …
- For me, the purpose of homework is to …

in your classroom that could be adapted or amended to encourage students to intentionally think about their learning processes. A change in focus, from considering mathematics content to incorporating conversations about learning, will enable students to become better learners of mathematics and lifelong learners.

References

Alberta Assessment Consortium (AAC). 2005. *A Framework for Student Assessment.* 2nd ed. Edmonton, Alta: AAC.

Baxter Magolda, M B. 1992. *Knowing and Reasoning in College: Gender-Related Patterns in Students' Intellectual Development.* San Francisco: Jossey-Bass.

Belenky, M F, B M Clinchy, N R Goldberger and J M Tarule. 1986. *Women's Ways of Knowing: The Development of Self, Voice, and Mind.* New York: Basic Books.

Boldt, G, and A Levine. 1999. "Adventures in Mathematical Inquiry." In *Developing Mathematical Reasoning in Grades K–12,* 1999 yearbook, ed L V Stiff and F R Curcio, 234–46. Reston, Va: National Council of Teachers of Mathematics.

Borasi, R. 1992. *Learning Mathematics Through Inquiry.* Portsmouth, NH: Heinemann.

Brown, C A, and M S Smith. 1997. "Supporting the Development of Mathematical Pedagogy." *Mathematics Teacher* 90, no 2 (February): 138–43.

Chazan, D. 2000. *Beyond Formulas in Mathematics and Teaching: Dynamics of the High School Algebra Classroom.* New York: Teachers College Press.

Chickering, A W, and L Reisser. 1993. *Education and Identity.* 2nd ed. San Francisco: Jossey-Bass.

Craig, C J. 1995. "Knowledge Communities: A Way of Making Sense of How Beginning Teachers Come to Know in Their Professional Knowledge Contexts." *Curriculum Inquiry* 25, no 2 (Summer): 151–75.

Freire, P. 2000. *Pedagogy of the Oppressed.* 30th anniversary ed, with an introduction by D Macedo. Trans M B Ramos. New York: Continuum.

Gordon Calvert, L M. 2001. *Mathematical Conversations Within the Practice of Mathematics.* New York: Lang.

Herbst, P, and J Kilpatrick. 1999. "Pour lire Brousseau." *For the Learning of Mathematics* 19, no 1 (March): 3–10.

Jackson, N. 2004. "Developing the Concept of Metalearning." *Innovations in Education and Teaching International* 41, no 4 (November): 391–403.

Manitoba Education and Youth. 1996. *Success for All Learners: A Handbook on Differentiating Instruction.* Winnipeg, Man: Manitoba Education and Youth.

Manitoba Education, Citizenship and Youth. 2006. *Rethinking Classroom Assessment with Purpose in Mind: Assessment for Learning, Assessment as Learning, Assessment of Learning.* Winnipeg, Man: Manitoba Education, Citizenship and Youth. Also available at www.edu.gov.mb.ca/k12/assess/wncp/index.html (accessed January 2, 2008).

Manitoba Education, Training and Youth. 2002. *Senior 2 Consumer Mathematics: A Foundation for Implementation.* Winnipeg, Man: Manitoba Education, Training and Youth. Also available at www.edu.gov.mb.ca/k12/cur/math/s2_cons_found/index.html (accessed January 2, 2008).

Marzano, R J, and D J Pickering. 1997. *Dimensions of Learning Teacher's Manual.* 2nd ed. Alexandria, Va: Association for Supervision and Curriculum Development.

Mason, R T, and P J McFeetors. 2002. "Interactive Writing in Mathematics Class: Getting Started." *Mathematics Teacher* 95, no 7 (October): 532–36.

McFeetors, P J. 2003. "Voices Inside the Classroom: Stories of Becoming in Mathematics." Master's thesis, University of Manitoba.

McLaren, P. 1994. *Life in Schools: An Introduction to Critical Pedagogy in the Foundations of Education.* 2nd ed. Toronto: Irwin.

National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics.* Reston, Va: NCTM.

Noddings, N. 1984. *Caring: A Feminine Approach to Ethics and Moral Education.* Berkeley, Calif: University of California Press.

Norton, L S, T Owens and L Clark. 2004. "Analysing Metalearning in First-Year Undergraduates Through Their Reflective Discussions and Writing." *Innovations in Education and Teaching International* 41, no 4 (November): 423–41.

Romano, R M. 2000. *Forging an Educative Community: The Wisdom of Love, the Power of Understanding, and the Terror of It All.* New York: Lang.

Schoenfeld, A H. 1987. "What's All the Fuss About Metacognition?" In *Cognitive Science and Mathematics Education,* ed A H Schoenfeld, 189–215. Hillsdale, NJ: Lawrence Erlbaum Associates.

Simon, M A. 1995. "Reconstructing Mathematics Pedagogy from a Constructivist Perspective." *Journal for Research in Mathematics Education* 26, no 2 (March): 114–45.

Tomlinson, C A. 1999. *The Differentiated Classroom: Responding to the Needs of All Learners.* Alexandria, Va: Association for Supervision and Curriculum Development.

Van Manen, M. 1986. *The Tone of Teaching.* Richmond Hill, Ont: Scholastic.

Verkaik, M, and B Ritsema. 2006. "Professional Development as a Catalyst for Classroom Change." In *Teachers Engaged in Research: Inquiry into Mathematics Classrooms, Grades 9–12,* ed L R Van Zoest, 253–71. Greenwich, Conn : IAP.

Ward, C D. 2001. "Under Construction: On Becoming a Constructivist in View of the Standards." *Mathematics Teacher* 94, no 2 (February): 94–96.

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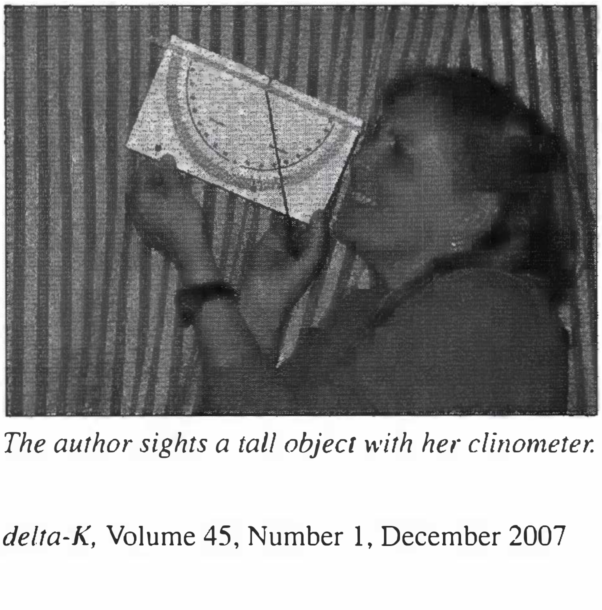
Taking Measurement Mathematics to New Heights with Student-Made Clinometers

Sherry Talbot

The use of tools in teaching and learning measurement allows students to experience mathematics in a direct and personal way. Tools help students connect diagrams and problems to the world around them and to their own experience. By the time they reach secondary school, students have a wealth of experience with rulers and measuring sticks, and they are beginning to look at how to use degrees to measure angles and construct polygons. We can build on these experiences by having students construct triangles to solve problems and use a clinometer, an instrument for determining angles of inclination or slope, to determine the heights of objects too tall to be measured with a measuring stick.

Building and using a clinometer to measure angles of elevation and depression helps students develop their understanding of length, height, degrees, right triangles, ratio and proportion, estimation, accuracy, trigonometry, and elevation and depression. The problems that students develop have solutions that help them determine whether their calculations are correct and accurate. Because they have constructed the problems themselves, students often gain a sense of efficacy and trust in their ability to work with mathematics.



The author sights a tall object with her clinometer.

Constructing and Using a Clinometer

My first experience with having students make clinometers was decades ago, when I was a student teacher. We used a copy of a protractor to make the clinometers. It worked, but the students had to add their readings to or subtract them from 90°.

More recently, I used Google Image Search and found several images that were much better. The one I chose, which works beautifully, can be found at www.learner.org/channel/workshops/lala/clinometer.html.

Students will have to work in pairs to use the clinometers, but they will benefit from each making a clinometer of their own.

To make the clinometer, each student will need a copy of the clinometer image (a half-circle with degrees marked on it), a piece of cardboard, a piece of string and a weight (a coin, pebble or washer will do) to make a plumb line, a straw for a sighting device, and tape and glue.

First, students will glue the half-circle to the cardboard.

Next, they will tape the string and the weight to the top centre (where the dot is). The plumb line should fall right across the 0 mark when the clinometer is held perpendicular to the ground.

Finally, the students will tape or glue the straw (the sighting device) right on the diameter of the clinometer.

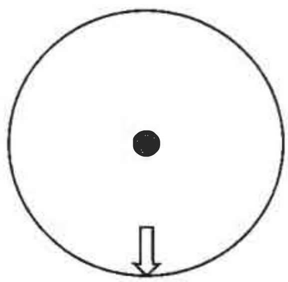
Voila! The clinometer is ready to be used for sighting angles.

Using a clinometer is a collaborative endeavour. One person sights the object through the straw, and the other reads the degrees of elevation or depression indicated by the plumb line. Suddenly, mathematical problems spring off the page and into students' lived experience.

Constructing and Using a Trundle Wheel

A complementary tool students can construct to help them measure triangles is a trundle wheel (see Figure 1). After measuring a variety of circles and determining π (circumference divided by diameter), students can construct a cardboard circle with a circumference of exactly 1 m. A pencil through the centre of the circle acts as an axle, and an arrow drawn on the circle indicates the starting point. The students then have a trundle wheel to measure distances along the ground.

Figure 1

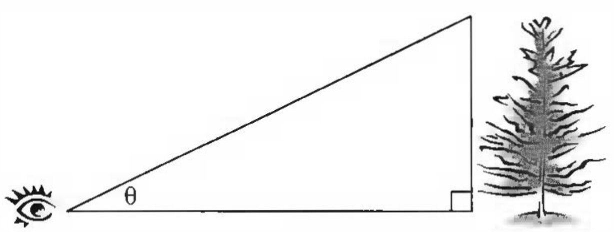


Now the students are equipped with tools to find and measure all kinds of triangles in their surroundings.

Finding the Heights of Objects Too Tall to Measure

Students who have not yet encountered trigonometry can use similar triangles and proportions to determine the heights of tall objects, such as flagpoles, tall buildings, trees and monuments (see Figure 2).

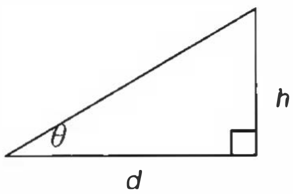
Figure 2



To use similar triangles, the students need to have a right triangle for which they know the measurements. In the classroom, gym or corridor, mark a spot on the wall at a specified height, h (3 m or higher is best). Put a measuring tape, or masking tape with metres marked out, on the floor from directly below the mark on the wall.

In pairs, students then choose a distance, d, and measure their angle, θ, by sighting the mark on the wall with the clinometer. Now they have a triangle for which they know the length of the two arms adjacent to the right angle, as well as the angle of inclination. Although they could determine all six angles and arm measurements, they need only d, h and θ (see Figure 3). This becomes the triangle that will be similar to all the triangles the student will measure outside the classroom.

Figure 3



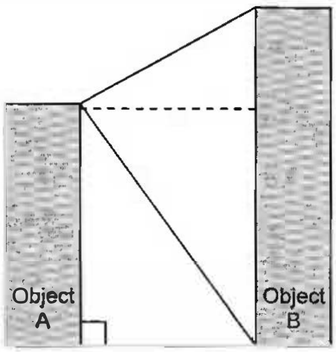
To make a large triangle similar to this reference triangle, students must sight the top of a tall object through the straw on the clinometer and then move back and forth until the angle of inclination is exactly θ. From that spot, they must run the trundle wheel to the base of the object to determine d. Once the students draw the triangle for the tall object beside the drawing of their reference triangle, they can see that their triangles are similar and that they can use ratios and proportions to calculate the height of the tall object.

When students find the length of the vertical arm of the right triangle, they are finding the height of the object from eye level. Therefore, the calculation of the object's total height is not complete. The height of the clinometer at the time of the sighting must be added to the calculated height of the object. Once students realize this, they must determine a way to do it. Some students measure the height of their eyes, and others lie on the ground to take their readings. One of the cleverest solutions I have seen was to use a metre stick as a stand for the clinometer; then, the students only had to add 1 m to the calculated measurement to find the total height.

As students move on to trigonometry, they no longer need to use a reference triangle. Once students sight the top of an object to get θ and measure d from the sighting spot, they need only to apply the tangent ratio to find the height. The challenge, then, is to determine the heights of objects using angles of depression, or a combination of elevation and depression. To do this, students can stand on a balcony or in front of a window on a floor above ground level. A building with an atrium also offers an opportunity to make measurements with elevation and depression.

Let's look at Figure 4. If students were able to measure the distance between Object A and Object B, and to take readings of elevation and depression with a clinometer from the top of Object A, many interesting problems involving angles and heights would arise. They would be able to work with the sum of angles in triangles and quadrilaterals, parallel lines, distances, the Pythagorean theorem and trigonometric ratios.

Figure 4



Experiences with Measurement Mathematics

Using a clinometer involves students in mathematics by situating them in measurement with their movements. Students not only become physically involved but also estimate, use mental math, communicate with their peers, make mathematics connections and use tools they have constructed themselves. In my experience, a wide range of students have done much mathematics with curiosity and a positive attitude while using these tools in problem-solving projects.

Although the mathematics that students use when working with clinometers and trundle wheels can be quite sophisticated, the tools themselves and the context of the problems are concrete, allowing students to get a personal feel for the mathematics.

The clinometer gives students direct experience with moving their heads to make different angles, connecting that movement with the number of degrees representing the angle measurement. When students draw the angle in a diagram, they can see how the problem looks on paper and relate it to what they did with the clinometer.

While using a trundle wheel to measure distances, students begin to make sense of large measures of length. These actions help them to visualize problems that they encounter, making connections between words, numbers and experiences.

These moving and feeling experiences give variety to students' connections to mathematics. For students who learn better through direct experience, this involvement can provide an entry point for learning mathematics. And diversity of experience can help all students remember mathematics more effectively.

Because working with clinometers requires students to work together to take measurements, students talk about what they are doing. They discuss whether they need to be more precise or if they may be doing something wrong. They use terminology that helps them work with and understand measurement and geometry problems. If one person makes a mistake, the other may be able to recognize and help rectify it.

Once the students have taken measurements, they must draw diagrams and solve the measurement problems. In doing so, they are writing about their experiences, describing them through mathematics. They are expressing through symbols what they have discussed and felt. While working together to draw their diagrams, discuss the process of solving the problems and do the calculations, students have opportunities to interact in a constructive mathematical fashion. At this stage, if students have made mistakes, they can recognize that there must be something wrong with their measurements or processes. If their result contradicts their sense of size (such as a school roof that is 2 m high or a flagpole that is 200 m high), they are able to see for themselves that they need to remeasure or recalculate.

Using clinometers gives students an opportunity to get out of the classroom to work with mathematics. It is a mathematical occasion when students can put their minds and bodies—and some simple tools—to work in producing and solving interesting measurement and geometry problems.

Sherry Talbot has taught mathematics to students in Grades 6–12 in Edmonton and at international schools in Munich and London. She is completing her master's degree in secondary mathematics education at the University of Alberta. Her research interests lie in looking at how students collaborate to learn mathematics. Over her many years of teaching, she has developed several projects that allow students to work together to build their mathematical understanding.

Alberta High School Mathematics Competition 2006/07

Andy Liu

The Alberta High School Mathematics Competition (AHSMC) is a two-part competition taking place in November and February of each school year. Book prizes are awarded for Part I (multiple-choice questions), and cash prizes and scholarships for Part II (extended-response questions).

The AHSMC is sponsored and helped by the following:

- ConocoPhillips of Canada
- Peter H Denham Memorial Fund
- A K Peters, Ltd
- Greenwoods' Bookshoppe
- Canadian Mathematical Society
- Mathematics Council of the Alberta Teachers' Association
- Pacific Institute for the Mathematical Sciences
- University of Calgary
- University of Alberta

For more information about the competition, visit www.math.ualberta.ca/~ahsmc/.

Report on Part I

Individual Results

Part I of the 51st AHSMC was written on November 21, 2006, by 731 students (290 girls and 441 boys). The number of students in Grades 8, 9, 10, 11 and 12 were, respectively, 1, 23, 192, 292 and 223.

Here are the top individual students:

Rank	Score	Student	School
1	100	Jarno Sun	Western Canada High School, Calgary
		Boris Braverman	Sir Winston Churchill High School, Calgary
		Jeffrey Mo	William Aberhart High School, Calgary
4	95	Jerry Lo	Ross Sheppard High School, Edmonton
5	91	Hunter Spink	Calgary Science School, Calgary
6	90	Danny Shi	Sir Winston Churchill High School, Calgary
		Gary Yang	Sir Winston Churchill High School, Calgary
		Andrew Zheng	Western Canada High School, Calgary
9	88	Wen Wang	Western Canada High School, Calgary
10	86	Yu Xiang Liu	Western Canada High School, Calgary
		Michael Wong	Tempo School, Edmonton
12	85	Melissa Chung	Harry Ainlay High School, Edmonton
		Linda Zhang	Western Canada High School, Calgary
14	84	Di Mo	Queen Elizabeth Junior/Senior High School, Calgary
15	83	William Wong	Ross Sheppard High School, Edmonton
16	81	Cindy Qian	Harry Ainlay High School, Edmonton
		Alex Sampaleanu	Saint Francis High School, Calgary
		David Ting	William Aberhart High School, Calgary
19	80	Philip Hogg	William Aberhart High School, Calgary
		Ye Jay	Henry Wise Wood High School, Calgary
		Yu Liu	Western Canada High School, Calgary
22	79	David Szepesvari	Harry Ainlay High School, Edmonton
		Lillian Wang	Western Canada High School, Calgary
		Kyle Boone	Western Canada High School, Calgary
		Victor Feng	Sir Winston Churchill High School, Calgary
		Victor Zheng	Harry Ainlay High School, Edmonton
27	78	Alex Chen	Sir Winston Churchill High School, Calgary
		Cindy Liu	Old Scona Academic High School, Edmonton
29	77	Li Han	Western Canada High School, Calgary
		Hyunbin Jeong	Old Scona Academic High School, Edmonton
31	76	Dustin Styner	Queen Elizabeth Junior/Senior High School, Calgary
		Kevin Trieu	Ross Sheppard High School, Edmonton
		Prudence Wu	William Aberhart High School, Calgary
34	75	Brett Baek	Western Canada High School, Calgary
		Patrick Pringle	Crowsnest Consolidated High School, Coleman
		Jie Yu	Western Canada High School, Calgary
37	74	Sophia Zhang	Western Canada High School, Calgary
		Stephen Portillo	Old Scona Academic High School, Edmonton
		Simon Sun	Sir Winston Churchill High School, Calgary
40	73	Tom Liu	Western Canada High School, Calgary
		Shervin Ghafouri	Western Canada High School, Calgary
		Matthew He	Western Canada High School, Calgary
		Graham Hill	Sir Winston Churchill High School, Calgary
		Sumayr Sekhon	Tempo School, Edmonton
45	72	Edward Choi	Sir Winston Churchill High School, Calgary
		David Liu	Winston Churchill High School, Lethbridge
		Darren Xu	Sir Winston Churchill High School, Calgary
		Jason Shin	William Aberhart High School, Calgary
		Travis Woodward	Sir Winston Churchill High School, Calgary
		Linda Yu	Western Canada High School, Calgary

Team Results

The contest was written by students from 33 schools, four of which did not enter a team. There were twelve schools from Zone I (Calgary), with 434 students; seven schools from Zone II (Southern Alberta), with 54 students; nine schools from Zone III (Edmonton), with 128 students; and five schools from Zone IV (Northern Alberta), with 115 students.

Here are the top teams:

Rank	Score	School and Team Members	Manager
1	280	Sir Winston Churchill High School, Calgary, with Boris Braverman, Danny Shi and Gary Yang	Mr Patrick Ancelin
2	278	Western Canada High School, Calgary, with Jarno Sun, Andrew Zheng and Wen Wang	Ms Renata Delisle
3	261	William Aberhart High School, Calgary, with Jeffrey Mo, David Ting and Philip Hogg	Mr Jim Kotow
4	254	Ross Sheppard High School, Edmonton, with Jerry Lo, William Wong and Kevin Trieu	Mr Jeremy Klassen
5	245	Harry Ainlay High School, Edmonton, with Melissa Chung, Cindy Qian and David Szepesvari	Ms Jacqueline Coulas
6	231	Queen Elizabeth Junior/Senior High School, Calgary, with Di Mo, Dustin Styner and R Wang/M Wanless	Ms Sharon Reid
7	229	Old Scona Academic High School, Edmonton, with Cindy Liu, Hyunbin Jeong and Stephen Portillo	Mr Lorne Pasco
8	227	Tempo School, Edmonton, with Michael Wong, Sumayr Sekhon and Maninder Longowal	Mr Lorne Rusnell
9	215	Henry Wise Wood High School, Calgary, with Ye Jay, Javier Romualdez and Patricia Rohs	Mr Michael Retallack
10	204	Bishop Carroll High School, Calgary, with Sean Heisler, Connor Kjersteen and Allison Yuen	Ms Susan Osterkamp

The other participating schools (and team managers) were as follows:

Zone I

- Calgary Science School, Calgary (Ms Martina Metz)
- Central Memorial High School, Calgary (Mr Gerald Krabbe)
- John G Diefenbaker High School, Calgary (Mr Terry Loschuk)
- John Ware School, Calgary (Ms Gail Slen)
- Rundle College High School, Calgary (Ms Rachel Hinz)
- Saint Francis High School, Calgary (Ms Allison van de Laak)

Zone II

- Canmore Collegiate High School, Canmore (Ms Patti Fairhart-Jones)
- Cardston High School, Cardston (Ms Debbie Fletcher)
- Crowsnest Consolidated High School, Coleman (Mr Bruce Kutcher)
- Foothills Composite High School, Okotoks (Ms Audra Schneider)
- Prairie Christian Academy, Three Hills (Mr Robert Hill)
- Strathcona-Tweedsmuir School, Okotoks (Ms Nola Adam)
- Winston Churchill High School, Lethbridge (Ms Terri Yamagashi)

Zone III

- Archbishop MacDonald High School, Edmonton (Mr John Campbell)
- Concordia High School, Edmonton (Ms Jenny Kim)
- Jasper Place High School, Edmonton (Ms Nadine Molnar)
- McNally High School, Edmonton (Mr Neil Peterson)
- St Francis Xavier High School, Edmonton (Ms Joanne Stepney)

Zone IV

- Archbishop Jordan Catholic High School, Sherwood Park (Ms Marge Hallonquist)
- École Secondaire Sainte Marguerite d'Youville, St Albert (Ms Lisa La Rose)
- Father Patrick Mercredi Community High School, Fort McMurray (Mr Ted Venne)
- Leduc Composite High School, Leduc (Ms Corlene Balding)
- Paul Kane High School, St Albert (Mr Percy Zalasky)

Report on Part II

Individual Results

Part II of the 51st AHSMC was written on February 7, 2007, by 65 students representing 13 schools. Here are the top performers:

Rank	Student	School
1	Jeffrey Mo	William Aberhart High School, Calgary
2	Jerry Lo	Ross Sheppard High School, Edmonton
3	Boris Braverman	Sir Winston Churchill High School, Calgary
4	Jarno Sun	Western Canada High School, Calgary
5	Linda Zhang	Western Canada High School, Calgary
6	Danny Shi	Sir Winston Churchill High School, Calgary
7	Tony Zhao	Sir Winston Churchill High School, Calgary
	Brett Baek	Western Canada High School, Calgary
9	Sherwin Ghafouri	Western Canada High School, Calgary
	Dustin Styner	Queen Elizabeth Junior/Senior High School, Calgary
11	Matthew Wang	Western Canada High School, Calgary
	Darren Xu	Sir Winston Churchill High School, Calgary
	Simon Sun	Sir Winston Churchill High School, Calgary
14	Kyle Boone	Western Canada High School, Calgary
	Annie Xu	Old Scona Academic High School, Edmonton
16	Yu Xiang Liu	Western Canada High School, Calgary
	Chong Shen	Sir Winston Churchill High School, Calgary
	Michael Wong	Tempo School, Edmonton
	Graham Hill	Sir Winston Churchill High School, Calgary
	Stephanie Li	Sir Winston Churchill High School, Calgary
	Cindy Qian Harry	Harry Ainlay High School, Edmonton
	David Ting	William Aberhart High School, Calgary

Congratulations to the above students, their schools and their teachers!

Problems and Solutions

Part I

1. The value of $2^4 4^8 8^{16}$ is
   (a) 2^{16}
   (b) 2^{32}
   (c) 2^{68}
   (d) 2^{84}
   (e) none of these

2. The number of noncongruent rectangles with integer sides and area $2006 = 2 \times 17 \times 59$ is
   (a) 3
   (b) 4
   (c) 6
   (d) 8
   (e) none of these

3. The number of pairs (m, n) of positive integers such that $m^4 + n = 100{,}000{,}001$ is
   (a) 100
   (b) 101
   (c) 200
   (d) 201
   (e) none of these

4. In a city, all streets run north–south or east–west, dividing the city into squares. A, B, C and D are four students who live at four street intersections that define a rectangle, with A and C at opposite corners of this rectangle. They all go to the same school, which is at some street intersection within this rectangle. Each goes to school by the most direct route along streets. A travels 10 blocks,

B travels 20 blocks and C travels 50 blocks. The number of blocks D travels is

(a) 10
(b) 20
(c) 30
(d) 40
(e) 50

5. When $(1 + x)(1 + x^2)(1 + x^3)(1 + x^4)(1 + x^5)$ is expanded, the coefficient of the term x^9 is

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

6. On a plane are two points, A and B, at a distance 5 apart. The number of straight lines in this plane that are at a distance 2 from A and at a distance 3 from B is

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

7. A country has three provinces, each province has three cities, each city has three wards and each ward has three electors. In a two-way election, a candidate wins a ward by getting more votes in the ward, wins a city by winning more wards in the city, wins a province by winning more cities in the province and wins the election by winning more provinces. Only electors may vote, and they must vote. The minimum number of votes needed to guarantee winning the election is

(a) 41
(b) 54
(c) 66
(d) 81
(e) none of these

8. The teacher asked, "What is the largest possible diameter of a circular coin of negligible thickness that may be stored in a rectangular box with inner dimensions $7 \times 8 \times 9$?" Ace said less than 8, Bea said 8, Cec said strictly between 8 and 9, Dee said 9 and Eve said more than 9. The one who was right was

(a) Ace
(b) Bea
(c) Cec
(d) Dee
(e) Eve

9. If a and b are two real numbers that satisfy $a + b - ab = 1$ and a is not an integer, then b

(a) is never an integer
(b) must be some positive integer
(c) must be some negative integer
(d) must be equal to 0
(e) may be either an integer or a noninteger

10. The nonzero numbers a, b, c, x, y and z are such that

$$\frac{x}{a} = \frac{y}{b} = \frac{z}{c}.$$

The value of

$$\frac{xyz(a+b)(b+c)(c+a)}{abc(x+y)(y+z)(z+x)}$$

is

(a) 0
(b) $\frac{1}{3}$
(c) 1
(d) 3
(e) dependent on the common value of $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$

11. If k is a rational number such that $\sqrt[3]{9\sqrt{3} - 11\sqrt{2}} = \sqrt{3} + k\sqrt{2}$, then the value of k is

(a) -2
(b) -1
(c) 1
(d) 2
(e) none of these

12. All positive integers that can be expressed as a sum of one or more different integer powers of 5 are written in increasing order. The first three terms of this sequence are 1, 5 and 6. The 50th term is

(a) 3751
(b) 3755
(c) 3756
(d) 3760
(e) 3761

13. Consider all polynomials whose coefficients are all integers, whose roots include $\sqrt{3}/2$ and $\sqrt{2}/3$, and whose degree is as small as possible. Among the coefficients of these polynomials, the smallest positive coefficient is

(a) 1
(b) 6
(c) 35
(d) 36
(e) none of these

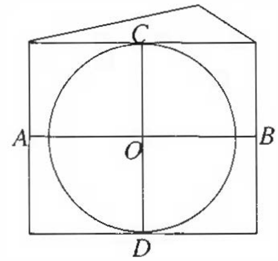
14. Colleen used a calculator to compute $(a + b)/c$, where a, b and c are positive integers. She pressed the buttons **a**, **+**, **b**, **/**, **c** and **=** (in that order) and got the answer 11. When she pressed **b**, **+**, **a**, **/**, **c** and **=** (in that order), she was surprised to get a different answer—14. Then she realized that the calculator performed the division before the addition. So she pressed **(**, **a**, **+**, **b**, **)**, **/**, **c** and **=** in that order. She finally got the correct answer, which is
   (a) 4
   (b) 5
   (c) 20
   (d) 25
   (e) none of these

15. The base of a tetrahedron is an equilateral triangle of side 1. The fourth vertex is at a distance 1 above the centre of the base. The radius of the sphere that passes through all four vertices of the tetrahedron is
   (a) $\frac{1}{3}$
   (b) $\frac{\sqrt{3}}{6}$
   (c) $\frac{1}{2}$
   (d) $\frac{\sqrt{3}}{3}$
   (e) $\frac{2}{3}$

16. If $p(x)$ is a polynomial of degree 4 such that $p(-1) = p(1) = 5$ and $p(-2) = p(0) = p(2) = 2$, then the maximum value of $p(x)$ is
   (a) 5
   (b) 6
   (c) 7
   (d) 10
   (e) none of these

Solutions

1. The value of $2^4 4^8 8^{16}$ is $2^{4+8\times2+16\times3} = 2^{68}$. The answer is **(c)**.
2. There are four such rectangles, namely 1×2006, 2×1003, 17×118 and 34×59. The answer is **(b)**.
3. Since $100^4 = 100{,}000{,}000$, (100, 1) is a desired pair. In fact, m can be any positive integer up to 100, and there is a unique positive integer n such that $m^4 + n = 100{,}000{,}001$ for this particular m. Hence, the total number of pairs is 100. The answer is **(a)**.
4. A and C together travel as many north–south blocks as B and D together. A and C together also travel as many east–west blocks as B and D together. Hence, A and C together travel as many blocks as B and D together. It follows that the number of blocks D travels is $10 + 50 - 20 = 40$. The answer is **(d)**.
5. We are counting the number of ways to make 9 from 1, 2, 3, 4 and 5, using each number no more than once. There are three ways, namely $1 + 3 + 5$, $2 + 3 + 4$ and $4 + 5$. The answer is **(d)**.
6. Draw a circle with centre A and radius 2. Draw another circle with centre B and radius 3. The lines we seek are the common tangents of these two circles, of which there are three. The answer is **(d)**.
7. We can steal the election by winning as few as 16 votes. We can win two provinces, two cities within each of those two provinces, two wards within each of those four cities and two electors within each of those eight wards. Hence, $(81 - 16) + 1 = 66$ votes are required to guarantee winning the election. The answer is **(c)**.
8. Let A and B be the respective midpoints of two opposite edges of length 9. Let C and D be the respective centres of the two 7×8 faces. (See the diagram below.) Then, AB and CD intersect at the centre O of the box. Place a coin of diameter 9 on the plane determined by AB and CD with centre O. Its circumference will pass through C and D, but A and B are not covered up since $AB = \sqrt{7^2 + 8^2} > 9$. Rotate the coin about AB so that it just comes off C and D. We can then expand the coin slightly and still have it fit inside the box. The answer is **(e)**. *Note: We do not know the actual maximum value. That is why the question is phrased in its current form.*



9. Solving for b, we have $b(1 - a) = 1 - a$. Since a is not an integer, $1 - a \neq 0$ and can be cancelled. Hence, $b = 1$ and is a positive integer. The answer is **(b)**.
10. Let

$$\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = r.$$

Then,

$$\frac{xyz}{abc} = r^3,\ x = ra,\ y = rb \text{ and } z = rc.$$

It follows that $x + y = r(a + b)$, $y + z = r(b + c)$ and $z + x = r(c + a)$, so that

$$\frac{x+y}{a+b}=\frac{y+z}{b+c}=\frac{z+x}{c+a}=r.$$

The given expression is equal to

$$r^3\left(\frac{1}{r}\right)^3=1.$$

The answer is **(c)**.

11. We have $9\sqrt{3}-11\sqrt{2}=\left(\sqrt{3}+k\sqrt{2}\right)^3=$ $3\sqrt{3}+9k\sqrt{2}+6k^2\sqrt{3}+2k^3\sqrt{2}$. From $9 = 3 + 6k^2$, we have $k = \pm1$. From $-11 = 9k + 2k^3$, $k < 0$. Hence, $k = -1$. The answer is **(b)**.

12. We change the base-10 number 50 to base-2 and obtain the number 110,010, representing $2^5 + 2^4 + 2^2$. We now interpret this as a base-5 number, representing $5^5 + 5^4 + 5^2$. Changing this number to base-10, we obtain 3755. The answer is **(b)**.

13. Since $\sqrt{2}/3$ and $\sqrt{3}/2$ are roots, we must also have $-\left(\sqrt{2}/3\right)$ and $-\left(\sqrt{3}/2\right)$ as roots in order to have rational coefficients. Thus, the degree of the polynomial cannot be less than 4, but 4 is sufficient. Of these polynomials, the one with 1 as the leading coefficient is

$$\left(x-\frac{\sqrt{2}}{3}\right)\left(x+\frac{\sqrt{2}}{3}\right)\left(x-\frac{\sqrt{3}}{2}\right)\left(x+\frac{\sqrt{3}}{2}\right)=\left(x^2-\frac{2}{9}\right)\left(x^2-\frac{3}{4}\right)$$

$$=x^4-\frac{35}{36}x^2+\frac{1}{6}.$$

Since we want integral coefficients, we clear the denominators to obtain $36x^4 - 35x^2 + 6$. Since the coefficients are relatively prime (though not pairwise relatively prime) and the smallest absolute value among the coefficients is 6, the smallest positive coefficient is also 6. The answer is **(b)**.

14. We have

$$a+\frac{b}{c}=11 \text{ and } b+\frac{a}{c}=14.$$

Adding the two equations yields

$$(a+b)\frac{c+1}{c}=25 \text{ or } (a+b)(c+1)=25c.$$

Since $c + 1$ and c are relatively prime, $c + 1$ must divide 25. Hence, $c = 4$ or $c = 24$. If $c = 24$, then $a + b = 25$, but at least one of a/c and b/c is not an integer. Hence, $c = 4$, $a + b = 20$ and $(a + b)/c = 5$. The answer is **(b)**.

15. Let the base of the pyramid be the equilateral triangle ABC. Let D be the midpoint of BC. Then,

$$AD=\sqrt{AB^2-BD^2}=\frac{\sqrt{3}}{2}.$$

Let G be the centre of ABC. Then, G lies on AD and

$$AG=\frac{2AD}{3}=\frac{\sqrt{3}}{3}.$$

Let V be the fourth vertex of the pyramid, and let O be the centre of the sphere passing through all four vertices. Then, O lies on VG, and let $r = VO = OA$ be the radius of the sphere. Since $VG-VO=OG=\sqrt{OA^2-AG^2}$, we have

$$1-r=\sqrt{r^2-\frac{1}{3}}.$$

Squaring both sides, we have $1 - 2r + r^2 = r^2 - \frac{1}{3}$. It follows that $r = \frac{2}{3}$. The answer is **(e)**.

16. Note that $p(-2) < p(-1) > p(0) < p(1) > p(2)$. Hence, the graph of this fourth-degree polynomial opens down. Let $p(x) = -x^4 + ax^3 + bx^2 + cx + d$. Then, $d = p(0) = 2$. From $p(\pm2) = 2$, we have $-16 \pm 8a + 4b \pm 2c + 2 = 2$. Hence, $4b - 16 = \pm(8a + 2c)$. This is only possible if both sides are equal to 0, so that $b = 4$. Similarly, from $p(\pm1) = 5$, we have $25a + 5c = 0$. Combining with $8a + 2c = 0$, we have $a = c = 0$, so that $p(x) = -x^4 + 4x^2 + 2 = 6 - (x^2 - 2)^2$. It follows that the maximum value of $p(x)$ is 6, occurring when $x^2 - 2 = 0$ or $x = \pm\sqrt{2}$. The answer is **(b)**.

Part II

Problem 1

Determine all positive integers n such that n is divisible by any positive integer m that satisfies $m^2 + 4 \leq n$.

Problem 2

The numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 and 15 are arranged to form a 5×3 table in each of the 15! possible ways. For each table, we compute the sum of the three numbers in each row, and record in a list the largest and the smallest of these sums. Determine the sum of the $2 \times 15!$ numbers on our list.

Problem 3

One angle of a triangle is 36° while each of the other two angles is also an integral number of degrees. The triangle can be divided into two isosceles triangles by a straight cut. Determine all possible values of the largest angle of this triangle.

Problem 4

Let a, b and c be distinct nonzero real numbers such that

$$\frac{1-a^3}{a}=\frac{1-b^3}{b}=\frac{1-c^3}{c}.$$

Determine all possible values of $a^3 + b^3 + c^3$.

Problem 5

A survey in Alberta was sent to some teachers and students, a total of $2006 = 2 \times 17 \times 59$ people. Exactly $a\%$ of the teachers and exactly $b\%$ of the students responded, yielding an overall response rate of exactly $c\%$, where a, b and c are integers satisfying $0 < a < c < b < 100$. For each possible combination of values of a, b and c, determine the total number of teachers and the total number of students who responded to the survey.

Solutions and Comments

Problem 1

This problem really consists of two parts, finding values for n and proving that there are no more. Most contestants got somewhere with the first part, but many faltered in the second.

For $n = 1, 2, 3$ and 4, there are no positive integers m such that $m^2 + 4 \le n$. Hence, these four values have the desired property vacuously. While not an essential part of the problem, these values should be included for completeness.

If the maximum value of m is 1, then $1^2 + 4 \le n < 2^2 + 4$ and $n = 5, 6$ or 7. Since 1 divides all of them, these three values have the desired property. If the maximum value of m is 2, then $2^2 + 4 \le n < 3^2 + 4$ and $n = 8, 9, 10, 11$ or 12. Of these, only 8, 10 and 12 are divisible by both 1 and 2. If the maximum value of m is 3, then $3^2 + 4 \le n < 4^2 + 4$ and $n = 13, 14, 15, 16, 17, 18$ or 19. Of these, only 18 is divisible by all of 1, 2 and 3. If the maximum value of m is 4, then $4^2 + 4 \le n < 5^2 + 4$ and $n = 20, 21, 22, 23, 24, 25, 26, 27$ or 28. Of these, only 24 is divisible by all of 1, 2, 3 and 4. It will turn out that no positive integers other than 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 18 and 24 have the desired property.

For the second part, **Jeffrey Mo,** of William Aberhart High School, argued as follows. Suppose the maximum value of m is k for some integer $k \ge 5$. Then, $k^2 + 4 \le n < (k + 1)^2 + 4$. In order for n to be divisible by just $k - 1$ and k, it has to be a multiple of $k(k - 1)$ since $k - 1$ and k are relatively prime. Now $k(k - 1) < k^2 + 4$, while $2k(k - 1) - [(k + 1)^2 + 4] = k^2 - 4k - 5 = (k + 1)(k - 5) \ge 0$ for $k \ge 5$. Hence, n cannot be a multiple of $k(k - 1)$, so that there are no solutions for $m \ge 5$.

Jerry Lo, of Ross Sheppard High School, argued as follows. Suppose we have solutions n for some $m \ge 5$. Then, n must be a multiple of m. Now $m^2 + 4 < m^2 + m < m^2 + 2m < (m + 1)^2 + 4 \le m^2 + 3m$, with equality holding in the last case only for $m = 5$. If $n = m^2 + 3m$, then we must have $m = 5$ so that $n = 40$, but 40 is not divisible by 3. Hence, $n = m(m + 1)$ or $m(m + 2)$. Note that n must also be a multiple of $m - 1$. Note that $m - 1$ and m are relatively prime. If $n = m(m + 1)$, then $m - 1$ must divide $m + 1 = (m - 1) + 2$. Hence, it must divide 2, so that $m \le 3$. If $n = m(m + 2)$, then $m - 1$ must divide $m + 2 = (m - 1) + 3$. Hence, it must divide 3, so that $m \le 4$. Either case contradicts $m \ge 5$. Hence, there are no solutions n for $m \ge 5$.

Problem 2

Far too many contestants did not know that the total number of tables is 15!. For those who did, most merely observed that the maximum sum of a row is $13 + 14 + 15 = 42$ and the minimum sum is $1 + 2 + 3 = 6$. From these, they concluded that the total of the two sums must be 48, in that if the maximum sum dropped, the minimum sum would rise and compensate. While this may be a loose description of what is the case, it does not explain why this is the case. The argument really rests on one simple fact. The following solution, by **Linda Zhang,** of Western Canada High School, is typical of those of the top contestants.

For each table A, there is a table B that may be obtained from A by subtracting each number in A from 16. Note that A and B are distinct tables. Now the row in A with the largest sum turns into the row in B with the smallest sum, and the row in A with the smallest sum turns into the row in B with the largest sum. The largest row sum of A plus the smallest row sum of B is 48, as is the largest row sum of B plus the smallest row sum of A. Since the 15! tables may be divided into 15!/2 such pairs, the sum of the $2 \times 15!$ numbers on our record is $48 \times 15!$.

Problem 3

This turned out to be the problem in which most contestants could make some progress. However, many approached it haphazardly and managed to find only some of the answers. Others found all the answers but did not prove that there are no more. The following is the solution by **Jarno Sun,** of Western Canada High School.

Let ABC be the triangle. Let $\angle ABC = 36°$. We may assume that $\angle CAB \ge \angle BCA$. Then,

$$\angle CAB \ge \frac{180° - 36°}{2} = 72° > \angle ABC.$$

In order for ABC to be divided into two triangles with a straight cut, the cut must pass through a vertex. We consider three cases:

CASE 1. The cut passes through B.

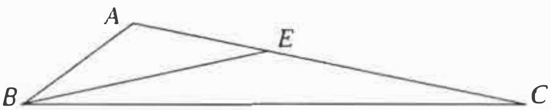
Let the cut meet CA at E. Since $\angle CAB > \angle ABC > \angle ABE$, $\angle BEA$ must be one of the equal angles in triangle BEA. It follows that $\angle BEA$ is acute so that $\angle BEC$ is obtuse. (See the diagram below.) Let $\angle EBC = \angle BCA = x°$. Then, $\angle BEA = 2x°$. We consider two subcases:

SUBCASE 1a. $\angle BEA = \angle CAB$.

Then, $\angle ABE = 180° - 4x°$ and $36° = \angle ABC = (180° - 4x°) + x°$. This yields $x = 48$, but then $\angle ABE = -12°$. This is impossible.

SUBCASE 1b. $\angle BEA = \angle ABE$.

Then, $\angle ABE = 2x°$ and $36° = \angle ABC = 2x° + x°$. This yields $x = 12$. It follows that ABC is a (132°, 36°, 12°) triangle.



CASE 2. The cut passes through A.

Let the cut meet BC at D. We consider three subcases:

SUBCASE 2a. $\angle BDA = \angle ABC = 36°$.

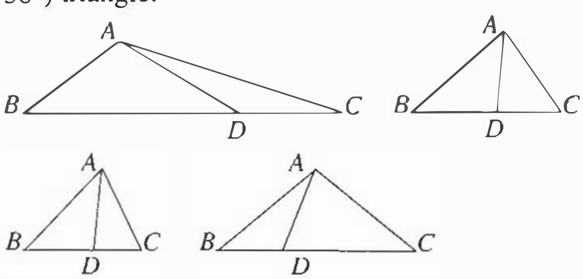
Then, $\angle ADC$ is obtuse. (See the first diagram below.) We must have $\angle BCA = \angle CAD = 36°/2 = 18°$, so that ABC is a (126°, 36°, 18°) triangle.

SUBCASE 2b. $\angle BAD = \angle ABC = 36°$.

Then, $\angle BDA = 108°$. If $AD = CD$, then $\angle DAC = \angle BCA = 108°/2 = 54°$ and $\angle CAB = 90°$. (See the second diagram below.) It follows that ABC is a (90°, 54°, 36°) triangle. If $AD = AC$, then $\angle BCA = \angle ADC = 72°$ and $\angle CAB = 180° - 36° - 72° = 72°$. (See the third diagram below.) It follows that ABC is a (72°, 72°, 36°) triangle. Finally, if $AC = CD$, then $\angle CAD = \angle ADC = 72°$. Hence, $\angle BCA = 36°$ and $\angle ABC = 108°$. (See the fourth diagram below.) It follows that ABC is a (108°, 36°, 36°) triangle.

SUBCASE 2c. $\angle BAD = \angle BDA = 72°$.

Then, $\angle ADC$ is obtuse. We must have $\angle BCA = \angle CAD = 72°/2 = 36°$, and ABC is again a (108°, 36°, 36°) triangle.



CASE 3. The cut passes through C.

Let the cut meet AB at F. Since $\angle CAB \geq \angle BCA > \angle ACF$, $\angle AFC$ must be one of the equal angles in triangle AFC. It follows that $\angle CFB$ is obtuse. It follows that $\angle BCF = \angle ABC = 36°$ and $\angle AFC = 72°$. Since $\angle CAB \geq \angle BCA$, ABC is again a (72°, 72°, 36°) triangle.

In summary, the largest angle of ABC, namely $\angle CAB$, may be 72°, 90°, 108°, 126° or 132°.

Problem 4

With greater reliance on graphing calculators and computer software, most students nowadays are uncomfortable with algebraic manipulations. A problem such as this has become inaccessible to most contestants. **Brett Baek,** of Western Canada High School, took the following approach.

From

$$\frac{1-a^3}{a} = \frac{1-b^3}{b},$$

we have $b - a^3b = a - ab^3$. Hence, $a - b = ab(b^2 - a^2)$ so that $ab(a + b) = -1$. Similarly, $bc(b + c) = -1$. From these two equations, we have $a^2 - c^2 = bc - ab$ or $(a + c)(a - c) = -b(a - c)$. Since $a \neq c$, we have $a + b + c = 0$. Hence, $abc(a + b) = -c = a + b$. Since $a + b + c = 0$ but $c \neq 0$, $a + b \neq 0$ and we have $abc = 1$. Now,

$$\begin{aligned} 0 &= (a + b + c)^3 \\ &= a^3 + 3a^2(b + c) + 3a(b + c)^2 + (b + c)^3 \\ &= (a^3 + b^3 + c^3) + 3(a + b + c)(bc + ca + ab) - 3abc \\ &= (a^3 + b^3 + c^3) + 0 - 3. \end{aligned}$$

It follows that the only possible value of $a^3 + b^3 + c^3$ is 3.

For those with more knowledge of algebra, this problem was practically trivial. **Jerry Lo** had the most succinct write-up.

The given conditions show that a, b and c are roots of the equation $x^3 + kx - 1 = 0$, where k is the common value of the three fractions. Hence,

$$\begin{aligned} x^3 + kx - 1 &= (x - a)(x - b)(x - c) \\ &= x^3 - (a + b + c)x^2 + (bc + ca + ab)x - abc. \end{aligned}$$

It follows that $a + b + c = 0$, $abc = 1$ and $a^3 + b^3 + c^3 = (a + b + c)(a^2 + b^2 + c^2 - bc - ca - ab) + 3abc = 3$.

Problem 5

This problem, which looks easy, is very annoying. Only one contestant gave a complete argument, and a handful of others came close. There is a relatively easy part of the problem that many contestants got—that is, showing that $c = 50$. Suppose z people in all responded to the survey. Then, $z/2006 = c/100$, or $50z = 1003c$. Since 50 and 1003 are relatively prime, c must be a multiple of 50. Since we are given that $0 < a < c < b < 100$, the only possible value is $c = 50$. After this, things get messy. What follows is the approach taken by **Jeffrey Mo**.

Let the total number of teachers be d and the number of those teachers who responded be x. Then, $x/d = a/100$ and $(1003 - x)/(2006 - d) = b/100$. From the first, we have $ad = 100x$. From the second, we have $2006b - bd = 100{,}300 - 100x = 100{,}300 - ad$. This may be rewritten as $(b - a)d = 2006(b - 50)$. It follows that $1003 = 17 \times 59$ divides $(b - a)d$. Now $b - a < 100 < 1003$, and we also have $d < 1003$ since $a < c = 50$. Hence, there are two cases.

CASE 1. 59 divides $b - a$.

This means, of course, that $b - a = 59$ and $d = 2 \times 17(b - 50)$. Hence, $50x = ad/2 = 17a(a + 9)$. Since 25 is relatively prime to 17 and to at least one of a and $a + 9$, it must divide either $a < 50$ or $a + 9 < 59$. We consider three subcases.

SUBCASE 1a. $a = 25$.

We have $b = 25 + 59 = 84$, $x = [17 \times 25(25 + 9)]/50 = 289$ and $d = 2 \times 17(84 - 50) = 1156$. Hence, $1003 - 289 = 714$ and $2006 - 1156 = 850$. It follows that 289 of 1156 teachers and 714 of 850 students responded to the survey.

SUBCASE 1b. $a + 9 = 25$.

We have $a = 25 - 9 = 16$, $b = 16 + 59 = 75$, $x = (17 \times 16 \times 25)/50 = 136$ and $d = 2 \times 17(75 - 50) = 850$. Hence, $1003 - 136 = 867$ and $2006 - 850 = 1156$. It follows that 136 of 850 teachers and 867 of 1156 students responded to the survey.

SUBCASE 1c. $a + 9 = 50$.

We have $a = 50 - 9 = 41$ and $b = 41 + 59 = 100$. This contradicts $b < 100$, and there are no solutions in this subcase.

CASE 2. 17 divides $b - a$.

Then, $b - a = 17n$, where $n \leq 5$. We have $nd = 2 \times 59(b - 50)$. Hence, $50nx = (and)/2 = 59a(17n - 50 + a)$.

We consider five subcases, none of which yield additional solutions.

SUBCASE 2a. $n = 1$.

We have $50x = 59a(a - 33)$, and 25 must divide either a or $a - 33$. The former means $a = 25$, but then $a - 33 < 0$. The latter means $a - 33 \geq 25$, but then $a \geq 58 > 50 = c$. Both lead to contradictions.

SUBCASE 2b. $n = 2$.

We have $100x = 59a(a - 16)$, and 25 must divide either a or $a - 16$. The former means that $a = 25$, but then $59a(a - 16)$ is odd. The latter means $a - 16 = 25$, but then $59a(a - 16)$ is again odd.

SUBCASE 2c. $n = 3$.

We have $150x = 59a(a + 1)$, and 25 must divide either a or $a + 1$. The former means that $a = 25$, but then $59a(a + 1)$ is not divisible by 3. The latter means $a + 1 = 25$ or 50. If $a = 49$, $59a(a + 1)$ is again not divisible by 3. If $a = 24$, then $x = 236$, but then $d = (100 \times 236)/24$ is not an integer.

SUBCASE 2d. $n = 4$.

We have $200x = 59a(a + 18)$, and 25 must divide either a or $a + 18$. The former means that $a = 25$, but then $59a(a + 18)$ is odd. The latter means $a + 18 = 25$ or 50. If $a = 7$, $59a(a + 18)$ is again odd. If $a = 32$, then $b = 32 + 68 = 100$, and this contradicts $b < 100$.

SUBCASE 2e. $n = 5$.

We have $250x = 59a(35 + a)$, and 25 must divide either a or $35 + a$. However, this means that $a \geq 25$ and $b = a + 85 > 100$, a contradiction.

A shorter approach goes as follows. Let s be the number of students and t be the number of teachers in the survey. Then, $s + t = 2006$, both $(ta)/100$ and $(sb)/100$ are integers, and

$$\frac{ta}{100} + \frac{sb}{100} = 1003.$$

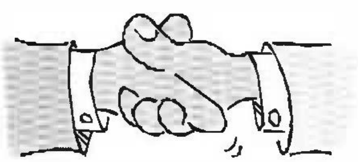
Note that 5 cannot divide both s and t. If 5 does not divide s, then b must be a multiple of 25. Since $50 < b < 100$, we must have $b = 75$. If 5 does not divide t, then a must be a multiple of 25. Since $0 < a < 50$, we must have $a = 25$. If 5 does not divide either s or t, then $a = 25$ and $b = 75$, and we have $t + 3s = 4012$. Subtract from this $s + t = 2006$, and we have $2s = 2006$, so that $s = t = 1003$. However, neither $(ta)/100$ nor $(sb)/100$ is an integer. Henceforth, we assume that 5 divides exactly one of s and t. We consider two cases. Suppose 5 divides t. Then, $b = 75$ and we have $ta + 75s = 2006 \times 50$. Subtracting this from $75t + 75s = 2006 \times 75$, we get $(75 - a)t = 2006 \times 25$. Since 5 divides t, $75 - a$ divides $2 \times 17 \times 59 \times 5$. Since $0 < a < 50$, $25 < 75 - a < 75$. Hence, we must have $75 - a = 34$ or 59. If $a = 41$, both t and a are odd, and $(ta)/100$ will not be an integer. If $a = 16$, we have $t = 850$. This leads to $s = 1156$. Thus, 867 students and 136 teachers responded to the survey, yielding a total of 1003, as required by $c = 50$. Finally, since $(100 - a)t + (100 - b)s = 100(s + t) - (at + bs) = 200{,}600 - 100{,}300 = 100{,}300$, the only other solution is $b = 100 - 16 = 84$ and $a = 100 - 75 = 25$, as indicated above since now 5 divides s instead of t. Solving $s + t = 2006$ and $84s + 25t = 100{,}300$, we have $s = 850$ and $t = 1156$. Thus, 714 students and 289 teachers responded to the survey, again yielding the desired total of 1003.

A Page of Problems

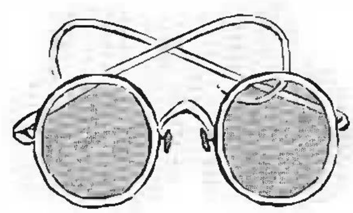
All the Same Down Deep

A Craig Loewen

At a special ceremony, every person present was asked to shake hands exactly once with each of the other people present. If there were 100 people at the ceremony, how many handshakes were there in all? How many people would have to be present to ensure that there were at least 10,000 handshakes?



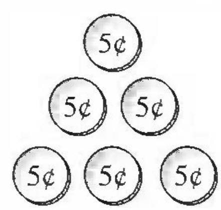
How many diagonals are there in a regular icosagon?



How many in a regular triacontagon?

Karl Friedrich Gauss (1777–1855) was a famous mathematician. He was good at mathematics even as a young child. One day his teacher was trying to keep him busy and assigned him the task of adding the numbers 1 through 100. It is said that he found the answer in under a minute! Can you?

Brad started arranging nickels in a triangular pattern as shown, using one nickel in the first row, two in the second row, three in the third row and so on.



He ran out of nickels when he completed the 30th row. How many nickels did he use? What was the value of his triangle?

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