Letter to the Editor

Dear Editor

I am aware that many of the teachers in your province are in the process of selecting textbooks to use when implementing the new curriculum. I have two issues with current textbooks that I feel teachers should be aware of: the eternal textbook error and the eternal textbook omission. I've described each of these below.

The Eternal Textbook Error

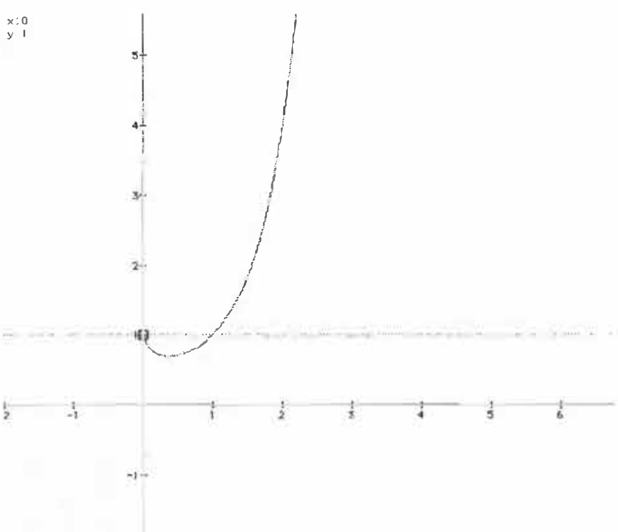
This statement, or a facsimile thereof, has appeared in every algebra textbook I have ever seen or used:

For any real number *a*, $a \neq 0, a^0 = 1$ (Any nonzero number raised to the zero power is 1.)

Consider the following function:

$$y = x^x$$
$$Ln\,y = xLn\,x$$
$$\textit{Limit as } x \Rightarrow 0,\ Ln\,y = 0$$
$$y = 1, \quad \therefore 0^0 = 1$$
$$\textit{Calculator test}:\ 0.000001^{0.000001} = 0.99999$$

While the statement is a fundamental error that may have minor consequences, it does not belong in a textbook on algebra. A graph of the function is shown below.



The Eternal Textbook Omission

It is a mystery why anyone would write a textbook on algebra and routinely fail to define the subject. So what is algebra? Algebra is the language of higher mathematics, but it is not just another language. Algebra is an abstract symbolic language plus reasoning; it is a language with the rules of logic built in. The structure of a language with the embedded rules of logic has the form *assumption–deduction–conclusion*. An equation in algebra is equivalent to a sentence in a written language that has a quantitative connotation; such sentences can be translated into equations. For example, the statement "John is five years older than Mary," translates into $J = 5 + M$, where the literals, J and M, are John and Mary's ages, the verb *is* translates to an equals sign (=), and the words *five years older* translate to 5 and the addition operator (5 +). Algebra is a tool for reasoning in that it enables the connection of one statement to another. The following statement provides the additional information needed to proceed: Two years ago, John was twice as old as Mary. Both statements can now be connected through a process of reformulation, and the unknown ages can be deduced from the one-to-one matching of terms.

First statement: $J = 5 + M >>$
$J - 2 = 5 + M - 2 + 2 - 2 >>$
$J - 2 = 7 + M - 4$

Second statement: $J - 2 = 2(M - 2) >>$
$J - 2 = M - 2 + M - 2 >>$
$J - 2 = M + M - 4$

The above method does not solve for the variable or use any abstract symbolic logic, but reformulates and compares the equations to deduce that Mary's age is 7, and John is 5 + 7 or 12. Two years ago John was 10 and Mary was 5; that is, John was twice as old as Mary.

The language analogy requires the following definitions: the set of real numbers is the *nouns* of the language. The set is infinite and unique in the sense that it is completely devoid of duplicate numbers. The set contains two numbers, 0 and 1, that have special properties that enable the process of reformulation.

The difficult, if not impossible, task of trying to define the numbers can be avoided by postulating that the numbers, whatever they are, satisfy certain basic assumptions or laws. These assumptions or laws are the foundation of algebra and are therefore the *grammar and syntax* of the language. Finally, the operators, $+, -, \times, \div, =, \sqrt{\ }$ and so on, tell us what action to take, so they play the role of *verbs* of the language.

Algebra is a human invention that deals with quantity (matter), space and time; like all languages, it is composed of concepts that attempt to capture the essence of reality. A concept is the basic element of thought; consequently, concepts such as the equator, the North Pole, the number 5 or any number do not exist in the physical reality. The most useful application of algebra is in the fundamental laws of nature. These laws can and must be expressed in abstract form so that the field of applicability is entirely unrestricted. In *The Mathematical Principles of Natural Philosophy*, by Isaac Newton, published by Cambridge University in 1686, the second law of motion is expressed in the form of analytical description and without any mathematical formulation: "The alteration of the quantity of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed." The same law in algebraic form: $f = \frac{d}{dt}(mv)$.

Science is analytical description—it is not mathematics; mathematics is a more precise and concise way of stating a principle. The equation states that the impressed force *(f)* is (=) ever proportional to the time rate of change (*d/dt*) in the linear momentum (*mv*). Newton's definition of momentum: "The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjunctly"; hence, the statement of the law and the algebraic equation have exactly the same meaning. Newton knew how to differentiate *(d/dt),* having invented fluxions (calculus) before his 25th birthday. Newton's book is completely devoid of his mathematical invention because he wanted people to understand what he wrote. Historically, this was the first book on physics that was correct; it was the catalyst for the Industrial Revolution.

I hope your readers will consider textbook omissions and errors when making important decisions about the textbooks they will use.

William G Mandras