Edmonton Junior High Mathematics Contest Winners

Susan Ludwig

Top Teams

First Place: Vernon Barford School—Kaiven Zhou, Joe Ou, Andrew Qi

Second Place: McKernan School—Jennifer Yu, Mariya Sardarli, Robert Luo

Third Place: Grandview Heights School—Lisa Wang, Qaasim Mian, Eric Xu, Stephanie Li

Top Three Individual Winners

First Place
Kaiven Zhou, Vernon Barford School
Jennifer Yu, McKernan School
Mariya Sardarli, McKernan School

Second Place
Joe Ou, Vernon Barford School

Third Place
Stephen Just, St Rose Junior High School

Edmonton Junior High Mathematics Contest 2008

Multiple-Choice Problems

1. The equation, shown below, which has **NO** solution is

A. $5x = 3x$

B. $x + 1 = x$

C. $\dfrac{x^2 - 1}{x - 1} = 0,\ x \neq 1$

D. $\dfrac{x + 1}{x} = 0,\ x \neq 0$

Solution B

Solution:
Subtracting x from both sides of the equation given in B gives $1 = 0$, which has NO solution in any set of numbers.

2. A quadrilateral drawn on the coordinate plane has the vertices R $(-4, 4)$, S $(3, 2)$, T $(3, -2)$ and U $(2, -3)$. The area of quadrilateral RSTU is

A. $49\frac{1}{2}$ units^2

B. $38\frac{1}{2}$ units^2

C. $38\frac{1}{2}$ units^2

D. $20\frac{1}{2}$ units^2

Solution D

Solution:
Area of surrounding square is 49 units^2
Subtract the areas of the triangles

$49 - 21 - 7 - \frac{1}{2} = 20\frac{1}{2}$ units^2

3. The Jones family averaged 90 km/h when they drove from Edmonton to their lake cottage. On the return trip, their average speed was only 75 km/h. Their average speed for the round trip is

A. 81.8 km/h

B. 82.5 km/h

C. impossible to determine because the distance from Edmonton to the cottage is not given

D. impossible to determine, because the driving time is not given

Solution A

Solution:

$d = st \qquad Average\ speed = \dfrac{\text{total distance}}{\text{total time}}$

$$Average\ time = \frac{2d}{t_{out} + t_{in}} = \frac{2d}{\frac{d}{90} + \frac{d}{75}} = \frac{2}{\frac{1}{90} + \frac{1}{75}} = 81.8 \text{ km/h}$$

4. The **four** answers shown below each contain 100 digits, with only the first 3 digits and the last 3 digits shown. The 100-digit number that could be a perfect square is

A. 512 ... 972
B. 493 ... 243
C. 793 ... 278
D. 815 ... 021

Solution D

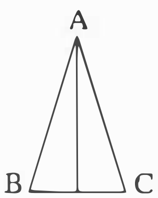
Solution:
By squaring each of the digits 0, 1, 2 ... 9 we see that squares cannot end in 8, 2, or 3.

5. Two sides of ΔABC each have a length of 20 cm and the third side has a length of 24 cm. The area of this triangle is

A. 192 cm^2
B. 173 cm^2
C. 141 cm^2
D. 72 cm^2

Solution A

Solution:



The triangle is isosceles; therefore the altitude drawn from A will bisect side BC at point D. The lengths of BD and CD are 12 cm. Apply Pythagorean Theorem to find the altitude.

$c^2 = a^2 + b^2$

$20^2 = 12^2 + b^2$

$256 = b^2$

$16 = b$

Calculate the Area $A = \frac{1}{2}ab$

$= \times 16 \times 24$

$= 192 cm^2$

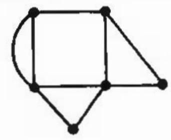
6. Only the **even** integers between 1 and 101 are written on identical cards, one integer per card. The cards are then placed in a box and mixed thoroughly. If a single card is drawn at random, then the probability that the number on the card is divisible by either 3 or 5, expressed as a decimal to the nearest hundredth, is

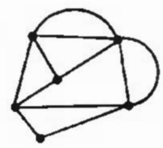
A. 0.50
B. 0.46
C 0.32
D. 0.20

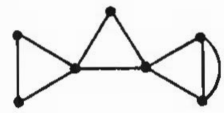
Solution B

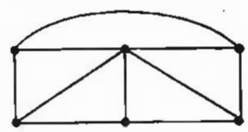
Solution:
The sequence 2, 4, 6, ..., 100 contains 50 terms, 23 of which are divisible by either 3 or 5.

7. Each of the following connected networks consists of segments and curves. A connected network is said to be *traversable* if we can trace the network without lifting our pencil from the paper. We must trace each of the segments or curves exactly once. Of the following connected networks, the one which is NOT traversable is

A.


B.


C.


D.


Solution: D

Solution:
A theorem of network theory is that if a connected network can be traversed, then it has at most two vertices of odd order. All of the networks above contain exactly two vertices of odd order, except for diagram D. The orders of the vertices in the diagrams (A though D) starting at top left and moving anticlockwise are

A. 3 – 4 – 2 – 4 – 2 – 3
B. 4 – 4 – 2 – 4 – 5 – 3
C. 2 – 2 – 4 – 2 – 4 – 3 – 3
D. 3 – 3 – 3 – 3 – 4

To traverse A, B or C, start at an odd vertex, and end at an odd vertex. Contest candidates may not know these theorems, but they should be able to traverse the networks by tracing.

8. The number of digits in the product $1^{2008} \times 25^{81} \times 2^{160}$ is

A. 2008
B. 2000
C. 162
D. 160

Solution C

Solution:

$1^{2008} \times 25^{81} \times 2^{160}$

$=1\times25\times25^{80}\times2^{80}\times2^{80}$

$=25\times(25\times2\times2)^{80}$

$=25\times100^{80}$

$=25\times10^{160}$ $\left(10^{160}\text{ is 161 digits}\right)$

$=2.5\times10^{161}$

9. Each side of equilateral triangle ABC measures 5 units. On the base BC, we draw point P. From point P, we draw perpendiculars to the other two sides. The sum of the lengths of the perpendiculars is

A. $\dfrac{5\sqrt{3}}{2}$

B. $\dfrac{5\sqrt{2}}{2}$

C. $5\sqrt{3}$

D. $5\sqrt{2}$

Solution A

Solution:
Draw in AP $= h$
The area of the equilateral triangle is $\frac{1}{2}(5)(h)$ where h is the height of the triangle, or the area can be expressed as $\frac{1}{2}(5)m+\frac{1}{2}n$, where m and n are the lengths of the constructed perpendiculars. These 2 expressions must be equal, so $\frac{1}{2}(5)(h)=\frac{1}{2}(5)m+\frac{1}{2}(5)n$

$h = m + n$

and $h=\dfrac{5\sqrt{3}}{2}$ by Pythagoras.

10. The sum of the ages of three brothers is 73. Tom is the oldest of the brothers, but he is less than 40 years old. The product of Tom's age and Michael's age is 750. The difference between Tom's age and Don's age is 7 more than the difference between Tom's age and Michael's age. Don is

A. 30 years old.
B. 21 years old.
C. 18 years old.
D. 8 years old.

Solution C

Solution:
Let T, D and M represent the age of Tom, Don and Michael respectively.

$$\begin{cases} T+D+M=73 \\ T<40 \\ TM=750 \\ T-D-7=T-M \end{cases}$$

is a system of equations that interprets the given information.
Solve the system to obtain T = 30, D = 18 and H = 25.
Don is 18 years old.

11. The 9-digit number 6❑8,351,962 is divisible by 3, where ❑ represents a missing digit. The remainder when this number is divided by 6 is

A. 3
B. 2
C. 1
D. 0

Solution D

Solution:
We are told that the number is divisible by 3; since it ends in 2, it is also divisible by 2. If divisible by 3 and 2, it is divisible by 6.

12. A set of six numbers has an average of 47. If a seventh number is included with the original six numbers, then the average is 52. The value of the seventh number is

A. 99
B. 82
C. 49.5
D 32.9

Solution B

Solution:

$\dfrac{282+n}{7}=52$

$282+n=364$

$n=82$

13. A set of N real numbers has an average of N. A set of M real numbers, where $M < N$, taken from the original set of N numbers has an average of M. The average of the remaining $N - M$ numbers is

A. M
B. N
C. N + M
D. N − M

Solution C

Solution:

Pick an arbitrary number for M and N.
Set N has 10 numbers with an average of 10; the sum of set N will be 100. Set M has 8 numbers with an average of 8; the sum of set M will be 64. There will be 2 numbers remaining with the sum of the set being 36. The average will be 18. 18 is 10 + 8, or $M + N$.

OR

Set N has N numbers with an average of N; the sum of set N will be N^2. Set M has M numbers with an average of M; the sum of set M will be M^2. The numbers remaining will be $N^2 - M^2$. The average will be $\frac{N^2 - M^2}{N - M} = N + M$.

14. A right-angled triangle has sides a, b and c, where c is the length of the hypotenuse. If we draw a line d from the right angle that is perpendicular to the hypotenuse, then an expression for d in terms of a, b and c is

A. $\frac{ab}{c}$
B. $\frac{bc}{a}$
C. $\frac{ac}{2b}$
D. $\frac{bc}{2a}$

Solution A

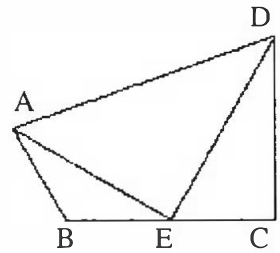
Solution:

Express the area of the right triangle in two ways:

$$\frac{1}{2}(a)(b) = \frac{1}{2}(d)(c)$$

$$d = \frac{ab}{c}$$

15. In the quadrilateral ABCD, AB = 1, BC = 2, CD = $\sqrt{3}$, $\angle$ABC = 120° and $\angle$BCD = 90°. E is the midpoint of BC. The perimeter of quadrilateral ABCD is



A. 5.16 units
B. 6.16 units
C. 6.38 units
D. 7.38 units

Solution D

Solution:

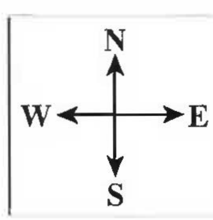
BE = 1 = EC since E is the midpoint of BC. A perpendicular from B to AE intersects AE at point F to create the 30° – 60° – 90° ΔBFE which gives EF = $\frac{\sqrt{3}}{2}$ and AE = $\sqrt{3}$. ΔECD is also 30° – 60° – 90° and since EC = 1 and CD = $\sqrt{3}$, we have ED = 2 by Pythagoras. In right triangle ΔAED, AD = $\sqrt{7}$ by Pythagoras. The perimeter of the quadrilateral is

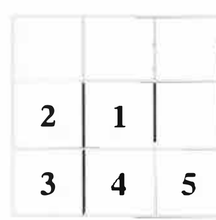
$$1 + 2 + \sqrt{3} + \sqrt{7}$$
$$= 3 + \sqrt{3} + \sqrt{7}$$
$$\approx 7.38$$

Answers-Only Problems

Problem 1

On the partial grid shown below, positive integers are written in the following pattern: start with 1 and put 2 to its west. Put 3 south of 2, 4 to the east of 3 and 5 to the east of 4. Now 6 goes directly north of 5 and 7 to the north of 6. Then 8, 9 and 10 follow in that order to the west of 7 and so on, always moving in a counter-clockwise direction.





We have made a south turn at 2, an east turn at 3, a north turn at 5, and a west turn at 7. At which integer will we be making the fifth west turn?

Solution: 111

49	48	47	46	45	44	43
26	25	24	23	22	21	42
27	10	9	8	7	20	41
28	11	2	1	6	19	40
29	12	3	4	5	18	39
30	13	14	15	16	17	38
31	32	33	34	35	36	37

Consider concentric squares centred on 1. The next square is $3 \times 3 = 9 = 1 \times 8 + 1$, and this square has the integers from 2 through $(2+7=)$ 99 on its perimeter, with 7 at the northeast corner as the first west turn point.

The next square is $5 \times 5 = 25 = 3 \times 8 + 1$, and this square has the integers from 10 to $(10+15=)$ 25 on its perimeter, with 21 at the northeast corner as the second west turn number.

The next square is $7 \times 7 = 6 \times 8 + 1$, and this square has the integers from 26 to $(26+23=)$ 49 on its perimeter, with 43 at the northeast corner as the third west turn point. That is, all west turn points are on the northeast diagonal, and form the sequence 7, 21, 43, ..., and we could include 1 in this sequence to obtain 1, 7, 21, 43, ...

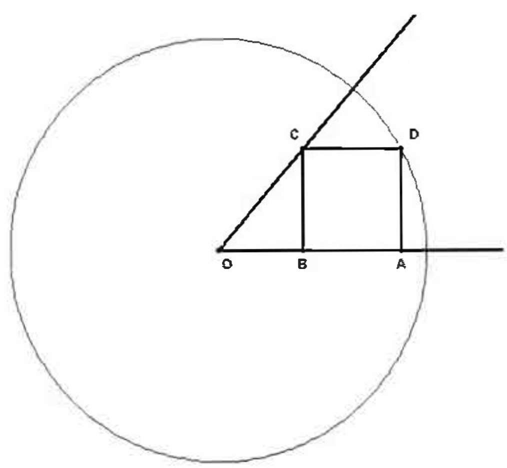
Considering the number sequence 1, 7, 21, 43, we can express this sequence as 1, 1+6, 7+14, 21+22, noting that the difference between successive terms is increasing by 8 units.

We can arrive at the same conclusion by noting that if A_n is the area of a given square, consider that $A_1 = 1$, $A_2 = 9$, $A_3 = 25$, $A_4 = 49, \ldots$, then $A_2 - A_1 = 9 - 1 = 8$, $A_3 - A_2 = 25 - 9 = 16$, $A_4 - A_3 = 49 - 25 = 24, \ldots$, which again shows that the difference between areas of successive squares is increasing by 8.

The difference, then, between 43 and the next integer in the sequence must be $22+8=30$, so that the fourth west turn point is $43+30=73$. The difference between 73 and next integer in sequence is $30+8=38$, so that the fifth west turn point is $73+38=111$.

Problem 2

Quadrilateral ABCD is a square. It is drawn so that points A and B are on $\overline{OA}$, and point D is on the circumference of a circle with its centre at point O. Point C is on $\overline{OC}$. If the radius of the circle is 10 units and $\angle COB = 45°$, then the area of square ABCD, to the nearest whole number, is



Solution: 20 units²

Put point O on the origin of a coordinate plane so that OA is along the positive x-axis. Let the distance $OB = x = BC = BA = AD$, since ABCD is a square and $\angle COB = 45°$. Draw a radius from O to D.
In $\triangle ODA$, we have $OD = 10$, $OA = 2x$ and so that $(2x)^2 + x^2 = 10^2 \rightarrow 5x^2 = 100 \rightarrow x^2 = 20$, which is the area of square ABCD.

Problem 3

Let a, b and c represent three different positive integers whose product is 16. The maximum value of $a^b - b^c + c^a$ is

Solution: 263

Using factors of 16, a, b, and c are 1, 2 and 8. The maximum value of c^a is 2^8. Hence the maximum value of $a^b - b^c + c^a$ is $8^1 - 1^2 + 2^8 = 263$.

Problem 4

Let a, b and c represent any positive integers. The value of $\frac{1}{a}+\frac{1}{b}\left(1+\frac{1}{a}\right)+\frac{1}{c}\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)-\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)\left(1+\frac{1}{c}\right)$ is

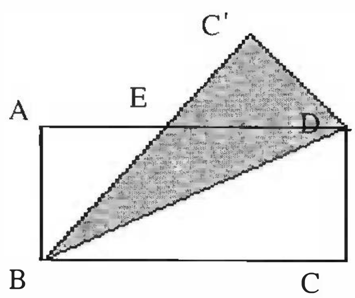
Solution: −1

$$\frac{1}{a}+\frac{1}{b}\left(1+\frac{1}{a}\right)+\frac{1}{c}\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)-\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)\left(1+\frac{1}{c}\right)$$
$$=\frac{1}{a}+\frac{1}{b}+\frac{1}{ab}+\frac{1}{c}+\frac{1}{ac}+\frac{1}{bc}+\frac{1}{abc}-\left(1+\frac{1}{b}+\frac{1}{a}+\frac{1}{ab}+\frac{1}{c}+\frac{1}{bc}+\frac{1}{ac}+\frac{1}{abc}\right)$$
$$=-1$$

Problem 5

A rectangular piece of paper ABCD is such that AB = 4 and BC = 8. It is folded along the diagonal BD so that triangle BCD lies on top of triangle BAD. C′ denotes the new position of C, and E is the point of intersection of AD and BC′.

The area of triangle BED is



Solution: 10

Triangles BAD and DC′B are congruent to each other. So triangles BAE and DC′E are also congruent to each other. Let AE = x. By Pythagoras's Theorem, BE = $\sqrt{4^2+x^2}$. We also have BE = C′B − C′E = $8 - x$. Squaring both expressions and equating them, we have $16 + x^2 = 64 - 16x + x^2$, which simplifies to $16x = 48$ and $x = 3$.

Hence the area of triangle BED is $\frac{1}{2}$DC × DE $= 2(8 - 3) = 10$.

Susan Ludwig is a secondary math consultant with Edmonton Catholic Schools.