Geometry, Geography and Equity: Fostering Global–Critical Perspectives in the Mathematics Classroom

Daniel Jarvis and Immaculate Namukasa

A significant part of the equity debate relates to the sensitivity that teachers show toward the different social, cultural and linguistic traditions that students bring to the mathematics classroom. This article combines geometry, geography and equity by discussing a variety of world map projections and five K–10 mathematics activities that promote increased global awareness and critical analysis skills.

Introduction

The fact that equity is at the top of the *Principles and Standards for School Mathematics* (NCTM 2000) underscores its importance: "Excellence in mathematics education requires equity—high expectations and strong support for all students" (p 11). Beyond curriculum content, pedagogical accommodation, differentiated assessment and technological access issues, another significant part of the equity debate relates to the sensitivity that teachers show toward social, cultural and linguistic traditions that students bring to the mathematics classroom (D'Ambrosio 1985, 1997).

Activities in the mathematics classroom that foster curricular connections, while raising awareness of global issues and traditions and promoting the development of critical analysis skills, represent worthwhile investigations. This article presents five student activities ranging from kindergarten to Grade 10. The first activity integrates geometry and geography as elementary students analyze the shapes of the continents, then position the seven polygons on an activity sheet and locate themselves on a world map template. The second activity integrates geometry and geography for Grades 4–6 students as they critically compare grid-based, student-generated representations of the common Mercator and less common Gall-Peters world map projections. In the final three activities, Grades 8–10 students further examine issues of cartographic distortion in terms of length, area, shape and angle as they explore various types of map projections. All five activities connect mathematics with the real world (and in this case with the "nonreal" projective world) and can form the basis for expanded discussions on cultural, historical and equity-based issues across the elementary and secondary school curriculum.

World Map Projections

Perhaps the most common world map projection, at least for North American, European and African students, is the Mercator projection (Figure 1).

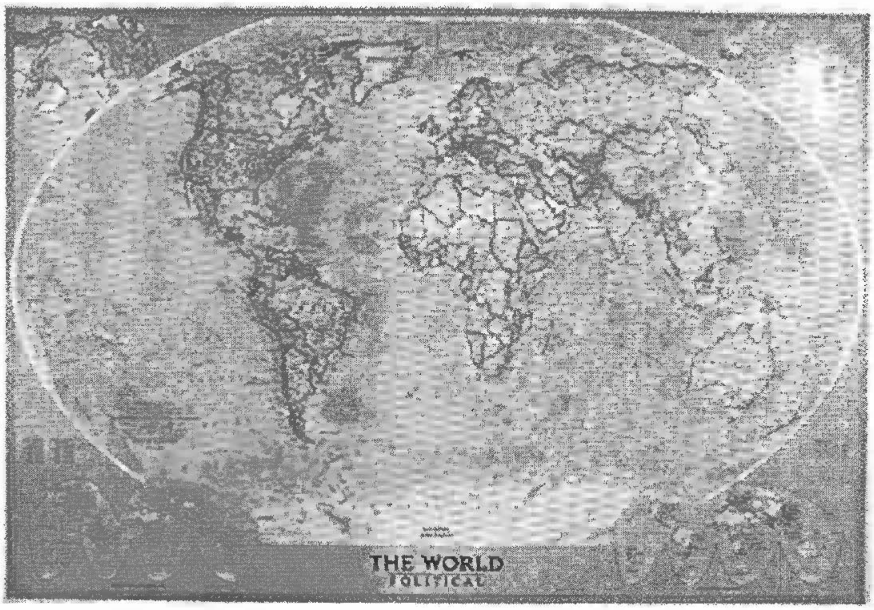

Figure 1: Common Mercator world map found in most North American schools

In fact, this particular representation of the planet Earth is so popular and pervasive that it is often difficult for students (and for most adults) to accept any other type of world map projection as legitimate. For example, one of the authors has the following three alternative world maps displayed in his office, and these maps never fail to elicit interest, surprise and questions from visiting students and colleagues.

The Upside-Down world map (Figure 2)---drawn in 1970 by a 12-year-old Australian student, Stuart McAuthur—is really a misnomer for two reasons:

1. From outer space there is no up or down in terms of the Earth's position—north and south poles have been arbitrarily selected to aid in navigation but are arguably rather meaningless in the larger universe context.
2. For many students living in the southern hemisphere, particularly in Oceania, this is the common world map displayed in schools and is therefore not considered upside down at all.

North Americans are surprised at the position of their country (that is, lower-left corner) and are uncomfortable with the map's strange and "incorrect" upside-down perspective. For Canadians, one small comfort is that the St Lawrence River flows down the map and into the Atlantic Ocean.

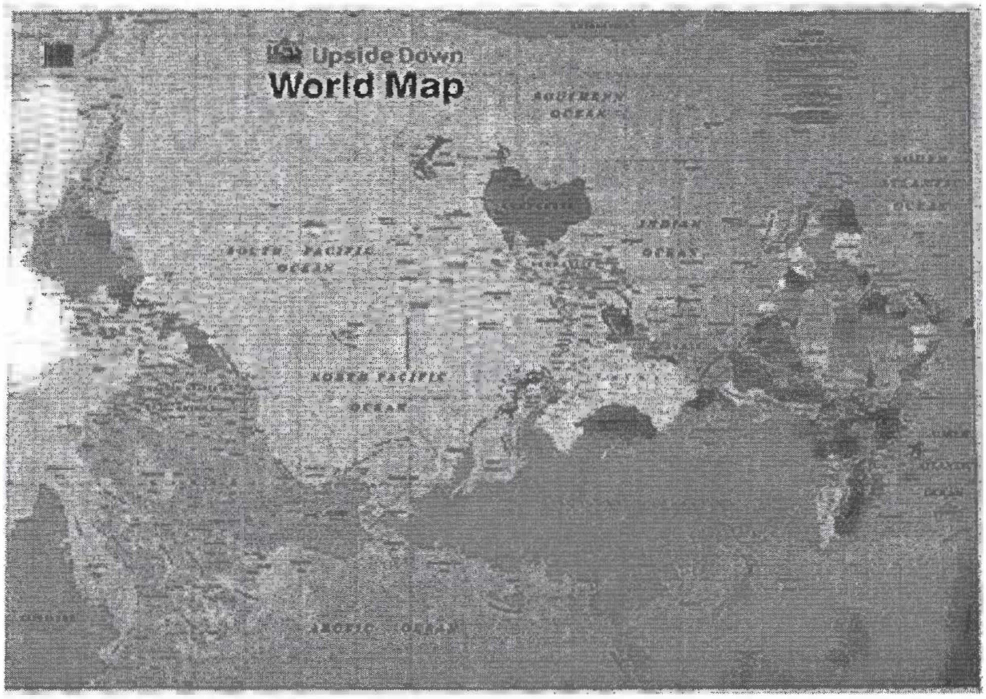

Figure 2: Upside-Down, Reversed or South-Up world map common in the southern hemisphere

The Pacific-centred map (Figure 3) is the second alternative world map projection in which the international date line located within the large Pacific Ocean is depicted as central to the map, rather than being the line selected to form both the peripheral boundaries, as in many Mercator projection maps. One of the authors first encountered this world map while teaching at an international school in northern Thailand. Many classrooms in Thailand, and presumably in other Asian and perhaps South American countries, would feature such maps as normal and familiar representations of the planet.

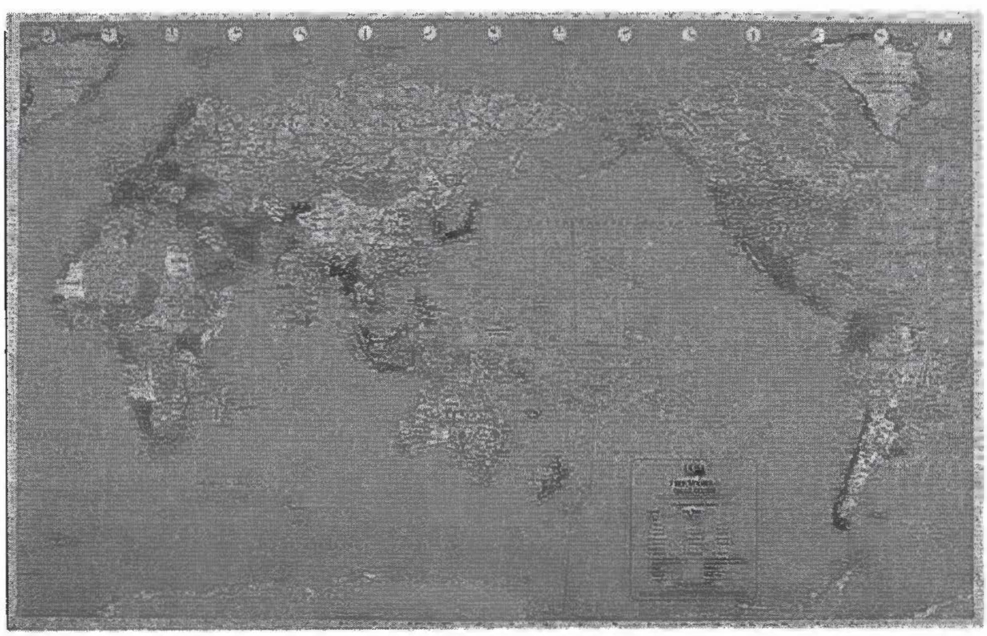

Figure 3: "Pacific-centred" map common in many Asian schools

The Gall-Peters equal area world map projection forms yet another significant and different representation of our three-dimensional earth on a two-dimensional surface.

In this equal-area cylindrical map—named after both the Scottish clergyman James Gall (1885) and the historian/activist Arno Peters (who presented his version in 1973)—the land masses are presented in their actual (or at least much closer to the actual) surface areas, without the major distortion that occurs toward the poles in other projections. For example, in the Mercator projection the size of regions is increasingly inflated according to their distance from the equator. As a result, Greenland sometimes appears larger than Africa, whereas in reality Africa is 14 times larger than Greenland. One of the authors, a native of Uganda, can understand Peters's emphasis on the political implications of such representations and how this often makes developing regions or countries near the equator appear much smaller and less significant than they really are. Although not entirely accurate in terms of its proportions, the Gall-Peters map presents a different and useful perspective of the world.

Taken together, the Mercator, reversed, Pacific-centred and Gall-Peters world map projections[1] can help students construct a more accurate and richer understanding of the earth. To further build on this increased understanding, and with a view to integrating geometry and geography, we present two mathematics activities that teachers can implement in the elementary curriculum.

Me and My Geometrical World

In this activity, K–3 students are escorted to the world map presumably hanging in a school hallway, library or classroom. Students have likely passed by this large map many times, but they are about to look at it in a new way that will make it difficult to ever pass by mindlessly again. Once the students are seated on the floor, the teacher asks them to name some basic geometric shapes; for example, triangles, circles, squares, rectangles and parallelograms. Then students are instructed to squint until the map details become fuzzy in order to identify any of these large geometric shapes. With a pointer, student volunteers take turns pointing out a few such perceived shapes on the map. This activity may also be done on an interactive whiteboard with an Internet connection.

Students then return to class and paste colourful, precut (or for older grades, student-generated and cut out) geometric shapes onto a template (Figure 6). While the template (see Student Activity: Me and My Geometrical World) obviously does not include direct visual reference to the world's 190 plus countries, it does provide an interesting perspective of the seven major continents and their approximate spatial relationship. Finally, students are asked to locate (in atlases, classroom maps or on the Internet) their school or city on the shapes they have pasted on the page and to mark the location with a coloured dot. If many international students or ESL immigrant children are in the class, the teacher may have students locate and mark their countries of origin on the world map,

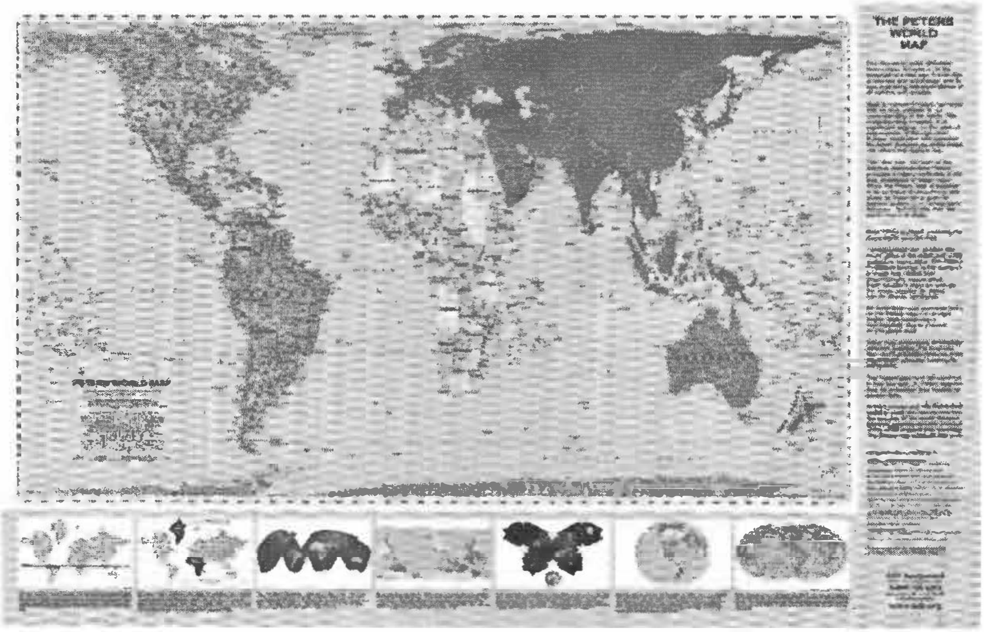

Figure 4: Gall-Peters equal-area map showing more accurate continental proportions

asking students if they would like to share their maps with partners or with the whole class. Having done this activity with a kindergarten class as an invited guest speaker, one of the authors was impressed with the students' keen curiosity and the teacher's positive comments during the lesson.

How Big Is My Country, Really?

In the second activity, Grades 4–6 students begin with Mercator and Gall-Peters world map projections. Using grid paper overlays (11-x-14-inch works better than 8.5-x-11-inch map/paper, but both are doable), students colour either full or partial squares (for example, half a square can be shaded for half a land mass area viewed within the square) to represent the approximate land mass areas found within the two different projections. For both projections, students count the total coloured-square area for the entire map and for each separate continent, then calculate the percentage of the world's land area for each continent using a simple division operation (see Figure 6). Class results can then be collected on the board or on a spreadsheet to examine average values. Average values may be rounded off to calculate areas of each continent in terms of fractions of the total world area. These average values may then be compared with the percentage of population for each continent (available on Wikipedia) and per capita figures. For enrichment, students can redraw the world map more critically and examine alternative maps based on population characteristics.

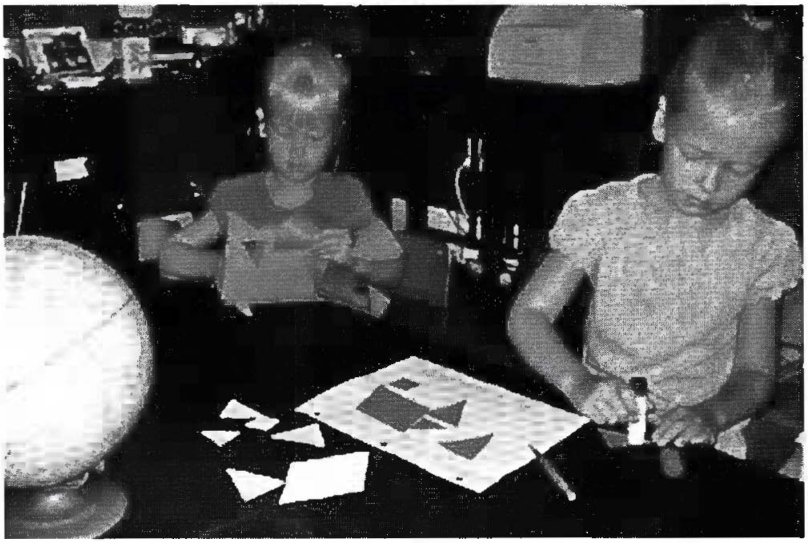

Figure 5: Students completing activity 1 template and locating self on world map

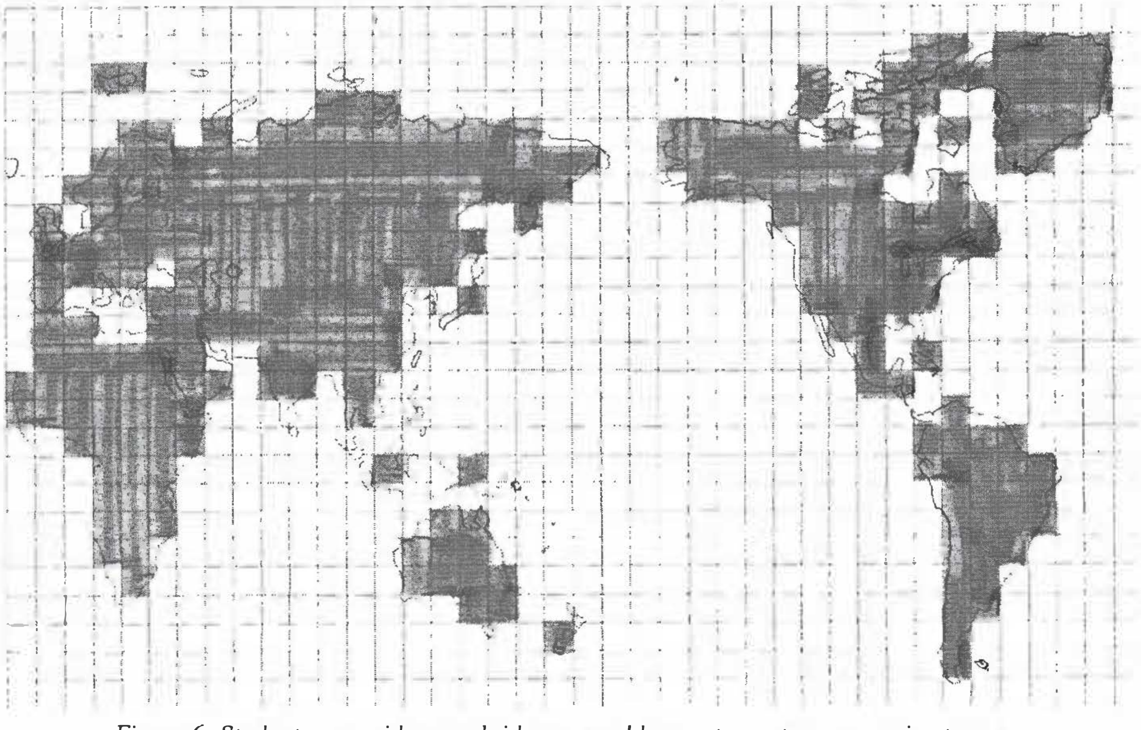

Figure 6: Students use grid paper laid over world maps to capture approximate areas

As part of an integrated geography, history, literacy and mathematics assignment, students can research the backgrounds of the Mercator and Gall-Peters world map projections focusing on when, where and why each map has developed and how each has been used internationally. This can lead to an interesting exploration, and the discussions that ensue can involve issues of politics, finance, trade, navigation, colonization, technology and social equity. This second can foster a deeper understanding of earth's land masses and an increased awareness that how people have chosen to visually represent these land masses has had a significant and often controversial effect on earth's various populations. For instance, students may begin to understand that even what appear to be fixed locations, such as west and east, north and south, are not really fixed, nor are they arbitrary, but that they represent given points of view.

Length, Area, Shape and Angle Distortion

The following three Grades 8–10 activities use different kinds of maps to measure angles, length and areas. The goal is to explore map projections as a mathematical activity, thereby understanding why cartographers and countries select various projections and the far-reaching consequences that these decisions can have.

In the first activity, the teacher asks how to flatten a globe; that is, how to project the globe onto a map. Students may be given spherical objects with which to work. A spherical- or ellipsoid-shaped ball (one that bulges more at the centre) or even an onion demonstrates this well. Cut one onion into halves vertically and remove layers to reconstruct several hollow onions. In attempting to flatten two halves of an onion, students soon realize that this produces an interrupted shape as the onions tear. Cutting the onion into quarters or eighths reduces the tear. A teacher shows an interrupted map of the world centred on or near the Greenwich meridian; for example, the Interrupted Mollweide (Greenwich-centred) map featuring two lobes in the northern hemisphere and three in the southern hemisphere (see Figure 7).

Using such a map, the teacher asks the following questions: What purpose does this map serve? Are there any distortions? What other centre longitudes could be used for interrupted maps? Students may look at other interrupted maps and state which ones they prefer, such as interruptions that do not cut across continents.

To expand on this activity, the teacher may introduce other (hollow) polyhedra and transparent plastic cylinders, such as a tumbler. The teacher explains that had the earth been a different sort of solid, such as a prism, pyramid or other polyhedron that has no curvature, then a map would have been drawn by opening up the polygons to form flat, connected patterns or nets. For example, a cube would be represented by six connected squares; a cuboid (rectangular prism) would form adjoining rectangles; a cylinder would feature rectangles and two circles; and a tetrahedron would form a series of connected triangles. Area and length would be preserved, and some angles and

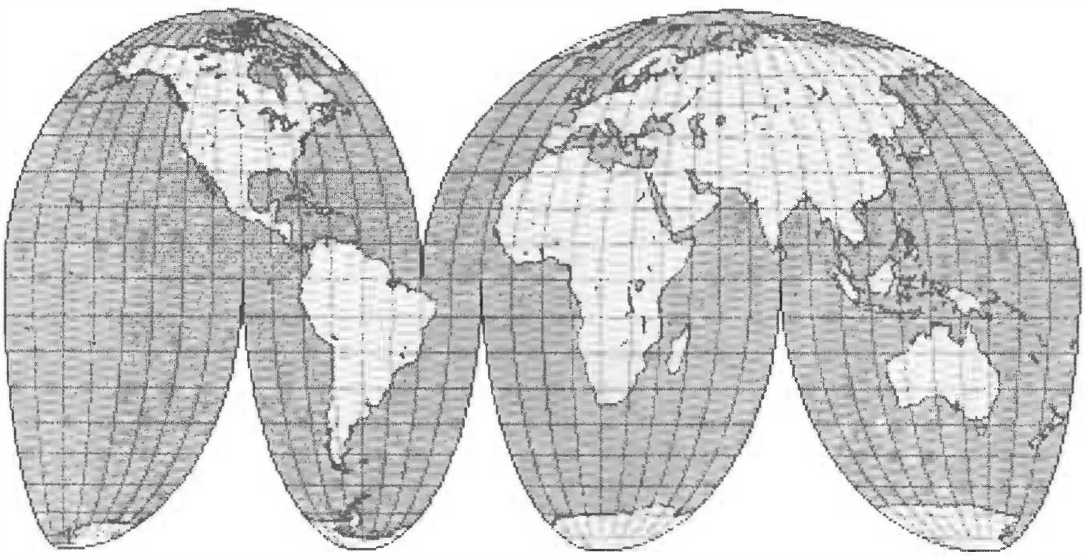

Figure 7: Greenwich-centred "Interrupted" Mollweide world map projection

directions would be preserved on these interrupted nets. Continents hypothetically located at the corners of these nets would be split apart as in other interrupted maps. A teacher may show Buckminster Fuller's Dymaxion map that is based on a 20-sided polyhedron—an icosahedron (see Figure 8)—and raise questions about purpose, distortions and other possible Dymaxion maps.

The second activity is based on two specific questions: What did cartographers do to avoid interrupted maps? and How can we produce a rectangular map from an interrupted map? Students may have noticed that if they stretch some parts and shrink others of a flattened sphere or ellipsoid, they may be able to fill the gaps or interruptions. A teacher may use a plastic ball to show that with some stretching, a rectangular shape can be formed from a flattened sphere. What are the different ways of stretching (and shrinking) that produce rectangular maps? A teacher may then show the class different maps that were produced with stretches farther away from the equator or those that stretch or shrink as one moves away from a meridian in the northern hemisphere; for example, 45° north (and its southern hemisphere counterpart—45° south). A teacher would introduce the concept of the standard latitude (no stretching or shrinking is done; for example, the Lambert [1772] map has standard latitude 0° and aspect ratio 3.141:1). Students would explore where the most distortion occurs on this map. For the Lambert map, students would likely notice that maximum distortion happens the farther one moves away from the equator, as widths are stretched to fill in the rectangle (that is, a horizontal line of equal length as one drawn on the equator will be shorter in reality the farther one is away from the equator). Horizontal lines at the north and south poles would be almost infinitely times longer. Students might then be asked, based on the Lambert map, to imagine a map with aspect ratio 1:1—a square map—and think about what the standard latitudes would be—nearer to or farther from the equator? What would happen to the shapes of the continents—would they be longer and thinner or shorter and thicker? What might be done with the lines of longitude that have been stretched to preserve area? To compensate for this distortion many cartographers use mathematics formulas to adjust the spacing of the meridians (that is, distort the length of vertical lines) so that in the end equality in area is maintained. This question introduces the idea of equal area maps such as the Gall /Peters map (standard latitude 45°; aspect ratio 1.571:1). Students then discuss the advantages and disadvantages of equal area maps.

The third activity introduces the Mercator map as an equal angular map constructed for navigation purposes. Students may estimate and then measure distances and bearings between their city and another using a Mercator map. The teacher looks up this distance and the relevant bearings beforehand in preparation for the activity. Students notice which measurement is in error (that is, Mercator will produce errors of distance but not angle). Students might also estimate, measure and compare the lengths of Greenland and Africa, and South America and Europe on this map. Although these two pairs of land masses appear to be in 1:1 ratios, the actual figures prove otherwise. Students can also be introduced to the concept of rhumb line (or loxodrome), which represents a path of constant bearing, and a conformal map which has a function that preserves angles.

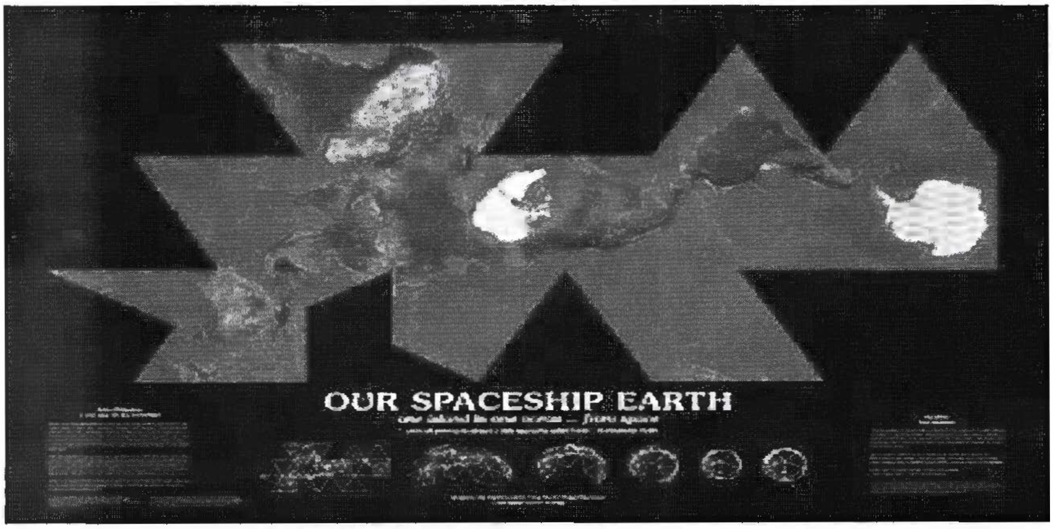

Figure 8: Buckminster Fuller's Dymaxion map world map projection

Highlights from the history of map projection may include early Egyptian recordings of the first estimates of the circumference of the earth; map drawings on Babylonian clay tables; the Greek mathematician Archimedes inscribing a sphere in a cylinder to figure out the surface area of the sphere; Eratosthenes' and Ptolemy's measurements of the earth's circumference; Mercator in the 16th century; Newton's 17th-century explanation that the earth is a spheroid; and the establishment of the Greenwich meridian in the 19th century. Also the teacher may relate, or have students research, actual stories that relate to map projection controversies. For example, the San Francisco federal court case in which the National Resource Defence Council (NRDC) fought against Navy and National Marine Fisheries (NMFS) to determine boundary lines, both parties presenting different map projections to substantiate their respective claims (NRDC used the Mercator projection; NMFS used the Behrman equal area map) (Propen 2007).

For other map-based mathematics activities we recommend Koirala and Goodwin (2002) on comparing areas of states or provinces using a map of a continent; Liben (2008) on helping students understand maps as representations; Haslam and Taylor (1996) and Wood, Kaiser, Abramms (2006) on hands-on activities for projecting globes and early maps, including Inuit coastal maps and a Toronto-centred circular map; Gutstein (2001) on maps and misrepresentations; Wilkins and Hicks (2001) on estimating the earth's water coverage using varied map projections; McLaughlin (2006) on creating maps for the areas in which students live; and Hawkins (2003) on lesson ideas that involve comparing proportions using varied maps. For more secondary school and university activities using maps, refer to Feeman (2000) who discusses how rectangular maps are constructed using calculus and trigonometry.

Ethnomathematics and the Global Student Citizen

Ethnomathematics educators encourage developing students' critical awareness in mathematics classrooms (see D'Ambrosio 2007). They examine interactions between culture, politics, economics and mathematics, and challenge those that perpetuate social injustice and inequities. D'Ambrosio observes that "social justice allows us not only to know what has been decided about ourselves and society (which is the objective of 're-productory' and imitative education), but calls us to participate in decisions about ourselves and society (which is the objective of creative critical education)" (Foreword).

This article illustrates how using a global artifact—a world map—can raise critical awareness among students. Historically, the map—originally conceived and drawn in the 16th century—was the first artifact to show the continents connected. The use of different maps, with one version becoming so dominant (that is, the Mercator), shows that the map, like mathematics, can significantly affect students' views of the world. The world map is an early facilitator of globalization. Using a map in a mathematics lesson is a way of drawing, writing and reading the world (Freire and Macedo 1987; Gutstein 2006). Mathematics in this case is used as a tool to interpret our society. Skovsmose and Valero (2002) refer to the types of activities described above as culturally and politically powerful mathematical ideas because they offer learners critical and emancipatory interpretations.

Concluding Thoughts

With impressive advancements in web-based technology, the availability of open-source (free) dynamic image software, such as Google Earth, and the growing availability of interactive whiteboards in many school districts, the above-described activities can be greatly enhanced within the classroom or computer lab. For example, lines or shapes can be drawn overtop of actual, updated satellite imagery, and the mouse can be used to rotate one's perspective at any level of magnification.

Such power placed in the hands of a young learner can make global explorations highly motivating and informative. The increasing international access to high-speed Internet connections and the use of such things as open-source software, no doubt will affect technology-based equity in mathematics education. Furthermore, the student-centred activities described in this article provide teachers with starting points from which integrated projects can grow, by which understanding of self and society can be deepened, and through which the equity principle of the National Council of Teachers of Mathematics can be realized in a variety of interesting and meaningful ways.

Note

1. For a full and more detailed discussion of the various world map projections, the reader is referred, for example, to the Wikipedia entry entitled, Map Projection (http://en.wikipedia.org/wiki/Map_projection).

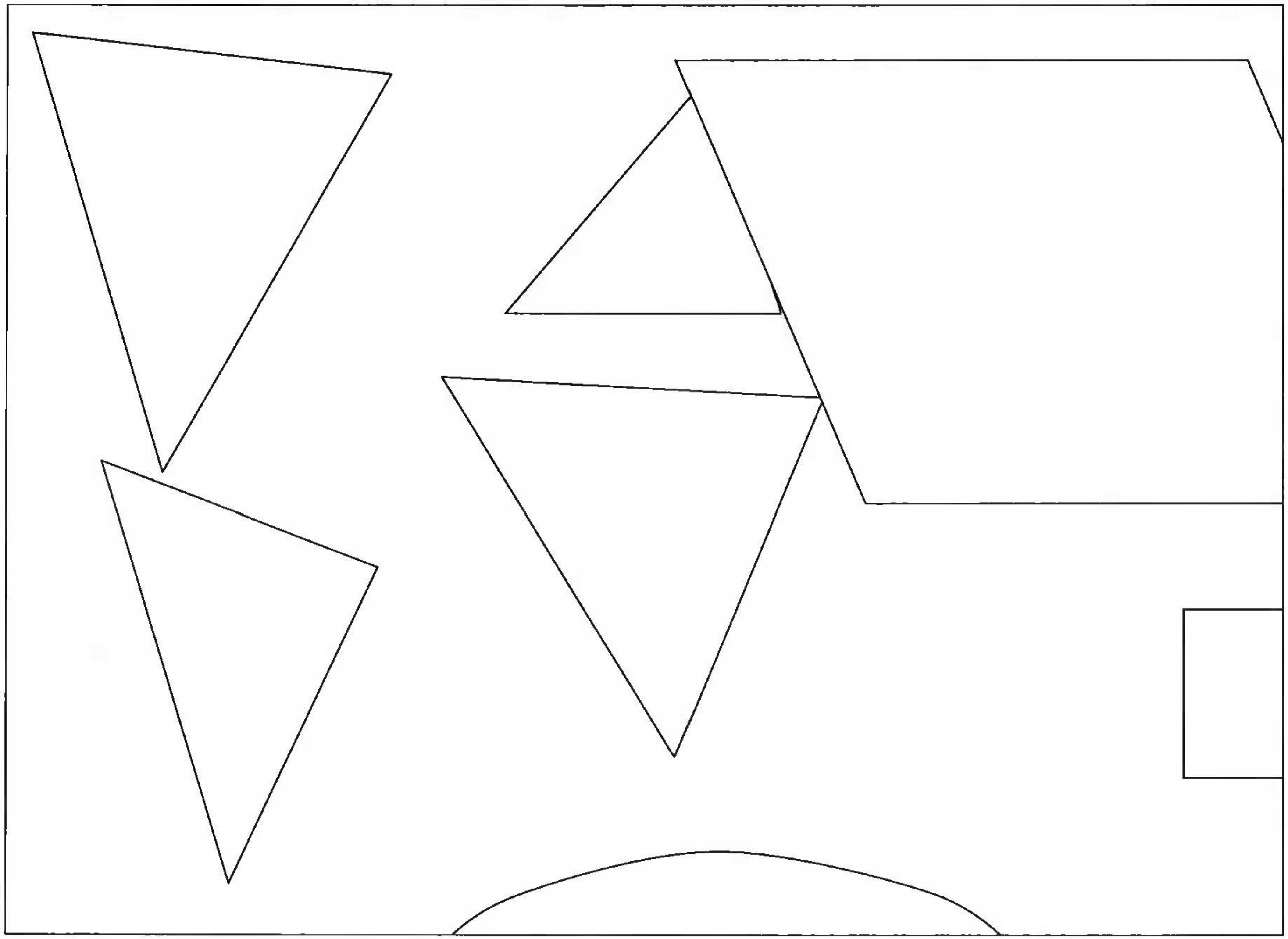

Figure 9: Student Activity: Me and My Geometrical World

Description: Analyzing the Mercator world map projection for large, common geometric shapes; labelling the seven major land masses (continents); and locating one's school on the world map.

Instructions: Once you have cut out, pasted and labelled the seven continents/polygons, find the approximate location of your school on the map and mark it with the small sticker provided. Turn your map around and view the world from different sides of the page.

References

Abramms, B. 2003. *Many Ways to See the World*. Videocassette. Amhurst, Mass: ODT Maps.

D'Ambrosio, U. 1985. "Ethnomathematics and its Place in the History and Pedagogy of Mathematics." *For the Learning of Mathematics* 5, no I: 44–48.

——. 1997. "Where Does Ethnomathematics Stand Nowadays?" *For the Learning of Mathematics* 17, no 2: 13–18.

——. 2001. "What Is Ethnomathematics, and Can It Help Children in Schools?" *Teaching Children Mathematics* 7, no 6: 308–10.

——. 2007. "Peace, Social Justice and Ethnomathematics." In *International Perspectives on Social Justice in Mathematics Education*, ed B Sriraman. The Montana Mathematics Enthusiast, Monograph 1. Charlotte, NC: Information Age.

Feeman, T G. 2000. "Equal Area World Maps: A Case Study." *SIAM Review* 42, no 1: 109–14.

Freire, P, and D Macedo. 1987. *Literacy: Reading the Word and the World*. South Hadley, Mass: Bergen and Garvey.

Gutstein, E. 2001. "Math, Maps, and Misrepresentation." In *Rethinking Mathematics: Teaching Social Justice by the Numbers*, ed E Gutstein and B Peterson, 111–16. New York: Rethinking Schools.

——. 2006. *Reading and Writing the World with Mathematics: Toward a Pedagogy for Social Justice*. New York: Routledge.

Haslam, A, and B Taylor. 1996. *Maps: The Hands-On Approach to Geography*. London: Franklin Watts.

Hawkins, J M. 2003. "Lesson Ideas for Using Conflicting Maps." *Social Studies* 94, no 1: 41–43.

Koirala, H P, and P M Goodwin. 2002. "Middle-Level Students Learn Mathematics Using the US Map." *Mathematics Teaching in the Middle School* 8, no 2: 86–88.

Liben, L S. 2008. "Understanding Maps: Is the Purple Country on the Map Really Purple?" *Knowledge Quest* 36, no 4: 20–31.

McLaughlin, C. 2006. "Mapping My World." *Technology and Children* 10, no 3: 13–14.

National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics*. Reston, Va: NCTM.

Propen, A. 2007. "Visual Communication and the Map: How Maps as Visual Objects Convey Meaning in Specific Contexts." *Technical Communication Quarterly* 16, no 2: 233–54.

Skovsmose, O, and P Valero. 2002. "Democratic Access to Powerful Mathematical Ideas." In *Handbook of International Research in Mathematics Education*, ed L D English, 383–407. Mahwah, NJ: LEA.

Wilkins, J L M, and D Hicks. 2001. "A Stimulating Study of Map Projections: An Exploration Integrating Mathematics and Social Studies." *Mathematics Teacher* 94, no 8: 660–73.

Wood, D, L Kaiser and B Abramms. 2006. *Seeing Through Maps: Many Ways to See the World*. Amherst, Mass: ODT.

Dr Daniel Jarvis is an assistant professor of mathematics education in the Faculty of Education, Nipissing University, North Bay, Ontario.

Dr Immaculate Namukasa is an assistant professor of mathematics education in the Faculty of Education, University of Western Ontario, London, Ontario.