A Mathematical Adventure with the Number Devil

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Introduction

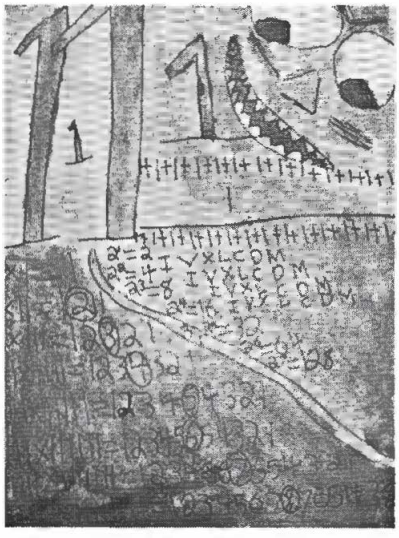
A university professor introduced me to Hans Magnus Enzensberger's *The Number Devil* and implored me to read the book at some point in my teaching career. A few years later, I picked it up as a summer read simply with the intention of honouring my professor's request. Before long—in fact, I'm sure it was within the first chapter—I knew I needed to share this book with children.

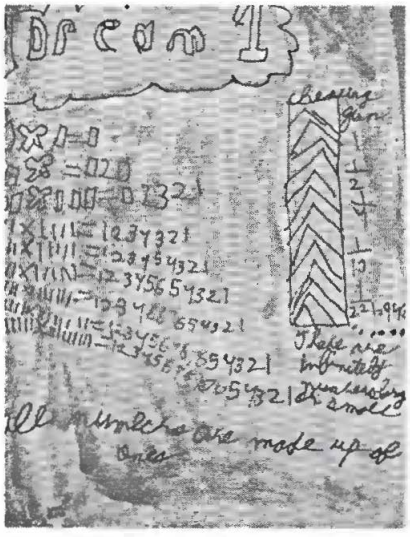
If you have read the book, you will know that some of the mathematics is insanely complicated. Knowing that I would be teaching Grade 4 students that fall, I was not sure how well the book would be received; nevertheless, whether it was an effective tool for teaching mathematics by promoting mathematical thinking or simply a way to spark students' imagination and interest, I decided to give it a try.

Mathematics aside, the book itself is a very easy read and is brilliantly illustrated. Its characters are Robert, a 12-year-old mathphobe, and the Number Devil, a lively and rather convincing little imp whom the students truly enjoyed following. A brief synopsis: Robert dislikes his math teacher, Mr Bockel, who gives his class boring math problems and does not let them use their calculators. Each night, as Robert drifts off to sleep, the Number Devil, who brings the subject magically to life for him, visits him. The Number Devil illustrates with wit and charm (and a few fun words like *nincompoop*, *prima donnas* and *rutabagas*) a world in which numbers can amaze and fascinate, and where math is nothing like the dreary, difficult process that so many of us dread.

Before I finished the first chapter, I knew that this book was to be shared with students. As I closed the book after reading the first chapter, I asked my class, "So ... there was a whole lot of math that just went on in those first few pages. Would you like me to keep reading, or do you want to explore some of this great math?" It was a unanimous decision to slow down and explore each of the concepts the chapters had to offer. And that was what we did.

Below is a description of how we explored with an open mind, and sometimes a calculator, all (well, most) of what the Number Devil shared with Robert.





Dream 1—Ones: Is Everything Really Made Up of Ones?

Students quickly realized they did not need to be afraid of large numbers, because they are simply made up of ones. They realized that if you really wanted to, you could count to "five million etcetera" starting with 1 + 1. Then 1 + 1 + 1. Then 1 + 1 + 1 + 1 ... and so on.

It was also suggested to the students that not only could they count to a number like five million, seven hundred twenty-three thousand,

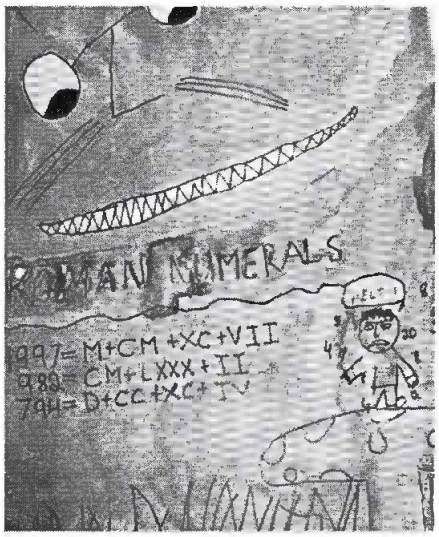
eight hundred twelve by ones, but the reverse is also true. Just as there are infinitely large numbers, there are infinitely small numbers.

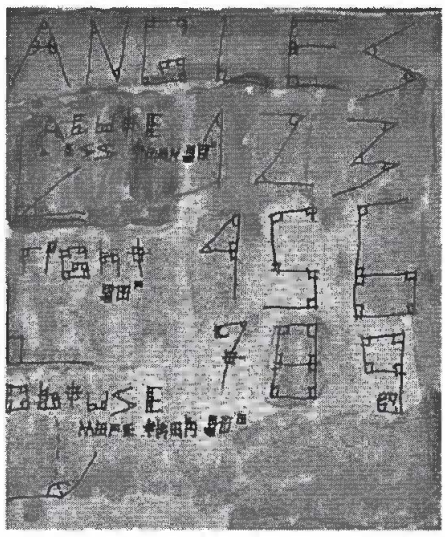
Just as students enjoy coming up with words such as *noon* or *racecar*, known as palindromes (a word that reads the same backward as forward), the Number Devil had us creating a similar pattern by simply using ones. This time, instead of adding them to make increasingly larger numbers, we were multiplying:

$1 \times 1 = 1$
$11 \times 11 = 121$
$111 \times 111 = 12321$

Students were fascinated as they conjectured and explored how these "palindrome products" were occurring, as well as how long the pattern would continue.

Dream 2—Place Value: Thank Goodness for Zero!



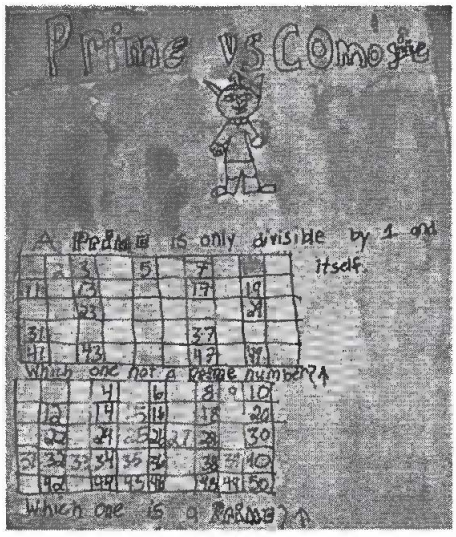


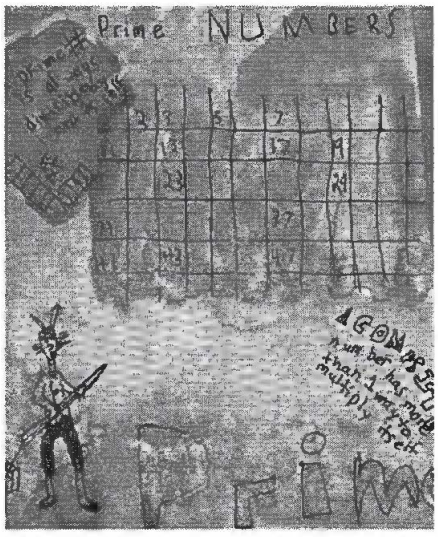
Most of my Grade 4 students were born in 1997. We know that in expanded form to be 1000 + 900 + 90 + 7, or 1997 (a great place-value teachable moment). To the Romans however, this would be much more complicated: MCMXCVII. M = 1000. C is 100, but because it comes before the second M you must subtract it to get 900. C, as we know, is 100, but because X comes before the second C you must subtract it to get 90. V is 5, which you add to two ones to get a sum of 7. So this is 1000 + (1000 − 100) + (100 − 10) + (5 + 2) = 1000 + 900 + 90 + 7 = 1997.

We explored the order of operations, which was not necessarily Grade 4 content, but fun just the same.

While we were back in time with the Romans, we also explored how, in the very early stages of our digits/number system, each digit (1 through 9) was made with the same number of angles as its value (see the artwork above). This was a great exploration, and we discovered that there are a number of ways to make a few of the digits. Also, this was an excellent opportunity to name the angles created within each of the digits.

Dream 3—Prima Donna Numbers: Working with Prime Numbers

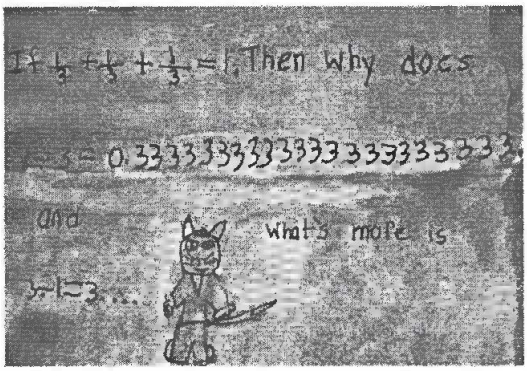




Students worked together to find all the prima donna numbers between 0 and 50. We were reminded that 0 doesn't count because when you divide any number by 0, you get 0! And that 1 also doesn't count because every number is made up of 1s!

The exploration of prime numbers brought out many

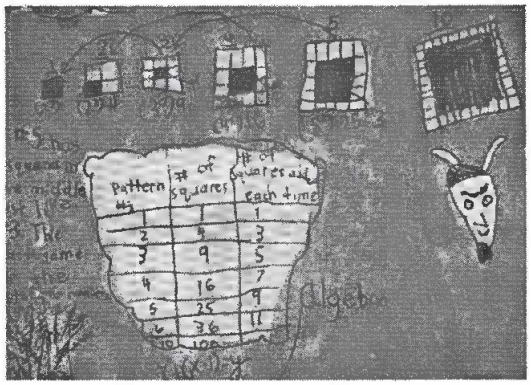
mathematical concepts, such as factors, composite numbers and square numbers. Students wondered why certain numbers had more factors than others, and figured out why square numbers had an odd number of factors. It also was a great way for the students to understand why 2, an even number, is a prime number and that not all odd numbers are prime.

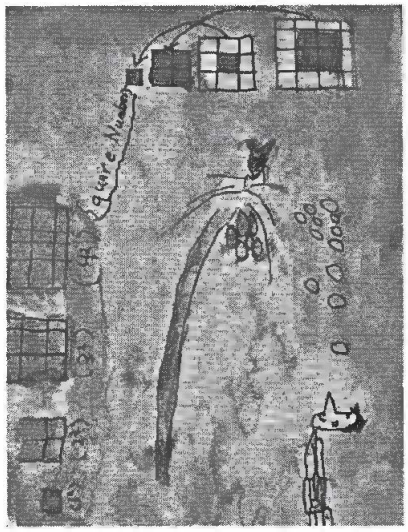


Dream 4—Powers: Unreasonable Numbers

Using the examples in the book, as well as the language introduced by the Number Devil, we took a quick look at powers ("hopping numbers") and square roots ("rutabagas"). Anyone can be confused by the unreasonableness of the following: if 1/3 + 1/3 + 1/3 = 1, then why when you divide 1 by 3 does this not come out quite so perfectly?

Dream 5—Square Numbers





Who knew that an innocent exploration of square numbers would lead to algebra? Starting with a visual representation of their hopping numbers (powers), students were asked the following questions:

- What patterns do you notice?
- How can you display these patterns?
- Can you predict the number of squares there will be in the 10th pattern?
- Can you come up with a rule that would work every time?

Also, it is no secret that if you close your classroom door and whisper to your students, "Now *don't* tell the Grade 6 teacher that you just learned about algebra! Can you believe you are doing math that some of your older brothers and sisters are doing?" you will get them into a mathematical space like never before. Bragging rights!

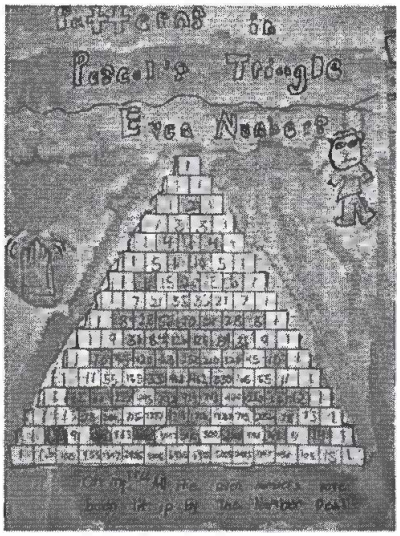
Dream 6—Fibonacci Rules!

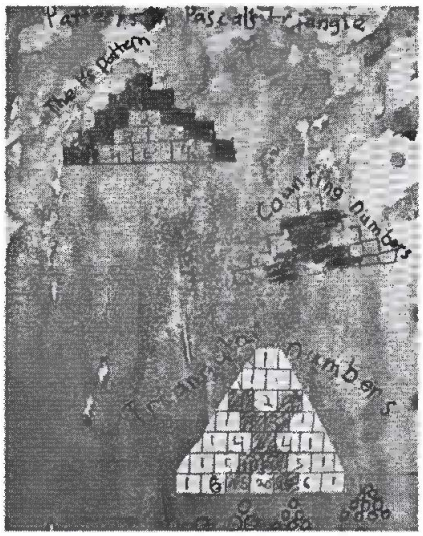
The day began with a brief discussion on patterns: What is a pattern? How might it grow? Students called out responses such as "ABABAB," "ABCABC," and "something that repeats or grows or both." Students were then asked to consider the next 8 numbers in the following sequence of numbers:

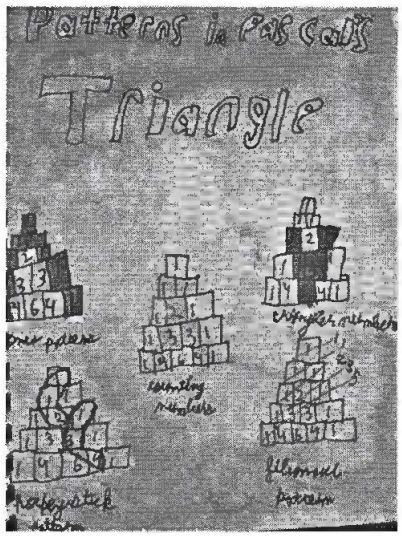
1, 1, 2, 3, 5, 8, 13, ___, ___, ___, ___, ___, ___, ___, ___.

Once the sequence was revealed and named as the Fibonacci sequence of numbers, I coupled this with the Number Devil exposing the truth about the rapid growth of rabbits. Then the students were given the opportunity to do some research on the amazing pattern. Discovering the beauty of the pattern in nature as well as in their own bodies was fascinating to them (see student work at http://projects.cbe.ab.ca/glendale/showcase/numberdevil/fibonacci.html).

Dream 7—Pascal's Triangle



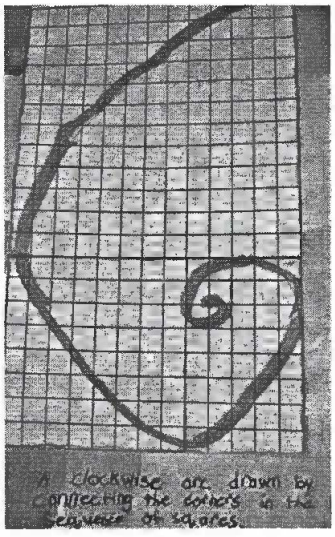




Playing with the Fibonacci sequence of numbers was only the beginning of looking at patterns. Within Pascal's Triangle we explored many other patterns, some recognizable and others simply interesting or perplexing, including

- counting numbers (1, 2, 3, 4, 5...),
- Fibonacci sequence (1, 1, 2, 3, 5, 8...),
- triangular numbers—coconuts! (1, 3, 6, 10, 15...),
- multiples of 2s (even numbers) and 5s,
- the hockey stick pattern and
- the sums of rows.

The list of patterns goes on and on, and there are excellent websites to support these explorations. The best way to begin is to start with just a few numbers on the triangle and have students complete the patterns they see.



Dream 8—Fibonacci Spiral

During the students' research on the Fibonacci sequence, the concept of the spiral came up everywhere for them—the seashells they examined, the pineapple they ate or pictures of the cochlea of an ear. This spurred on the next exploration—squaring Fibonacci numbers to create the spiral for themselves.

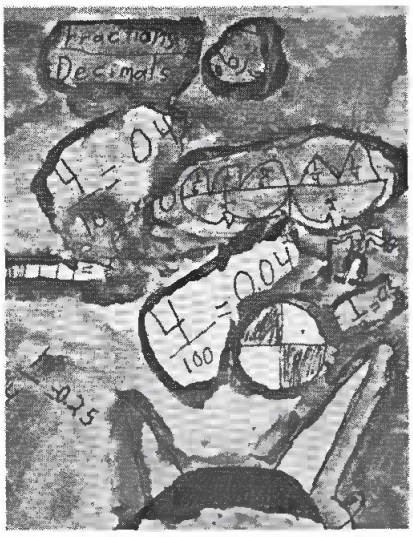
The equiangular spiral can be created by drawing an arc through a series of squares that grow in size by following the Fibonacci sequence (1×1, 1×1, 2×2, 3×3, 5×5, 8×8, 13×13). Students first used tiles to generate the spiral, and then were asked to find it again using grid paper.

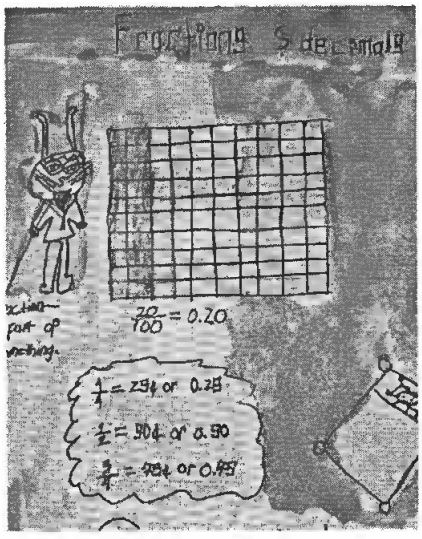
Taking a look at how the Fibonacci spiral approximates the Golden Spiral was something that we explored only briefly, but this could definitely take students down another amazing path.

Dream 9—Fractions

While Robert was dreaming about fractions, we were eating them! I find it interesting that when food is involved, the concepts of equivalence and the importance of comparing the same vs different wholes come to light. With respect to equivalence, watching a student eat, for example, two pieces of licorice from an original piece that had been cut into fourths and another eat four pieces from a piece that had been cut into eighths

demonstrated that while it appears someone has more (and they do in number of pieces), if placed side by side, the two students in fact had an equivalent amount of licorice.





Very often, when comparing fractions we consider the same whole and, for the most part, it is important that we do so. However, when I asked my students, "Can 1/4 ever be greater than 1/2?" their first inclination was to say, "No." On one particular day I asked my students to bring in a variety of food items that could be shared with either the entire class or only a few students. The conversations and explanations about how their item could be cut and shared were very rich. It was also on this day that the concept of comparing the same whole came to light. For example, half of a store-bought chocolate chip cookie was, in fact, much smaller than a fourth of a pizza-pan-sized chocolate chip cookie. Aha!

Bringing It to Life: Our Number Devil Performance

I cannot remember exactly when in the reading of the novel the next unanimous decision came about, but when I asked the students, "How can we ever share all that we've been learning with others?" they excitedly replied (as if they had already been conspiring with one another), "A play! We can act it out!"

I used *The Number Devil* as well as *A Place for Zero* (another excellent piece of literature for bringing math to students) and essentially turned them into a screenplay by simply summarizing chapters or following the text verbatim (the script we created follows as Appendix A). Tryouts were conducted, sets were made and, with the help of a resident artist, songs were generated to turn our play into a musical. It was an evening many teachers, students, parents and administrators will never forget.

Conclusion

Through our reading, rereading, exploring, conjecturing, drawing, creating, designing, singing, building and performing, the mathematical understanding generated by this group of Grade 4 students far exceeded my expectations. While it may appear that these 10-year-olds were simply great at memorizing and reciting prime and triangular numbers, the Fibonacci sequence and patterns within Pascal's Triangle, I assure you that there was nothing superficial about their level of understanding. I, too, have a greater appreciation of and ongoing fascination with the numbers and patterns that surround us.

After watching and participating in the performance of *The Number Devil: A Mathematical Adventure*, many parents were quoted as saying, "Now *that's* the way to learn mathematics!"

References

Enzensberger, H M. 1997. *The Number Devil: A Mathematical Adventure.* New York: Holt.

LoPresi, A. 2003. *A Place for Zero: A Math Adventure*. Watertown, Mass: Charlesbridge.

Karen Cleveland is currently a learning leader for the Calgary Board of Education where she supports teachers in the area of mathematics, primarily with the program of studies changes. Prior to this, she taught in an inquiry-infused environment at Glendale School, in Calgary, Alberta. She believes that working collaboratively can generate growth and understanding. While she does not identify herself as a mathematics expert, she has a passion for mathematics and it is in sharing her passion that she continues to learn.

Appendix A

The Number Devil Script

Robert: Mom, I just don't get this question.
Mom: Here, let me help you.
Robert: Okay, if 2 pretzel makers can make 444 pretzels in 6 hours, how long does it take 5 pretzel makers to make 88 pretzels?
Mom: Oh my ... gee ... I was never very good in math, Robert. How about you ask your dad in the morning? Can you get ready for bed, please?
Robert: Oh man ... Mr Bockel is going to be so mad at me if I don't have my homework done.

Number Devil's (ND) first entrance ... ND song. Note that songs have not been added to the script, but simply shown where they were placed within the performance.

ND: Hey, Pretzel Boy ...

Wagon #1

Robert: What? Who are you? Am I dreaming?
ND: Where does your pretzel tale come from? School, I bet.
Robert: Where else? Mr Bockel, he's our teacher. He's always eating pretzels and always making up dumb questions about pretzels.
ND: I see. I have nothing against your Mr Bockel, but do you want to know something? Most genuine mathematicians are bad at sums. Besides, they have no time to waste on them—that's what calculators are for. I assume you have one.
Robert: Well, yes, but we're not allowed to use them in school.
ND: I see. That's alright; I guess there's nothing wrong with a little addition and subtraction. It's good to know what to do if ever you don't have your calculator. But mathematics, my boy—that's something else!
Robert: Oh, just go away. The last thing I want to do is dream about math! Shoo! Scram!
ND: That's no way to talk to a devil!
Robert: I'm sorry. I'm listening.
ND: All right then. The thing that makes numbers so devilish is precisely that they are simple. And you don't need a calculator to prove it. You need one thing and one thing only: one.
Narrator #1: The Number Devil went on to explain to Robert that really, to make big numbers such as 5,723,812, all you have to do is start with 1 + 1 and go on until you have 5 million ... etcetera.
Narrator #2: The Number Devil also convinced Robert that once you got to 5 million ... etcetera, you could keep on going if you wanted to, because there is an infinite number of numbers.
Narrator #3: Poor Robert couldn't believe he was actually dreaming math. He pulled the sheets over his head hoping the Number Devil would go away. And he did...until the next night.
ND: Hey, Pretzel Boy! Tell me what you know about hopping numbers.

Wagon #2

Robert: I ... I don't know what you mean. Numbers don't hop!
ND: Want me to tell you?
Robert: Fine.
ND: Good. Okay, let's go back to square one. Or rather, the number one.
Narrator: #1: The Number Devil wrote this on Robert's wall.

Overhead turns on ... music

$1 \times 1 = 1$
$1 \times 1 \times 1 = 1$

ND: Take as many 1s as you like and you still get 1 for the answer.
Robert: Sure, but what's your point?
ND: You'll see if you try the same thing with 2s.
Narrator #2: The Number Devil erased all the 1s and wrote this.

Overhead ... music

$2 \times 2 = 4$
$2 \times 2 \times 2 = 8$
$2 \times 2 \times 2 \times 2 = 16$
$2 \times 2 \times 2 \times 2 \times 2 = 32$

Robert: Wow, that goes fast! If you go much faster I'll need my calculator!
ND: It goes even faster if you start with 5. Watch.
Narrator #3: In a flash, the 2s were down and the 5s were up.

Overhead ... music

$5 \times 5 = 25$
$5 \times 5 \times 5 = 125$
$5 \times 5 \times 5 \times 5 = 625$
$5 \times 5 \times 5 \times 5 \times 5 = 3125$

Robert: Whoa!
ND: Why do large numbers scare you? I assure you they're perfectly harmless.
Robert: Says you! Plus that's just too many 5s to have to write.
ND: Aha. I thought you'd never ask. You see, instead of writing all those 5s, I write

Overhead ... music

$5^1 = 5$
$5^2 = 25$
$5^3 = 125$

and so on. Now do the same with 10. You can throw your calculator away. Make the 10 do one hop, and it remains exactly the same

$10^1 = 10$

Make it hop twice and you get

$10^2 = 100$

Make it hop three times and you get

$10^3 = 1{,}000$

Robert: So if I make it hop five times, I get 100,000. Once more, and I get a million!
ND: You've got it, my boy! With the help of my friend zero and a bit of hopping you can produce any number you please. I'd say that's enough for tonight.
Robert: A hopping zero ... now that's a good one.
Narrator #1: Before Robert woke up he dreamed of the concept of a hopping zero.

Zero, zero, zero, zero ...
Set change.

A Place for Zero

Narrator #1: Not long ago, Zero lay floating on the calm waters of Central Lake. He could hear the happy cries of the other numbers, 1 through 9, as they played in the meadow. Zero didn't play Add-em-up because he had nothing to add. He felt he had no place among the other digits.
Narrator #2: Zero lived in Digitaria, a curious country ruled by King Multiplus and Queen Addeleine.

Digitaria song

Narrator #3: Count Infinity, the King's trusted advisor, was the one who shaped all the numbers. When old digits retired, he replaced them with shiny new numerals. Every number knew its place.
Narrator #1: A 7 was the number of days in a week, and a 5 was the number of points on a star.
Narrator #2: A 2 was handy for counting the wheels on a bicycle.
Narrator #3: The 1s were important because Count Infinity added them together to make the other numbers.
Narrator #1: Every number had a place except Zero. Count Infinity had been experimenting when he formed the strange new digit. But Zero meant nothing, and no one was sure what his job would be.

Zero song

Narrator #2: King Multiplus declared that no more zeros would be made until they found a purpose for this one.

King Mult: I, King Multiplus, King of Digitaria, do declare that no more *zeros* be made. Until we find a purpose for this *zero* we will not waste Count Infinity's precious time!

Narrator #3: Nowadays no one mentioned Zero much. The few times someone tried to count with him, they always ended up with nothing.

Counter: Okay, let's see what we have here. Zero? No, no ... let's try that again. Zero. What?! Aaahhh...

Zero: This just isn't fun anymore. I need to find my place. Count Infinity must have thought of something by now.

Narrator #1: Count Infinity made incredible things. He made knickknacks, whatsits and thingamajigs. The most impressive thing he ever made was an enormous machine—the Numberator!

Narrator #2: The Numberator had a vacuum tube on one side and a large curved spout on the other.

Narrator #3: When Count Infinity placed digits under the tube, they were sucked up with a great clanking and whirring until, finally, they emerged from the spout—with a totally new number!

Narrator #1: Count Infinity could put in two 1s to make a bouncy baby 2, or three 1s to make a roly-poly little 3.

Count Infinity (CI) song

CI: Ah, young Zero. You're just in time to help me figure out a better way to make a 1.

Zero: What do you mean?

CI: Well, most numbers are easy to make. I simply pop a handful of 1s into my trusty Numberator. Making a 1, though—that's trickier.

Zero: Why don't you just stick one 1 into the Numberator?

CI: I've tried that, but the Numberator adds. You have to put in at least two numbers. No two numbers that I can think of add up to ...

Narrator #2: Count Infinity eyed Zero in a strange way.

CI: Zero, could you put this 1 into the Numberator for me? Just help the 1 stand right under the vacuum tube.

Narrator #3: The Count made sure that Zero was right where he wanted him to be.

CI: Perfect! Just stand right there.

Narrator #3: As Zero did as he was told; he was sucked up into the Numberator along with the 1.

Narrator #1: Zero barely had time to realize what was happening before he was out the other side. Sitting next to him were the old 1 and a shiny new 1.

C.I: Wonderful! We have discovered your additive identity. I bet my binomials that if we add you to any number, we will always come up with the same number. You can help replenish the supply of digits in Digitaria!

Zero: Well, gee. I suppose I should be thanking you for helping me find my purpose, but 1s can make digits, too. I want to have a job that no one else has.

Narrator #2: As Zero made his way home he had another idea.

Zero: I wonder what would happen if I were multiplied with another number? I will ask the King.

Narrator #3: Everyone knew that multiplication was a powerful thing, and only the King could use it. Often the product of the multiplied numbers would be too large for anyone to understand.

Narrator #1: At the gates of the castle, Zero was stopped by two sharp-looking 7s.

Guard #1: What kind of number are you? You look like a 9 that someone squished!

Zero: I need to ask the King about multiplication.

Guard #2: Well, good luck to you.

Narrator #2: Zero entered the castle. He saw King Multiplus and Queen Addeleine on their thrones. Crowds of numbers filled the hall, waiting to address them. The 5 bowing next to Zero was rather surprised to see him there.

Five: Why are you here?

Zero: I'm asking the King to multiply me.

Five: Are you kidding?

Narrator #3: Suddenly, the numbers began to whisper.
Nine: That's absolute nine-sense!
Two: This is just two much for me to handle.
King Mult: What is the meaning of this?
Queen Add: It's Zero ... and something about multiplication.
King Mult: Multiplication! That is serious business. Let him come forward and explain himself.
Zero: I would like to be multiplied. I thought it might help me find my place and purpose.
King Mult: Yes. I have wondered myself what would happen. Guards, fetch me my Multi-tube.
King Mult: We need to find someone who is willing to be multiplied with Zero.
King Mult: Will no one come forward?
Brave One: I'll do it ...
Queen Add: Oh what a brave little 1.
Narrator #1: The guards led Zero and the 1 into the Factor end of the tube. All that could be heard was the clanking and whirring of the tube.
Narrator #2: Suddenly, Zero and the 1 shot out of the other end of the tube. As Zero looked around, he saw beside him a small, round figure who looked strangely familiar.
King Mult: Five alive! It's another zero!
Queen Add: Why, we must try that again.
King Mult: Yes, indeed. This time, we will multiply zero with a 7. Guard 7, step forward.
Narrator #3: The same thing happened with Zero and the 7.
King Mult: Well, Zero, you have your answer. No matter what number we multiply you with, we get zero!
Narrator #1: This was big news, but Zero still didn't seem pleased.
Zero: But King, now there are just more zeros with no real place. I want to make new numbers that no one has ever seen before.
Queen Add: Wait! What number do we get when we add 1 to 9?
King Mult: As everyone know, anything bigger than 9 is ... *many*. We don't think about 1 plus 9 because there is no digit for a number that big.
Zero: Sire! Look at us! We can make a new number to represent what you get when you add 1 and 9.
Zero: When I stand in this place, next to my friend 1, as a zero I can represent zero 1s. But he now represents 9 + 1.
King Mult: I like your thinking, Zero. But I can't call the new number *9 + 1*. Quick, think of a new name.
Zero: This new number will *attend* to a lot of business, Sire. We could call it Attend?
King Mult: I proclaim that this new number will be called a *TEN*!
Narrator #2: The numbers all loved it.
Numbers: Hooray for Ten! Hooray for Ten!
Narrator #3: The numbers crowded around Zero and asked if they could stand beside him. Zero gave each digit a turn and soon they were making wonderful new numbers like 20, 30, 60 and 90.
Narrator #1: Before long they were standing next to each other in different combinations, to make all kinds of new numbers, like 32, 47 and 89. Big numbers didn't seem quite so scary anymore.
Baby Zero: What about me? Is it my turn to stand next to you?
Zero: I'm sorry, little guy. Two 0s standing next to each other still equal nothing.
Narrator #2: The little 0 looked so sad that the brave 1 and Zero each put an arm around him. Suddenly, Zero had a thought.
Zero: Look, Sire! What do we look like?
King Mult: Why, you look like a new number, bigger than any I've ever seen. Bigger even than the new number 90.
Zero: We can represent what you get when you have ten 10s.
King Mult: Splendid!
Narrator #3: Soon, all the other digits were clamouring to stand next to the two 0s. They made numbers like 500 and 700.
CI: Well, Zero, did you find what you were looking for?
Zero: I just wanted to mean more than nothing. And now I think I've finally found my place.

Zero song

Zero song continues during set change.

Mom: Good morning, dear.

Robert: Good morning, Mom. Hey, Mom, did you know that if you wanted to count to 5,723,812 you can do so just by using 1s? And did you know numbers hop, that if you hop a 10 six times you get a million? And did you know that Zero means WAY more than nothing!

Mom: Robert, sometimes you just say the oddest things.

Narrator #2: In time, Robert grew accustomed to dreaming of the Number Devil.

Narrator #3: He even came to look forward to it.

ND: Good evening, Pretzel Boy! Ready for some more?

Wagon #3.

Robert: You bet!

ND: Alright then, I'll tell you a secret. There are two types of numbers. The garden variety, which can be divided evenly, and the rest, which cannot. Wonderful numbers like 11, 13 or 17.

Robert: Oh, that's easy—I know this one. They're all odd numbers.

ND: Oh, no, my young man. The numbers I'm talking about are more than just odd; they are numbers that are only divisible by one and itself. They are *prima donna* numbers.

Robert: Prima donnas? Why?

ND: Because from the very first they've caused mathematicians no end of trouble. Watch this. Would my numbers from 0 to 50 please come forward.

Parents in the crowd come forward. Music.

ND: Take my cane, Pretzel Boy, and tap the first prima donna number.

Robert: Well, that's simple—zero.

ND: WRONG! Zero is forbidden. We know the importance of our zero, but if we divide anything by it, we'll end up with zero. So, no. Off with you ...

Robert: All right, then—one.

ND: One doesn't count. Everything is made up of ones. Haven't you been listening to anything I've taught you? Off with you ...

Robert: Okay, okay. Calm down. Two. Two can be only be divided by one and itself.

ND: Oh, you're on a roll now!

Robert: Well, any other even number from now on will be divisible by 2, so they can't be prima donnas.

ND: You're right. All the even numbers ... off with you.

Narrator #1: Robert had ruled out 25 numbers.

Narrator #2: The Number Devil had given him one hint.

Narrator #3: That there were exactly 15 numbers between 0 and 50 that were prima donnas.

Narrator #1: Robert knew that once he passed by number 3, all other multiples of 3 should leave.

Narrator #2: He gently tapped them on the shoulder, and off they went.

Narrator #3: He did the same with 5, leaving it in the lineup, but asking all the multiples of 5 to please leave.

Narrator #1: 7 can stay, but all other multiples of 7 can't.

Narrator #2: Robert was pretty sure that he was done.

Narrator #3: But to make sure, he counted, hoping to have only 15 remaining.

Robert counts his numbers ...

ND: Well done, Robert.

Prima Donna song

ND: Prima Donnas, please have a seat.

ND chuckles to himself.

Robert: What's so funny?

ND: It's not so hard if you stop at 50. But what if it's a number like 421,356,237,307? Is it a prima donna or isn't it?

Robert: I … I don't know.

ND: If you only knew how many mathematicians have racked their brains over this issue. Why, even I have come to grief over it.

Robert: No offence, but if *you* can't look at a number and tell if it's a prima donna or not, there's no way I can. And I'm too tired anyway. I should be sleeping. I have a math test tomorrow.

Narrator #1: After explaining to Robert that he actually was sleeping, because in order to be dreaming him, Robert had to be sleeping, he decided to leave him be.

Narrator #2: The next night Robert went to bed without a bedtime snack.

Narrator #3: "It's all right, Mom," Robert called from his bedroom. "I don't want a snack; I just want to go to bed."

Mom: Oh, my! What has gotten into my boy? No bedtime snack? I promised him it wouldn't be pretzels!

ND: I bet you've forgotten your calculator again, eh, Pretzel Boy?

Wagon #4

Robert: Look, how many times do I have to tell you? I can't take all my stuff to bed with me at night. Do *you* know what you're going to dream the night before you dream it?

ND: Of course not. But still, if you dream of me, you can just as easily dream of your calculator.

Robert: Where am I? It looks like I'm in someone's vegetable garden.

ND: Perhaps you are. You recall our hopping game, I'm sure. What we did with the 2 and the 5 and the 10. Ten times ten times ten equals a thousand, which we write

$10^3 = 1{,}000$

because it's faster.

Robert: Right. And when we hop with 2s we get 2, 4, 8, 16, 32 ... and so on until the cows come home.

ND: Perfect! So now, let's do the first hop in reverse. Hopping backward, so to speak. Only when you go backward you don't really hop, we call the step *taking a rutabaga*. As if we were pulling one of these fine root vegetables out of the ground.

Rutabaga song

ND: So, what's the rutabaga of 4?

Robert: Two?

ND: Right. What's the rutabaga of 25?

Robert: Five!

ND: Rutabaga of 100?

Robert: Ten!

ND: Rutabaga of 36?

Robert: Six!

ND: Rutabaga of 5,929?

Robert: Are you crazy or something? How do you expect me to do that one? Mr Bockel asks me enough dumb problems in school. I don't need to dream about them.

ND: Calm down, calm down. This is what a calculator is for.

Narrator #1: The Number Devil showed Robert how to find the rutabaga of 5,929 using a calculator.

Narrator #2: As thrilled as Robert was to have learned something new, all this backwards hopping was making him tired. Or so he thought.

Narrator #3: Tired of convincing Robert that he was actually sleeping, the Number Devil tiptoed off, careful not to awaken him.

Look out---a coconut …

Main character change and coconut catchers set up.

Narrator #1: No sooner had Robert closed his eyes the next night than he found himself wandering through a desert.

Narrator #2: There was no shade or water, and he was wearing nothing but his shorts and a T-shirt.

Narrator #3: On and on he trudged, thirsty and sweaty, until at last he made out a few trees in the distance.

ND: Hello there, Pretzel Boy!

Wagon #5

Robert: I'm dying of thirst!
ND: How about some coconut milk?
Narrator #1: The Number Devil tossed down a coconut from the tree and Robert took a long drink.
ND: Heads up!
Robert: What?
ND: I'm going to throw down a few coconuts. Just toss them on the ground.
Robert: Anywhere in particular?
ND: No, just down.
Narrator #2: Robert threw the first coconut into the sand. It looked much like a dot.
Narrator #3: Robert continued to catch and toss until there were no more coconuts.
ND: What do you see?
Robert: Triangles! Funny, they fell into such neat patterns. And I wasn't even aiming. I'd never have been able to do that if I tried.
ND: You're probably right. Now, would you count up the number of coconuts in each of the triangles?
Robert: All right. But the first is not triangle at all. It's just a dot. And there is only 1.
ND: Ah yes, our good friend one. Next?
Robert: The second triangle has 3 coconuts, the third 6, the fourth 10, and the fifth … I'm not sure. Wait. Let me count.
ND: Why? You don't need to count. You can calculate it.
Robert: No I can't.
ND: Yes you can. Look—the first triangle consists of one coconut. The second has two more—the second row—which comes to three.
The third has exactly three more. $3 + 3 = 6$.
The fourth row has another row with four more. $6 + 4 = 10$.
Robert: Wait. Wait. I know. The fifth will be a row of 5 added. So $10 + 5 = 15$. And the next triangle would have 21 coconuts, 15 from the last one, plus the six new ones.
ND: You've got it, Robert!
Narrator #1: Robert felt quite pleased with himself as he made his way back into his bed, thankful to be out of the heat.
Narrator #2: Before Robert drifted off to sleep, the Number Devil snuck in one last trick. If you pick a number, any number, it can be shown to be the sum of two or three triangular numbers.
Narrator #3: Don't take our word for it. Try it!

Pick a number, any number, go ahead and try it.

Narrator #1: The next night, Robert just couldn't wait for the Number Devil to visit him.
Narrator #2: There was something he had been meaning to ask him.
Narrator #3: Something you may have also been wondering about.
ND: Wakey, wakey…

Wagon #6

Robert: Oh good, you're here!
ND: Wow. Someone's ready for some numbers.
Robert: No, no. It's not that. I've been meaning to ask you about all those rabbits that keep following you. It seems as if every time I see you, there are more rabbits.
ND: Like I said. You're ready for some numbers.
Robert: Huh?
ND: You see, in order to explain the rabbits, I need to tell you about a few numbers and about an Italian named Bonacci. He's been dead for years now, poor Bonacci, but he came up with what we call Bonacci numbers. A capital idea! And like most good ideas, it begins with …
Robert: One?
ND: And not just one, but two 1s.
Narrator #1: The Number Devil scrolled the sequence out on the wall.

Narrator #2: He explained that by adding the two previous numbers, you'd get the next number in the sequence.
Narrator #3: Robert was definitely intrigued, but he still didn't understand what this had to do with rabbits.
Robert: This is all fine and dandy, but tell me—what are these numbers good for, your Bonacci numbers?
ND: You don't think mathematics is for mathematicians only do you? Nature needs numbers too.
Robert: Come on. You don't expect me to believe that.
ND: I expect you to believe that every living thing uses numbers. Or at least behaves as if it did.
Robert: Well I don't believe it, and this still isn't explaining the rabbits.
ND: Okay then, let's talk rabbits. I bet there are rabbits all over this field.
Robert: I don't see any.
ND: Look, there are two now!
Narrator #1: Sure enough, two white rabbits hopped up to Robert and plunked themselves at his feet.
ND: A male and a female I think, or one couple. And as you know, one is all we need to get things rolling.

...talking to the rabbits...

Robert: He wants me to believe you can do arithmetic.
Narrator #1: The Number Devil explained to Robert that it would only take one month for these two rabbits to grow up, and then their fur turns brown.
Narrator #2: They are now old enough to have babies; and they do—two of them.
Narrator #3: Then their babies grow up and do the same.
ND: But don't forget about the original couple. They continue to have babies every month as well.
Robert: Whoa! I get the feeling there are going to be hordes of rabbits romping around here real soon!
Narrator #1: The Number Devil threw a chart up on the wall.
ND: Do you remember these numbers?
Robert: 1, 1, 2, 3, 5, ... Of course. It's obvious. Bonacci numbers all the way.
ND: You never cease to amaze me, Pretzel Boy! *Now* will you admit that rabbits behave as if they have learned their Bonacci numbers by heart?
Robert: Fine. Great. Anything you say. I admit it.
ND: You still don't sound convinced. Let me show you where else the Bonacci numbers are found in nature.
Narrator #2: The Number Devil took Robert over to a nearby tree.
ND: Look how the branches follow the Bonacci sequence.
Narrator #3: Robert noticed how the tree's branches grew in accordance with the sequence. As he made his way back to bed, Robert was quite intrigued by the tree and started to rethink that what the Number Devil was telling him about the Bonacci and nature connection was true.

Bonacci song

Mom is on the phone ...

Mom: I'm terribly worried. I don't know what's wrong with my boy. He used to spend all his time in the park, playing ball with his friends, but now he just shuts himself up in his room painting pictures of trees and rabbits!
And the numbers he keeps muttering to himself. Numbers, numbers and more numbers. It's not normal.
Don't worry? I've got to go ...
Robert, time for bed.
Robert: Woo-hoo!
Mom: Aaahhh ...
Narrator #1: No sooner did Robert's head hit the pillow than the Number Devil was on the scene.
ND: Today I have something extraordinary to show you.
Robert: Anything you like! Just no more rabbits
ND: What do you say we build a pyramid?
Narrator #2: The Number Devil took Robert over to a huge wall and handed him his walking stick.

Narrator #3: Following his mentor's directions, Robert outlined the shape of a very large triangle, and with a final zap …

ND: Voila!

Robert: This isn't a pyramid. Pyramids have triangular, or rectangular or square bases. This thing is flat.

ND: Fine. Then we built a triangle. Now take a look at the numbers and tell me what you notice.

Robert: I can see that the numbers along the sides are all 1s.

ND: Good.

Robert: And that the next diagonal rows on either side of the 1s are the perfectly normal counting numbers.

ND: What about the next diagonal row.

Narrator #1: Robert read down the row from right to left.

Robert: 1, 3, 6, 10 … Hey, they look familiar.

ND: Coconuts!

Robert: Right, right. Now I remember. The triangular numbers.

ND: You have been paying attention!

Robert: So how do you know what numbers go where?

ND: I thought you'd never ask. Take a look at any row and the row directly above it and see if you notice anything.

Narrator #2: Robert studied two rows.

Robert: Oh, I see it.

Narrator #3: Robert used the walking stick to show that numbers in each cube were the sums of the two cubes directly above it.

Robert: Fun!

ND: Oh, but there's more, Pretzel Boy!

Robert: There always is …

ND: Would you mind adding up the sums of each row for me?

Robert: Um … 1, 2, 4, 8, 16 … Oh, I know this one—these are the hopping 2s.

Narrator #1: As the Number Devil walked Robert back to his room, he told him about all the other patterns he'd be guaranteed to find in their magic triangle.

Narrator #2: Patterns like magic 11s, multiples of 5, hockey sticks and even the Bonacci numbers!

Narrator #3: Robert was once again thrilled with this evening's number show.

Narrator #1: He tucked himself into bed with a smile and closed his eyes in anticipation of his friend, the Number Devil's next visit.

Parade music

All students in hallway preparing for number parade.

ND #1: Robert, we've got just one final show for you. Would my hopping numbers please come in?

ND #2: Otherwise known as *powers*.

ND #1: Would my prima donna numbers please come in.

ND #2: Otherwise known as *prime numbers*.

ND #1: Would my coconut numbers please come in.

ND #2: Otherwise known as *triangular numbers*.

ND #1: Would my Bonacci numbers please come in.

ND #2: Otherwise known as the *Fibonacci sequence*.

Robert: I don't think we'll ever look at numbers the same way!

Parade music and out.