# Perceptions of Problem Solving in Elementary Curriculum

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## Introduction: A Look at the Meanings of Problem Solving

Recent research with intermediate teachers[1] indicates that the phrase *problem solving* often evokes multiple meanings in mathematics teaching and learning (Kajander and Mason 2007). While some teachers support the vision of problem solving espoused by the National Council of Teachers of Mathematics (NCTM) (2000) as "engaging in a task for which the solution method is not known in advance" (p 51), others view it as something to be done after students are taught and only if there is time (Kajander and Mason 2007). The goal of this article is to examine the usefulness and intent of the various meanings of this "problematic" phrase, while shedding light on the best way to engage in effective problem solving with students.

The inclusion of problem solving in mathematical learning is not new. Polya's (1957) famous model is even included in some provincial curricula (for example, Ontario's Ministry of Education 2005), as shown in Figure 1, and is one of the best known outlines of the possible processes involved in problem solving. What is perhaps new in many classrooms is that effective problem solving should be more than having students solve a problem using formulas or methods that *the teacher has previously shown*.

As a Grade 2 teacher in the United States, I was able to experience first-hand the effects of new legislation that required us to drill mathematics facts and algorithms into the minds of our students so that they could survive testing. Each year the same concepts had to be reviewed, because retention was minimal if at all. We met each year as a school staff to discuss ways to better meet the needs of the students with information handed to us from our school board. We discussed at length using problem solving to improve our mathematics test scores. Yet what this actually entailed, according to what we were told, was handing students a sheet of word problems to solve using the algorithm the teachers had already given them to use. It was suggested that we do a similar word problem with the students so that they would follow a similar process. Similar understandings of the implementation of problem solving have also been found in some Canadian classrooms (eg, Kajander and Mason 2007; Kajander and Zuke 2008).

Elementary mathematics curricular goals may refer to problem solving without exploring what the phrase really means. In Alberta, for example, the curriculum endorses the importance of using a problem-solving approach noting that "students need to explore problem-solving strategies in order to develop personal strategies and become mathematically literate" (Alberta Education 2007, 1). Ontario, as well, mentions that "problem solving forms the basis of effective mathematics programs and should be the mainstay of mathematical instruction" (Ontario Ministry of Education 2005, 11). These statements could be interpreted in multiple ways. From these statements in the mathematics curriculum guides, teachers could assume that problem solving means having students read a problem and find a correct solution using a given, previously taught method. Research has shown that this is not the most effective way to use problem solving in mathematics classrooms (Bay-Williams and Meyer 2005; Boaler and Humphreys 2005; Buschman 2004), nor is it the most effective method for teaching mathematics (Askey 1999; NCTM 2000; Van de Walle and Lovin 2006). This article will further examine some alternatives to this common interpretation.

## Problem Solving *as* Learning

I take the stance that *true* problem solving involves students *really learning* something new and not just applying a previously taught strategy to a new example or task. This position underscores the importance of problem solving *as* learning. As Bay-Williams and Meyer (2005) note, "teacher-directed instruction may help a teacher feel that more topics have been covered, but it reduces the chances that students are (1) making connections with other mathematical ideas and (2) understanding the concepts related to the skill" (p 340). In fact, students should engage in rich problem-solving tasks ***in their daily** mathematics classroom experience* in order to construct new knowledge and understanding by connecting it to their previous knowledge.

This interpretation of effective problem solving differs from the belief that students must be taught the concepts before they can engage in problem solving (Kajander and Mason 2007; Kajander and Zuke 2007, 2008). One view might be that by assigning the problem-solving questions in the textbook for homework, exercises, and on tests, students have the opportunity to problem solve. Typically, such problems are really *applications* of known formulas or methods to new examples. However, the goal with problem-solving tasks should be to allow students to figure out *how* they will solve the problem. The importance is placed on the *method for determining the solution*, as opposed to the solution itself. As McGatha and Sheffield (2006) point out, in problem-solving classrooms "students are pushed beyond simply *finding a right answer to questioning the answer*" (p 79; emphasis in original). The one single right answer is no longer the singular goal of the mathematics classroom; rather, the process taken to find an answer is where the real learning lies. Students should subsequently be given opportunities to discuss how they solved the problem so that they can learn from each other and see different ways of arriving at a possible solution. This is very different from classrooms in which the teacher tells the students how to go about solving problems so that they can arrive at the single right answer in this so-called correct way. True problem-based learning involves students constructing new ideas based on their experiences with appropriate problems, *not* applying known methods to new contexts.

## How Effective Problem Solving Is Accomplished

Effective problem-solving tasks can be implemented as part of a three-part lesson plan (Van de Walle and Lovin 2006). It is important to consider that the actual lesson may take more than a single mathematics class period to finish, depending on the students. In the first part of the lesson, the teacher sets up the current problem to be worked on. The teacher acquaints the students with any previously unknown vocabulary at this time. This portion of the lesson does *not* include the teacher showing the students a similar problem and how to solve it. After the teacher sets the stage for learning, the students begin to explore the given problem.

The second stage of a problem-solving lesson requires teachers to set up an environment and procedures that are conducive to exploratory learning. While exploring the problem, students may work individually, in pairs or in groups. Students need to be arranged in a way that allows them to share their ideas with each other. During this phase of the lesson, the students work with the problem to figure out a solution method that makes sense to them. As students work with the mathematics concepts embedded in the problem, they should record their thoughts to share during the final portion of an effective problem-solving lesson—the discussion.

Discussion is an absolutely essential phase of the problem-solving method because it allows students to come together and share while explaining their thinking. As Boaler and Humphreys (2005) note,

> students are not asked to present their answers; they are asked to show representations of their ideas and to justify why they make sense. None of the audience members will have the exact same answer, and all the students have a role. (p 50)

Not only are students more engaged while discussing ideas with their peers, they also learn more from each other and discover new ways of thinking about a problem. Students need to be able to put their solutions into words and discuss how they solved the problem so that they can explain their methods to others. This forces students to get at *how* their solution was found, not just what they decided was the correct answer. It is important that students learn "to *question the answers* by posing additional questions when solving the original problem [because this] is one way that teachers and students can develop mathematical power" (McGatha and Sheffield 2006, 79; emphasis in original). It is this power that helps further students' understanding of and learning in mathematics. Boaler and Humphreys (2005) suggest using the method of "convincing a skeptic" when trying to explain the solution the students came up with (originally from Mason, Burton and Stacey 1982). Their belief is that "this strategy ... helps place responsibility on the person who is explaining to make his (sic) explanations understandable and gives permission for anyone who doesn't understand *yet* to play the role of being unconvinced rather than being just slow to catch on" (p 67; emphasis in original). Students are given the opportunity to question each other and refine their thought processes until everyone sees why the solution method works. Seeing alternative solutions is important because "if their knowledge is limited to the computational procedure without any idea why the procedure works, this is also not enough to build on. Students need both" (Askey 1999, 3). Through exploring a problem and discussing the solution, students learn how and why their method and procedures work and gain deeper mathematical understanding. At this point, teachers can help students see the generalizations or the procedures that are being developed through examining the students' solutions. Teachers play an important role in fostering this development of ideas. Since students are sharing their knowledge and understandings, or even misunderstandings, during this portion of the lesson, the teacher must create an environment where all contributions are valued and allowed to be expressed.

In order to use the problem-solving method effectively, students must be given opportunities to share their solution methods so that the teacher can see where any misunderstandings or confusions lie. These essential discussions also allow students to learn from each other. This very important aspect can be the deal breaker for the success of problem-based lessons if the teachers do not allow time for sufficient sharing of ideas. Sometimes issues or difficulties that arise during the discussion can prompt the teacher to suggest a new problem for the next class.

In a problem-solving lesson as just described, problem solving is the vehicle for knowledge and learning instead of simply the way that students showcase what they have learned. One issue with doing problem solving *after* the teacher has *taught* a concept is that students have trouble switching from a teacher-directed lesson one day to a lesson in which they control the learning path the next (Van de Walle and Folk 2007). Also, if students are to truly engage in problem solving, they need to know that the teacher is not about to step in and tell them the strategy or the answer eventually. If they feel that this will happen, my experience is that some students will simply wait for the instruction or answer to come from the teacher and therefore will not deeply engage in the task. In other words, they feel that their work will be devalued eventually when the teacher provides the right answer or method, and they have simply learned to wait for this to happen. By engaging in problem-solving lessons as the main curricular vehicle, students learn that their thoughts and ideas are important and are correct ways to solve a problem.

Teachers should choose problems that allow students to explore and construct knowledge for the big ideas or the overall expectations of the grade level. This allows the teacher to address multiple curricular expectations in one problem while, at the same time, addressing the needs of different learners. The benefit of well-chosen problems is that they "can be solved at different levels of sophistication, enabling all children to access the powerful mathematical ideas embedded in the problem" (English, Fox and Watters 2005, 156). For example, a problem like the handshake problem could be used with a class: "There are 20 students in a class. On the first day, the teacher asks each student to shake hands with each other student. How many handshakes were there?" (Small 2008, 567; similar problem in Kajander 2007). In order to make the problem accessible to all students, the teacher could make several different size classes, starting with 5 students and ranging up to 20. Students who are more advanced could begin looking for patterns in the different numbers in order to make the problem more challenging, while students who are struggling can simply tackle what would happen if five students shook hands. By having students solve the problems from their ability level, all students are engaged and learning from each mathematics lesson.

The problem-solving approach also allows all students to be included in the discussions. Students choose methods to solve the problem that make sense to them, which is more meaningful than just repeating what the teacher has said. Using problem solving in the classroom allows all students to reach mathematical understanding at a level that they are comfortable with. Since the goal is to have students use their prior knowledge, all students will be able to work with the problems using what they already know to build their own new ideas. As the Alberta curriculum asserts, "students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics" (Alberta Education 2007, 1). Teachers can also use this baseline knowledge to help students to come up with new ideas and more effective solution methods instead of teaching formulas that students apply without really understanding. For example, the solution to the handshake problem could be arrived at in many different ways, including drawing a picture, acting out the problem with children, looking for patterns or even solving algebraically. Students would be able to solve the problem with their own solution methods, but during the discussion would be exposed to all the different methods and thereby learn from the other students. The problem itself can be used to teach or review addition, multiplication, division, geometric patterning, numeric patterning, pattern rules and iterative patterns (Kajander 2007), depending on how the teacher guides the students through the discussions and what areas are highlighted as students present their solution methods.

One caution does need to be made in choosing effective problems to solve in order for students to gain the most benefits. Teachers should avoid forcing too much content into a single lesson; therefore, each lesson "focuses on investigating one rich problem, probing deeply into a different mathematical content strand each day" (McGatha and Sheffield 2006, 79). By narrowing the focus to one main concept each day, teachers can allow students to look further into the problem in order to reach a deeper understanding. For example, simply introducing a simple problem like the handshake problem with different-sized classes would allow the students to explore the solution methods; the teacher could then guide discussions to accomplish the necessary curriculum goals. By focusing on one problem, students are not overwhelmed by a worksheet full of problems and could be challenged to come up with multiple solution methods. Another important consideration is that, as one teacher said, "too much choice could be overwhelming for the children and difficult for me to manage" (Whitin 2004, 181). Putting too much into one lesson is not only hard for a teacher to organize and observe, but it can confuse students and prevent them from delving deeply into the topic being explored.

Another benefit of using problem solving extends beyond the mathematics classroom. Since the goal is not for teachers to show students a formula and the exact method to solve the problem, students use their *own* problem-solving skills to solve the problem. This can affect students' lives—not only do students learn mathematical concepts with deep understanding, they also gain skills that enable them to solve problems in their daily lives. The benefits of using problem solving and allowing students to learn how to solve a problem in their own daily lives are great. I turn now to showing how this can fit within the mathematics curriculum.

## Examples of Using Problem Solving with Ontario and Alberta Curriculum

It is my experience that curriculum guides mention problem solving while not explicitly laying out how to use problem solving in a classroom. For example, in the curriculum I am most familiar with, the Ontario Ministry of Education sets out several mathematical processes that should be included in the elementary curriculum: "problem solving; reasoning and proving; reflecting; selecting tools and computational strategies; connecting; representing; communicating" (Ontario Ministry of Education 2005, 11). These processes are listed as separate entities; yet an effective problem-solving approach to teaching would encompass all of these processes and would, therefore, be the only method necessary to accomplish these curricular goals. After being given a problem, students would have to *select tools* and the *computational strategies* needed to solve the problem. Since the students would be using prior knowledge to pursue a solution, they are *connecting* the new concept to previous knowledge and skills. Students would then be required to *reason* through their solution and *prove* that it works to the class and teacher. Through the discussion of the solution, students would have to *reflect* on whether or not their method makes sense in order to solve the problem. By sharing their solution with others, students would be required to *communicate* their thought processes and *represent* the solution so that others can see how they solved the problem. Using problems with a similar focus on different days would allow students to practise their skills and create more in-depth conceptual knowledge. By using a problem-solving approach to teaching, teachers are able to simplify their planning while meeting all the goals of the curriculum.

In Alberta, the curricular goals identified are that students will "use mathematics confidently to solve problems; communicate and reason mathematically; appreciate and value mathematics; make connections between mathematics; make connections between mathematics and its applications; commit themselves to lifelong learning; [and] become mathematically literate adults, using mathematics to contribute to society" (Alberta Education 2007, 2–3). As with the Ontario curriculum, the curriculum guide provides the goal of using problem solving in the classroom as part of the routine. Effective problem solving would also help to accomplish the other goals by giving students ample opportunities to use mathematics in meaningful ways that will benefit students throughout their lives. Where Alberta's curriculum differs from Ontario is that it explicitly states that students "must realize that it is acceptable to solve problems in a variety of ways and that a variety of solutions may be acceptable" (p 1). This statement lends itself more to the problem-solving methods described within this article, but teachers should keep in mind that this should not mean giving students different solution methods but allowing them to discover multiple solution methods. The two curriculums mention the importance of problem solving, but it is my experience that teachers are often left to their own devices to locate the problems that would address these goals. Some resources that I have used and found successful with students include *50 Problem-Solving Lessons: Grades 1–6*, by Marilyn Burns, and *Big Ideas for Small Mathematicians*, by Ann

Kajander. The problems in these books mention different curriculum strands addressed by the problems, and a single problem could be used in a lesson to give students a chance to delve deeply into the topics. In order to encourage more problem solving, the Ontario Ministry of Education has also created its own lessons that relate to the curriculum for Grades 7 to 10, called *Targeted Implementation and Planning Supports* (TIPS). Textbooks might also be a helpful tool—teachers could choose or create a single problem from the lesson that students could explore on their own in order to determine their own solution methods.

In Ontario, the Ministry of Education (2005) does provide a valuable framework that can be used with students while exploring a problem. During the exploratory phase of problem solving, the Ministry of Education suggests using Polya's problem-solving model (see Figure 1) to guide the students in thinking about how to solve a problem. The belief is that teachers should guide students in Grades 1 and 2 through the model without directly teaching the steps, whereas students in Grade 3 and above should be taught the terminology of each step of the model directly. For Grades 1 and 2, a simpler way of remembering the steps can be beneficial.

Thomas (2006) has suggested the use of the THINK strategy to get students organized in their thinking, which could be a useful mnemonic for Grades 1 and 2. First, students *talk* about the problem. Second, students look at ***how*** the problem could be solved. Third, students *identify* a strategy for solving the problem. Fourth, students *notice* how the strategy helped solve the problem. Finally, they ***keep*** thinking about the problem. As students continue working on the problem, they may need to cycle through this framework several times until they arrive at a solution that makes sense to them. According to the research study, Thomas notes that "students who used THINK demonstrated greater growth in problem solving than students who did not use the framework" (p 86). The use of a model is beneficial because

> a teacher who is aware of the model and who uses it to guide his or her questioning and prompting during the problem solving process will help students internalize a valuable approach that can be generalized to other problem solving situations, not only in mathematics but in other subjects as well. (Ontario Ministry of Education 2005, 13)

Students could be coached using this model while they are exploring the problem. The Polya model should be directly taught to higher grade levels (Ontario Ministry of Education 2005). While older students are working with the problem, the first step is for them to understand the problem. According to Outhred and Sardelich (2005), "understanding the problem requires children: to be able to read the problem; to comprehend the quantities and relationships in the problem; to translate this information into mathematical form; and to check whether their answer is reasonable" (p 146). Students begin by rereading the problem and deciding what the problem is asking them to figure out. Next, students make a plan for deciding how to solve the problem through examining different strategies to solve it. As Askey (1999) discovered when working with teachers, "the teachers argued that not only should students know various ways of calculating a problem [solution] but they should also be able to evaluate these ways to determine which would be the most reasonable to use" (p 6). Third, students enact the plan that they decide

**Understand the Problem**
**(the exploratory stage)**

- reread and restate the problem
- identify the information given and the information that needs to be determined

*Communication:* talk about the problem to understand it better

**Make a Plan**

- relate the problem to similar problems solved in the past
- consider possible strategies
- select a strategy or a combination of strategies

*Communication:* discuss ideas with others to clarify which strategy or strategies would work best

**Carry Out the Plan**

- execute the chosen strategy
- do the necessary calculations
- monitor success
- revise or apply different strategies as necessary

*Communication:*

- draw pictures: use manipulatives to represent interim results
- use words and symbols to represent the steps in carrying out the plan or doing the calculations
- share results of computer or calculator operations

**Look Back at the Solution**

- check the reasonableness of the answer
- review the method used: Did it make sense? Is there a better way to approach the problem?
- consider extensions or variations

*Communication:* describe how the solution was reached, using the most suitable format, and explain the solution

*Figure 1. Polya's problem-solving model. (Ontario Ministry of Education 2005, 13).*

to use to solve the problem. Finally, students assess whether or not the solution is reasonable through re-examining the problem. If the solution is determined to be unreasonable, students would then go back through the model. This model is important because when students are used to traditional instruction they typically do not have the skills and strategies developed to effectively problem solve (Van de Walle and Folk 2007; Kajander and Zuke 2007). Once students are given a problem, they need to be given a way of organizing their thinking in order to solve the problem.

## Summary

Problem solving may have varying definitions for different teachers, but effective problem solving should allow students to explore a problem for themselves to find a solution. I have argued that problem solving does *not* involve giving students a method or formula for how to get the answer; rather, it involves giving them a framework to think through the problem and work to develop their own method. Students need a structure to develop problem-solving skills, and this must be supported by peer and teacher-facilitated discussion at certain points in the learning. Neither of these can take place when problem solving is attempted in isolation as homework or on tests. True problem solving cannot happen for most students (or even most mathematicians!) in a time-limited situation such as a test. Students need time to reflect, discuss, and try possibilities. Tests are simply *not* good places to attempt problem solving. While tests might play a role in efficiently assessing procedural skills, learning and assessment tasks are much better vehicles for learning through problem solving. Teachers in an effective problem-solving environment are no longer disseminators of knowledge but facilitators and coaches who help students create their own knowledge. In my experience, one of the most rewarding experiences can be watching students grapple with a problem and come to a solution after they have worked on the concepts within the problem. The excitement and feelings of accomplishment that accompany the final product can be empowering to their mathematical abilities, as well as foster the idea that they, too, can do mathematics! While using problem solving and discussion may be uncomfortable at first, the long-term benefits for both student learning and engagement are phenomenal (NCTM 2000). The goal of mathematics classrooms is to have students learn and understand mathematics, and engaging in effective problem-solving tasks is the best way to accomplish these goals.

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## References

Alberta Education. 2007. *Mathematics Kindergarten to Grade 9*. Program of studies. Edmonton, Alta: Alberta Education.

Askey, R. 1999. "Knowing and Teaching Elementary Mathematics." *American Educator* 23, no 3. Also available at www.aft.org/pubs-reports/american_educator/fall99/amed1.pdf (accessed January 25, 2007).

Bay-Williams, J M, and M R Meyer. 2005. "Why Not Just Tell Students How to Solve the Problem?" *Mathematics Teaching in the Middle School* 10, no 7: 340–41.

Boaler, J, and C Humphreys. 2005. *Connecting Mathematical Ideas: Middle School Video Cases to Support Teaching and Learning*. Portsmouth, NH: Heinemann.

Burns, M. (1996). *50 Problem-Solving Lessons: Grades 1-6*. Sausalito, Calif: Math Solutions Publications.

Buschman, L. 2004. "Teaching Problem Solving in Mathematics." *Teaching Children Mathematics* 10, no 6: 302–9.

English, L D, J L Fox and J J Watters. 2005. "Problem Posing and Solving with Mathematical Modeling." *Teaching Children Mathematics* 12, no 3: 156–63.

Kajander, A. 2007. *Big Ideas for Small Mathematicians*. Chicago: Zephyr.

Kajander, A, and R Mason. 2007. "Examining Teacher Growth in Professional Learning Groups for In-Service Teachers of Mathematics." *Canadian Journal of Mathematics, Science, and Technology Education* 7, no 4: 417–38.

Kajander, A, and C Zuke. 2007. *Factors Contributing to Success for Intermediate Students of Mathematics: The Needs of the Teacher and the Characteristics and Needs of Students at Risk. Final Report*. Thunder Bay, Ont: Northern Ontario Educational Leaders (NOEL). Available at http://noelonline.ca/depo/fdfiles/KajanderNOEL%20FINAL%20report%20APRIL%2030%202007.pdf (accessed September 21, 2009).

———. 2008. "Listening for Their Voices: A Study of Intermediate Students at Risk in Mathematics." *delta-K* 45, no 2: 21–24.

Mason, J, L Burton and K Stacey. 1982. *Thinking Mathematically*. London: Addison-Wesley.

McGatha, M B, and L J Sheffield. 2006. "Mighty Mathematicians Using Problem Posing and Problem Solving to Develop Mathematical Power." *Teaching Children Mathematics* 13, no 2: 79–85.

National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics*. Reston, Va: NCTM.

Ontario Ministry of Education. 2005. *The Ontario Curriculum Grades 1–8: Mathematics*. Rev ed. Ontario: Queen's Printer.

———. *Targeted Implementation and Planning Supports for Revised Mathematics*. Available at www.edu.gov.on.ca/eng/studentsuccess/lms/tips4rm.html (accessed September 21, 2009).

Outhred, L, and S Sardelich. 2005. "'A Problem Is Something You Don't Want to Have': Problem Solving by Kindergartners." *Teaching Children Mathematics* 12, no 3: 146–54.

Polya, G. 1957. *How to Solve It*. 2nd ed. Garden City, NY: Doubleday.

Small, M. 2008. *Making Math Meaningful to Canadian Students, K–8*. Toronto, Ont: Nelson Education.

Springer, G T, and T Dick. 2006. "Connecting Research to Teaching: Making the Right (Discourse) Moves: Facilitating Discussions in the Mathematics Classroom." *Mathematics Teacher* 100, no 2: 105–9.

Thomas, K R. 2006. "Students THINK: A Framework for Improving Solving." *Teaching Children Mathematics* 13, no 2: 86–95.

Van de Walle, J, and S Folk. 2007. *Elementary and Middle School Mathematics: Teaching Developmentally*. 2nd Canadian ed. Toronto, Ont: Pearson.

Van de Walle, J A, and L H Lovin. 2006. *Teaching Student-Centered Mathematics: Grades 3–5*. Boston, Mass: Allyn & Bacon.

Whitin, P. 2004. "Promoting Problem-Posing Explorations." *Teaching Children Mathematics* 11, no 4: 180–86.

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