Possibilities for Understanding Children's Mathematics Knowledge

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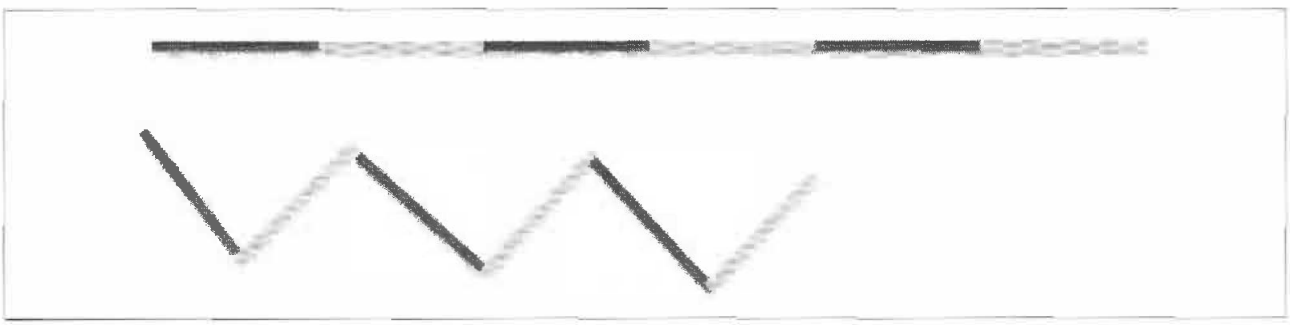
During the question about conservation of length, Michael, a child in Grade 1, was shown two rows of six rods laid end to end in a straight line. Michael was told that a bunny rabbit was hopping along each row, then he was asked if each rabbit had the same distance to go; Michael responded, "Yes." For the next question I altered one road by changing its path from straight to a zigzag pattern, using the same six rods (see Figure 1). Michael was again asked if the rabbits had the same distance to go and responded, "No."

I think Michael saw the two roads as a race—he stated his concern that the slight break between rods in the zigzag road might cause the rabbit to fall off the road, thus making the rabbit slower. He could not understand what *same length* meant. Length and equality were concepts I thought he understood from his work with length and distance months earlier in the classroom. He continually looked to me for some type of cue to help him understand what I was looking for in this question. I found it hard to explain what the question was asking without prompting him. I believe Michael thought I was trying to trick him. Often in my classroom I ask students how many ones are in the ones' house when there is zero or how many nickels are in a piggy bank that only has quarters and dimes. Because of this classroom trickery I feel Michael was looking for much more than the question asked (field note, February 10, 2009).

In this opening field note Michael and his teacher, Raina (the *I* in the field note), illustrated for us the complex understanding necessary when teachers and children are involved in assessment. Michael and Raina were in the process of demonstrating their knowledge, his of length or distance, and hers of mathematics education. This field note was a piece of an interview that was part of a three-year inquiry into children's and teachers' identity-making possibilities in mathematics assessment.[1] In the above field note both student and teacher wish to be understood as mathematical and knowledgeable, a common feature in any assessment.

The video of this assessment moment shows that Raina worked diligently to elicit a response from Michael that was correct with respect to what the assessment was focused on—the conservation of length. Michael, however, brought a number of contexts into play. He was concerned about the welfare of the rabbit, he saw the breaks between the rods as important and, in response to the question, "Did the rabbit travel the same distance?" he replied, "No." As our research team, which now includes Raina in a different role from that of assessor, watched and discussed the video, it was evident to us that the rabbit on the angled rods did not get as far on the surface of the desk as the one on the straight rods. A further complication, in Raina's thinking, was that Michael thought she was trying to trick him, which is an aspect of Raina's teaching that she uses to provoke the

Figure 1



children's thinking. Raina thought that Michael imagined that something else was at play in this assessment moment.

After video-recording Michael and Raina in the assessment, a member of our research team videoed Michael watching the assessment to see if he could comment about what he was doing; this was followed by an interview with Raina. In the interview, Raina raised concerns about the misunderstanding of distance and length. Raina knows Michael as a thinker. She knew that he was involved in an intellectual negotiation about length and distance, and she knew that he knew there would be a reason for the questions Raina was asking. Later, as our research team viewed both the assessment videos and the interview with Raina, we discussed what might have been happening in the assessment moment. In this space for thinking we began to see the complexity in understanding distance and length.

Distance and length, while seemingly similar, ask us to attend to different conceptions. Distance is defined as "the length of the line segment joining two points" (James and James 1992, 130) or "the separation between two things measured in units of length, or the length of a path joining two points" (Fyfield and Blane 1995, 70). Length is "the number of times a unit interval will fit in the line segment" (James and James 1992, 246) or "one-dimensional extent measured in units defined by a line segment" (Fyfield and Blane 1995, 125).[2] This may seem straightforward, but notice the nuance in the definition of *length* and consider that the line segment between the two points is no longer straight—therefore, the distance travelled by Michael's rabbit changes. We understood that the intent of the question was to learn whether or not a child could comprehend the notion of the conservation of length; that is, the length of the path that the rabbit travels does not change when the rods are angled.[3] But when Michael compared the angled rods to the straight rods, he could see that the rabbit did not travel as far on the desk. The straight rods got the rabbit further ahead on the desk. Michael saw it as kind of contest. In fact, in the assessment interview he said that the rabbit on the straight rods would win. This would indicate that Michael does not yet have an in-depth understanding of length and distance because he is attentive to the context. In fact, we do not know about the level of his understanding of length and distance because he is working so hard to help Raina understand the importance of the context.[4]

In the list of achievement indicators for Grade 1, an indicator of meeting the measurement outcome is "determine which of two or more given objects is longest/shortest by matching, and explain the reasoning" (Alberta Education 2007, 61). For Michael, the story of the rabbit trumps the comparison of the length of the rods. In relation to this achievement indicator, while Michael might be seen as struggling with the first part of the indicator, he is proficient at the second. His reasoning is sound.

The implication for educators is to be able to elicit knowledge from children about their understanding of concepts such as distance and length in ways that attend to the complexity of their thinking. The strength of the one-on-one assessment interview is that it allows us to more fully engage with the child's reasoning, which might mediate our evaluation of a child's knowing.

This research takes up the work of Dr Grayson Wheatley (Wheatley 1990, 1991, 1992, 2002; Wheatley and Reynolds 1999) and his deep interest in the complex thinking of children and the possibilities for encouraging this thinking in mathematics classrooms. We used an assessment instrument designed by Wheatley to be conducted in a one-on-one interview between a child and a teacher. We hoped that using this assessment interview would help us discover opportunities for children to demonstrate more complex mathematical thinking not typically found in paper-and-pencil assessments.

As we conducted this inquiry, we came to realize that the interview gave us a different understanding of the children and their mathematical knowledge. There have been many calls for teachers to differentiate instruction in order to meet the diverse educational needs of children. We began to realize that there is also a need to differentiate assessment. Differentiated assessment necessarily goes hand in hand with differentiated instruction. We awakened to this notion as we watched children and teachers work together in their assessment making (Clandinin et al 2006) during the mathematics interview. We saw how the act of conversing with children helped teachers know the children with whom they worked in more complex ways. This has implications for teaching styles and how teachers teach different mathematics concepts. Through the interviews, teachers reached an important realization of how they began to understand the children as knowers and sense makers.

Differentiated assessment does ask teachers to consider more in their work alongside children in mathematics classrooms. We are not suggesting that paper-and-pencil assessments be replaced, but, rather, that they are not the only way of understanding what children know. Wheatley's interview assessment was a tool we chose to use in our work. It is a multiquestion assessment that takes approximately 45 minutes, though using only a few questions derived from

planning or a more traditional assessment might suffice. In fact, we can see how a planned conversation (Glanfield et al 2003) about parts of a paper-and-pencil assessment would provide a deeper understanding of the child and provide the teacher with more information to communicate to parents.

Notes

1 This inquiry was supported by a grant from the Dr Stirling McDowell Foundation for Research into Teaching.

2 As our research team discussed meanings of distance and length, we learned that many definitions of *distance* include the word *length*. We began to wonder about the ways in which we use these terms in our own practices and how each of us had come to make sense of the terms.

3 Our research team also noted that the manipulative used in this assessment task was Cuisenaire rods. The nature of the rods actually shows that the length of the path might change when the rods are angled, depending on where the rabbit travels on the rod. For example, if the rabbit travels down the middle of the rods then the rabbit would have to hop over a small gap between rods when they are angled.

4 Our research team also discussed the questions that Raina might have used to learn more about Michael's understanding of distance and length, and we discussed how Raina might use what she now knows about Michael's conceptions of distance and length to plan for future instruction. We do not include the discussion of these items in this paper, because the focus of the paper is on what can be learned from children about their mathematical knowledge in an assessment interview.

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