Subitizing: Part-Part-Whole of a Big Idea

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A continual cause for concern among many math teachers is how to help struggling students in our math classrooms. Some children who arrived in my Grade 4 or 5 math classes over the years struggled with number sense, and many came without the ability either to compose or decompose numbers or see part-part-whole relationships. The obvious breakdown in students' conceptual understanding of part-whole reasoning often occurred when students were asked to break up numbers into different parts and then recompose them or represent them in a different way. This was a common weakness for many students in my math classes and seems to still be an issue in many math classrooms today. After much reflection and searching, I have determined that one big idea in mathematics seems to provide more insight into why students struggle with number sense. This is my journey towards greater understanding of subitizing.

Subitizing in Relation to Counting

One of the most significant changes to my ideas of mathematical instruction in early years is the shift in emphasis away from the count as the first step to numeracy and quantifying. This change in focus is very difficult for many early childhood educators and parents to accept because it is a definite shift from the way we were taught as children. In the past 20 years of my career I was of the same mindset: students should count first, and then they can recognize quantity meaningfully—but this is not necessarily true. I was first inspired to look at a change in thinking regarding counting about three years ago, when I read an article by John Marshall in *Phi Delta Kappan* (2006), in which he stated

> Children come to understand these numbers as complete entities before counting. Counting comes after the numbers have been placed in order and when children know why three is more than two. Matching a set of three (cups) with two (saucers) in one-to-one correspondence will show which set has more members. (p 359)

Marshall's claim made me rethink my understanding of early learners' capacity to identify number and quantity as well as question the importance of the rote count first versus matching, comparing and subitizing along with counting. Having worked primarily with older children, I had evidence that children who could readily count and read number words could not always quantify number. Many of the struggling students in my classroom over the years had had extreme difficulty breaking apart numbers (decomposing) and putting numbers back together (composing). I had incorporated some visualization or subitizing activities into a mental math portion of my lessons as starters. This visualization focus seemed to significantly change the way students saw and understood number. Clements and Sarama (2009) would identify this change as an incorporation of "conceptual subitizing—seeing the parts and putting together the whole" (p 9). It had a substantial impact on the older students' ability to visualize number as a composition of parts. This led me to pose yet another question: How could using subitizing activities with very young children affect their learning of number?

Recollecting my experiences with Division II students, I had observed that many of the struggling students could recognize regular arrangements of number, like those on dice or playing cards, but were unable to relate this and apply their recognition of number to irregular arrangements of dots. Students who had difficulties with the visualization or subitizing activities were often unable to apply or bridge the part-whole reasoning to operations like addition/subtraction or multiplication/division of whole numbers. It was not until the students actively engaged in subitizing activities and made explicit connections to the operations through the "flash method" of dot pattern cards (regular and irregular arrangements) and ten frames (Wheatley and Reynolds 1999) that these students in fact attended to number, quantity and number operations with greater understanding. By using the subitizing activities, some students began to see that numbers had more than one composite. Multiple compositions to produce whole numbers were possible. Students began to recognize different ways to compose and decompose numbers from whole to parts and from parts to whole. For example, 100 could be constructed in multiple ways using many different arrangements; a link to 10 was recognized by some students (4+6 could be recognized as linking to 40+60). Again, having had success with older children, I questioned why educators were not implementing these strategies at a younger age. Over the past few years I have witnessed more use of subitizing in the early primary levels—the Alberta K–9 mathematics program of studies (2007) now includes subitizing in the specific outcomes for kindergarten and Grade 1. I am anxious to see the effects of the implementation of this strategy longitudinally.

Details of Children's Learning from a Pedagogical Documentation: Subitizing

My participation in a University of Alberta early childhood mathematics course earlier this year strengthened my belief in the power and benefit of incorporating subitizing activities into young children's math lessons. In this course, I was asked to document classroom research described as a "pedagogical documentation." This process draws on Reggio Emilia practices in early childhood education. This form of documentation not only prompts teachers to think about children's work but encourages them to use the information to plan further activities with children. The project was intended to benefit the teachers, the children and the parents involved. I chose to study the effects of using subitizing with a group of 21 Grade 1 students in a rural area of northeastern Alberta.

The research project involved two rounds with students in both whole-class and small-group settings. Round one involved the whole class drawing what they saw when the image in Figure 1 was flashed. In round two, small groups of students were asked to participate by verbally responding to flashed images like the ones in Figures 2, 3 and 4. Round two also involved small groups of students drawing what they saw when the arrangements for 9 and 10 shown in Figures 3 and 4 were flashed.

The Questions Posed:

What do you see? How do you see it?

How many dots do you see?

How do you know?

Can you see it in another way?

Two students' responses in round one of the pedagogical documentation gave me some interesting insight and a good opportunity to view some students' ability or lack of ability to see part-part-whole relationships in arrangements of dots. These responses caused me to reflect on the limitations of students who have limited and/or emerging skills with subitizing and the ability to see parts in relation to whole number arrangements. Mary's[1] ability to subitize parts of the whole was evident—she saw 2s correctly to form 6 (see Figures 1 and 1a).

However, Mary could not answer without looking back at the picture she had drawn and counting each dot. I found this shocking. Mary proceeded to do this again with 9 in round two. She was able to subitize 3s in 9, but could not readily quantify the whole arrangement, 9, until she took part in the small-group discussion in round two. Mary frequently subitized parts as

Figure 1

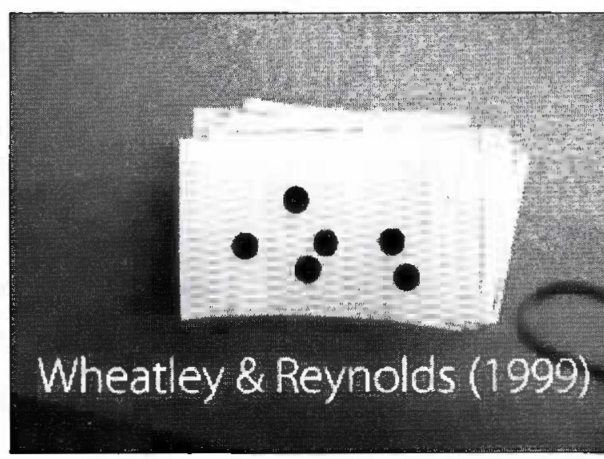
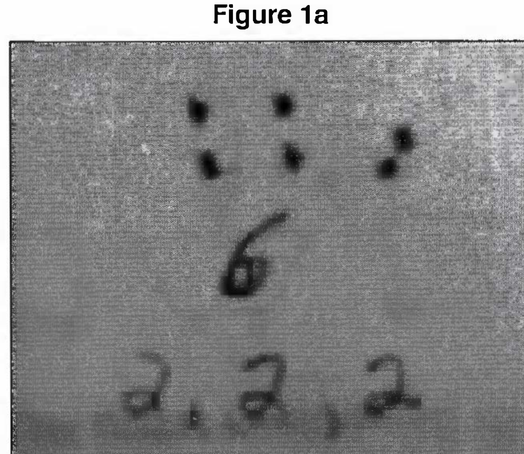


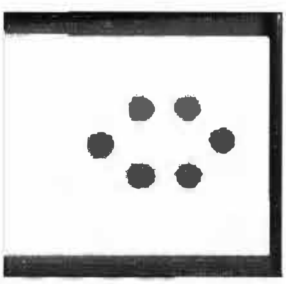
Figure 1a



2s or 3s, but only infrequently did she quantify without a count to verify the quantity. I would have to judge that this student's number sense was emerging; she saw the parts and was only beginning to relate the parts in relation to the whole arrangement.

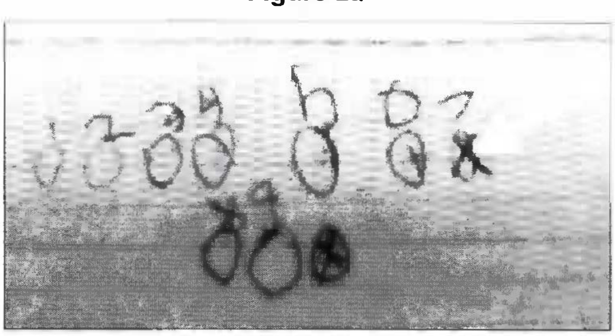
Edward was another student that I thought was subitizing the arrangements in the first round of the pedagogical documentation. However, looking back at the video allowed me to recognize that Edward was sharing facts that he knew. He was not subitizing the images and could not recognize what parts made up the whole. This was proven to be the case in both rounds: in the first round Edward stated that he saw 7 for an arrangement of 6. This prompted me to ask, "How do you know?" He stated, "Two plus five equals seven," emphasizing his knowledge of this fact with five fingers and a counting-on action of two more. When I asked, "Where did you see the 5?" Edward replied, "On the bottom" (referring to the bottom of the dot arrangement—he was not able to readily identify where the 5 was). Even after I showed Edward the image again, he could not identify the quantity or parts for the arrangement of 6 that was shown (see Figure 2).

Figure 2



Edward's actions and statements illustrated the idea behind the quote/question posed by Hunting (2003), "Are finger sets used to represent visualized material, or simply used as a standard symbol set because visualization alone was too great a cognitive task? This we do not know" (p 232).

Figure 2a



In Edward's case, I believe that his finger sets represented rote learned facts that he knew and that did not accurately reflect what he was seeing in the dot patterns. Edward's inability to see parts of the whole unit was again evident in the second round with his drawn images for 9 (see Figures 2a and 2b). He repeatedly attempted to change the images he had drawn after the irregular dot images were repeatedly flashed; however, he was not successful in making correct notations of the parts that he saw in the image or the whole arrangement of 9. Edward could not see parts of 9 or verify the quantity of the whole arrangement of 9, and he stated that he was "not sure" how many dots were in the irregular images. As evidenced in round one and again in round two, Edward knew some math facts (2+5 = 7) and could draw the regular square geometric arrangement of 9, but conceptually he did not see 9 as a composition of parts and a whole within irregular arrangements the dot patterns.

Visualizing and Verbalizing—Is It Enough?

Verbal responses alone did not provide enough information about what and how the students saw the dot patterns. Having the students draw the images and talk about their perspectives in round two provided much more information about student accuracies or inaccuracies of the subitized images. Having the students talk about their perspectives allowed me to note what and how they were seeing the parts in relation to the whole after they had created their personal drawings. For some students, just seeing the arrangement was a simple task—they "just saw" the arrangements for the smaller numbers. For these same students, combining the parts of an image to form the more complex arrangements was also a simple task. However, for others, seeing the parts required multiple viewings (up to three) and even with multiple views these students struggled with breaking up the irregular arrangements of 9 or 10 (see Figures 3 and 4).

Figure 2b

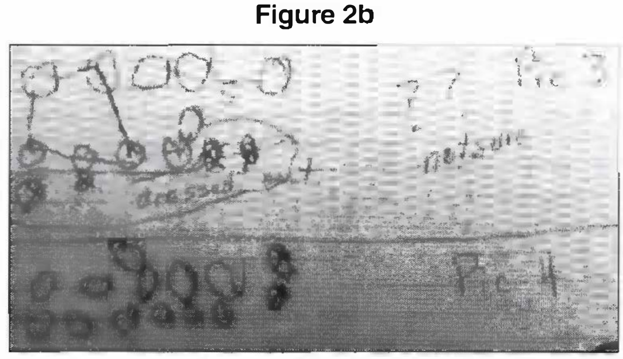


Figure 3

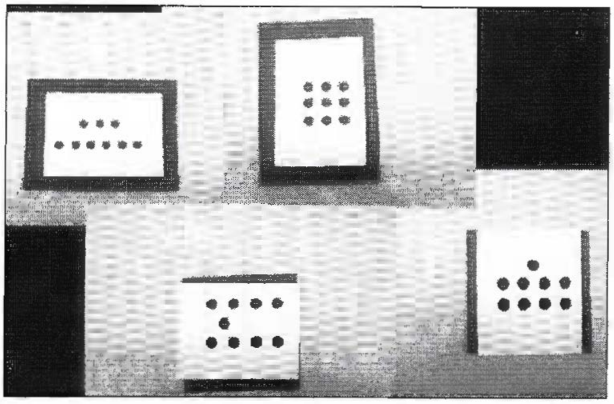
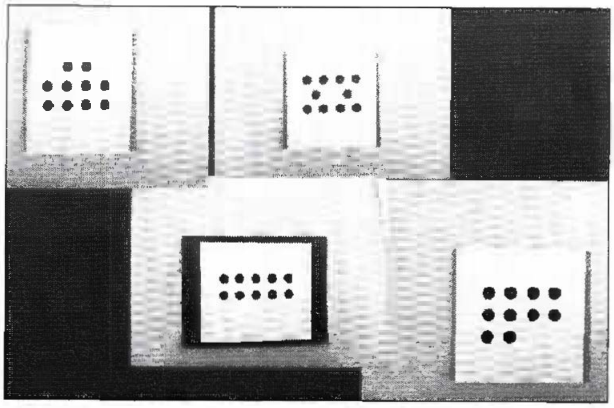


Figure 4



For other students, attending to the arrangement was a combination of seeing and drawing the parts to form a whole picture—recognizable shapes identified in the image helped some of these students see and then successfully draw the arrangement. Adapting this activity to have manipulatives available for the students to create, as opposed to drawing, the image may alleviate some of the frustration that some students had with the motor skills required to draw the image.

Content Analysis of the Mathematical Concepts: Subitizing—Where Do We Go from Here?

This pedagogical documentation dealt with the importance of subitizing and its role in students' ability to identify parts of a whole unit. What came to light were issues of students' inaccuracy in quantifying number even if they could accurately identify parts of the whole. This was also new learning for me. Prior to the pedagogical documentation, I had held the belief that if students could accurately subitize parts of a whole they could simultaneously quantify the whole. Additionally, I learned that students who could quantify the whole image did not always see the parts that made up the whole. This pedagogical documentation highlighted a key understanding: to be more successful in mathematics, students need to be able to both subitize parts and quantify number as a whole. Hunting (2003) alludes to this connection as well as an extension to mathematical operations:

> We are aware of the dynamics of part-whole reasoning where a subset is cut out from the whole while the whole set is kept in focus. We suppose that the logical operations of class inclusion are important here. The reverse situation, where a whole is rendered a part, by the conjoining of other items to make a new enlarged whole, prefigures the symbolic statements we know as addition of whole numbers. (p 232)

After completing this pedagogical documentation, there are three additional questions that require further exploration with young mathematics learners:

- Will just isolated work with conceptual subitizing dot patterns be enough to improve student ability to see part-part-whole relationships?
- Which subitizing activities make the most impact on student conceptualization of part-part-whole relationships as a link to number operations?
- How important are student discussion, discourse and sharing of personal perspectives to improving conceptual subitizing of number?

Conclusion

Pedagogical Documentation: Subitizing was an extremely powerful learning experience. The project brought to light several important issues and personal misconceptions around conceptual subitizing. First, the documentation reinforced the importance of children having multiple experiences with conceptual subitizing, with both regular and irregular arrangements of dot patterns for number. Surprisingly, accurately subitizing the dot patterns did not automatically mean that students could accurately quantify the arrangement they were seeing. Additionally, quantifying the whole did not mean that students could see parts of the whole. Students need to engage in active discussions about what they see and how they see the arrangements. Discussing their perspectives with peers and pointing out how they are comparing and combining parts to the whole are powerful. Actively encouraging student discourse in the math classroom allows children to develop alternative perspectives for the abstract arrangements, further highlighting the possible different arrangements of the parts in relation to the whole. Last, observing children and reflecting on their learning are powerful experiences that help us modify and adapt pedagogy to best suit student learning in mathematics. All in all, this was an incredible experience for me as a teacher in the role of teacher/researcher.

Note

1. Students' names have been changed throughout to protect privacy.

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Sylvia Malo is a teacher with more than 20 years of experience in teaching Grades 1–9. She is presently on secondment from Northern Lights School Division, acting as a full-time learning network math consultant. In 2010, she completed her master's in elementary education at the University of Alberta, with a curricular focus on elementary mathematics. Her enthusiasm for mathematics teaching has been recognized by parents, peers and students; she was awarded the 2006 APEGGA Math Teacher Award for the Lakeland Region. Sylvia is passionate about changing administrator, teacher and student attitudes toward teaching and learning mathematics. She believes that reforming teachers' beliefs about how students conceptually learn mathematics will affect their success in math. Her research interests involve the use of subitizing and incorporating mental math strategies and problem solving to promote student success in mathematics learning.