Integrating History of Mathematics into High School Curriculum: To Be or Not to Be?

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Introduction

A decade ago, the National Council of Teachers of Mathematics (NCTM) created principles and standards for school mathematics, called *Standards 2000*, as an effort to provide benchmarks of achievement as well as consistency in content and rigour for all mathematics students in the United States. These documents were designed to give students a well-rounded, enriched and useful mathematics education that would foster conceptual understanding, appreciation and achievements in mathematics for all students (NCTM 2000, 13–14). While there was a strong emphasis on values and appreciation of mathematics, the history of mathematics (HOM) was ignored. Recently, the new Common Core Mathematics Standards[1] have been issued and adopted by many states. Not unlike the NCTM principles and standards, the goal in the adoption of the Common Standards is to ensure that students across the United States are provided with a curriculum that is unified in rigour and content. And again, unfortunately, nowhere in the Common Core Standards is there any mention of learning HOM.

This continuing omission is puzzling. It can be demonstrated that many of the goals outlined in the principles and standards would be better met by incorporating HOM into the curriculum. There is significant evidence that including HOM in math instruction is not only worthwhile but also an enrichment to the mathematics curriculum and can help students reason critically.

Does the absence of HOM from a national document not send a political message to teachers that learning HOM plays no role in learning the subject? If at the state and national levels there exists the implicit message that learning HOM is not important, why would a teacher consider the contrary?

It is fair to say that most teachers make every effort to stimulate their students' interest in mathematics and try to teach mathematics as a social construct—an activity that makes sense through its usefulness and utility. To appreciate mathematics, students should have a variety of experiences related to the cultural and historical aspects of the evolution of mathematics so they can value the role of mathematics in the development of our society. Unfortunately, the historical dimension of teaching mathematics is either totally absent, ignored or viewed as an "exotic luxury," as Whitrow (1932) suggested.

While HOM may not be appreciated in the United States, its importance is recognized in many other countries. According to Fasanelli et al (2000), "the experience of many mathematics teachers across the world (is) that the history of mathematics makes a difference: that having history of mathematics as a resource for the teacher is beneficial" (p 1). If mathematics teachers around the world see value in the inclusion of HOM, why doesn't the United States? Research into teachers' perspectives on the inclusion of HOM in the classroom is scarce, but some studies (Philippou and Christou 1998; Schram et al 1988; Siu 2004; Smestad 2009; Stander 1989) indicated that teachers' interest in and valuing of mathematics increased when they were introduced to HOM.

An interesting dichotomy revealed by the work of Fasanelli et al (2000) is that, while the importance of HOM is widely recognized, these studies emphasized that teachers found no interest in *using* HOM with the mandated curriculum. This failure of inclusion is important for us to explain and contradicts what our study shows; some enthusiast teachers do include HOM in their instruction and therefore must see benefit in doing so.

In light of these considerations, the purpose of our large-scale study was to gain understanding of high school mathematics teachers' perceptions and beliefs about the integration of HOM into their instruction. While we documented multiple findings, this paper addresses some of the factors that affect high school teachers' decision whether to include HOM as a systemic part of their mathematics courses.

Background

History of Mathematics

Several key assumptions served as a foundation of our research. First, by learning about the evolution of at least some mathematics concepts—which are inescapably linked to mathematicians, who, through years of sacrifice, trials and tribulations, created the mathematical concepts and the language to communicate them—a student looks into the past and can trace the intellectual development of humankind. Second, HOM provides a background for gaining a rich and deep understanding of the development of mathematical concepts. Third, there may exist an implicit relationship between the learning of HOM and students' attitude toward mathematics. Although there is no empirical proof for such assumptions related to mathematics, certainly the teaching of historical development of basic concepts is fundamental in the natural and social sciences. Since the work of Thomas Kuhn (1996), the history of science has been a rapidly growing field of research and publication. The popularity of the science histories of Stephen Hawking, in physics, and Stephen Jay Gould, in biology, suggests that history is useful even in abstract and complex disciplines. If the history of a discipline is important in the natural and social sciences, why should it not also be in mathematics? In the words of J W L Glaisher (1848–1928), uttered more than 100 years ago, "I am sure that no subject loses more than mathematics by any attempt to dissociate it from its history" (Glaisher 1890).

The importance of HOM in the school curriculum has been supported by research (Siu 2004; Ho 2008) and scholarly writings (Kline 1972; Wilder 1968; Katz 1997). There is even a debate about whether the notion of "using history of mathematics" should be replaced with "integrating history of mathematics" to encourage the view that the history of mathematics is inseparable from the subject itself (Siu and Tzanakis 2004).

It seems common sense to believe that it is beneficial for all mathematics learners to be aware of the purposes and intellectual struggles of those who created mathematics and to appreciate the process of invention. Further, as Swetz (1994) said, "The history of mathematics supplies human roots to the subject. It associates mathematics with people and their needs. It humanizes the subject and, in doing so, removes some of its mystique" (p 1).

We support the position that mathematics is a "cultural phenomenon" (Wilder 1968, xi), and that meaningful learning of school mathematics must be facilitated by studying the cultural significance of mathematics, the role of the evolution of mathematical concepts and scientific thought. According to Wilder (1968), "The anthropologist George P Murdock listed 'numerals' as one of 72 items that occurred, so far as was known, in every culture known to history or ethnography" (p 33).

Swetz et al (1995) suggested that exposure to HOM at the high school level can have "... a profound effect! For it is at the secondary or high school level that students first experience the power of mathematics and begin to realize the wide scope of its application and possibilities" (p 1). Barbin et al (2000) posited that

> the conviction that the use of history improves the learning of mathematics rests on two assumptions about the process of learning. The more a student is interested in mathematics, the more work will be done; and the more work that is done, the greater will be the resulting learning and understanding. (p 69)

Fasanelli et al (2000) reported that many countries have a magistrate of education who outlines the educational goals for the entire country. Countries such as Austria, Brazil, China, Denmark, France, Greece, Italy, New Zealand, Norway and Russia have a national set of frameworks that explicitly incorporate HOM into the learning standards. Fasanelli et al (2000) asserted, "These decisions are ultimately political, albeit influenced by a number of factors including the experience of teachers, the expectations of parents and employers, and the social context of debates about the content and style of the curriculum" (p 1). It is highly probable that a primary influence on a teacher's decision about whether or not to include HOM in the classroom is the set of government-created learning objectives provided in the national or state curriculum requirements.

Clearly, there are gaps between what is espoused in the professional and scholarly arena about the possible benefits of students learning HOM; the curriculum standards, which have no trace of HOM; and teachers' views on the integration of HOM in curriculum.

About the Nature of Mathematics

The historical dimension of mathematics and its evolution as a living organism suggest that mathematics is viable, ever changing and socially constructed. There are numerous examples supporting the value of understanding the evolution of mathematics (for example, the act of counting goes back to primitive civilizations and their predecessors; the need to record the process of division led to the creation of new type of numbers [that is, fractions and their symbolic representation]; the advancement of the numeration system led to the development of a whole new language to communicate newly invented and discovered mathematical ideas in a more precise and elegant way). We emphasize that referring to mathematics as a language is critical to accentuate that mathematics is a result of social practices of people, rather than an objective realm that is metaphysical and superhuman.

Due to humans' natural tendency to "symbolic initiative" (Wilder 1968, 5), it is logical to accept that, historically, mathematics was born during the process of the invention of a language to communicate the patterns observed in the real world and the internal structures in mathematics.

From an anthropological perspective, such invention was possible because of humans' natural ability, borne out of necessity, to think abstractly, manipulate mental objects at a conceptual level, and establish connections between ideas and communicate them. "Mathematics is something that man himself creates, and the type of mathematics he works out is just as much a function of the cultural demands of the time as any of his other adaptive mechanisms" (Wilder 1968, 4).

The dual nature of mathematics and its evolution is subtle but critical to understand. A lack of understanding might cause misconceptions about the nature of mathematics as a discipline, which could lead to further misconceptions about the teaching and learning of mathematics.

Method

To investigate the factors that influence whether teachers include HOM into their classroom instruction, we used SurveyMonkey, a web-based software program (www.surveymonkey.com) to design and administer a comprehensive survey scale instrument. Having weighed the advantages and the disadvantages of both online and postal mail, we chose to disseminate a web-based survey and had the participants send responses to the company's server.

Participants

This study took place in a US state that had 372 operating public high schools, including charter schools, and about 2,909 mathematics teachers. To encourage participation, we sent e-mails to all public and seven private high school principals (total 379) requesting them to forward to their mathematics teachers an invitation to participate in an anonymous, online survey. The number of the private high school mathematics teachers remains unknown. Only six principals declined the request. We also used our extended personal contacts and sent e-mail messages to high school mathematics teachers, encouraging them to participate. The invitation letters sent to the principals and directly to the teachers included a description of the study, detailed instructions on how to access it and contact information should they have any questions. The survey was available on the Web for several weeks. A total of 367 teachers participated in the online survey, which is about 12 per cent of all high school mathematics teachers in the state.

Instrument

The survey's six parts—(1) Attitudes Toward Mathematics as a Discipline, (2) Philosophical Perspectives of Mathematics, (3) Philosophical Perspective on the Nature of Mathematics, (4) Perceptions of History of Mathematics, (5) Reasons For or Against the Inclusion of History of Mathematics and (6) Teacher Background Variables—included 110 items that would require about 25 to 30 minutes to complete. Some of the items were designed by the researchers. Others were adapted from surveys of previous studies (Tapia and Marsh 2004; Dutton 1962; Shulman 1986; Alken 1974; Charalambous, Panaoura and Philippou 2009; Tzanakis et al 2000), and some were extracted from the National Assessment of Educational Progress Mathematics Teacher Background Questionnaire (National Center for Education Statistics 2009).

We established a Likert scale consisting of five declarative sentences, with choice responses varying from strongly disagree, with value of 1, through disagree, neutral and agree, to strongly agree, with value of 5. We used skip logic to direct the teachers to a custom path of the survey that, depending upon their response of yes or no to the inclusion of HOM into their teaching, led them to the statements pertaining to reasons for or against.

Reliability Analysis and Scales Formation

Our analysis of the data focused on teachers' perceptions about HOM and the factors that affect their decision to integrate HOM into their curriculum. Of the 110 questions, 74 Likert (ordinal) questions were used to run a reliability analysis, which yielded a Cronbach's alpha value of 0.94.

Subsequently, a total of seven constructs were formed as the result of a factor analysis of all 74 Likert (ordinal) data items. For each construct, scales were developed by averaging the responses (1—strongly disagree to 5—strongly agree) for the variables that loaded into the corresponding construct. Three of the seven constructs, Perceptions of HOM, Reasons for Including HOM and Reasons for Not Including HOM, and their relationship are described below.

Data Analysis

Our primary purpose was to evaluate whether teachers' perceptions of the utility, benefits and importance of HOM influence their decision to incorporate HOM into their instruction.

We analyzed three sets of data, shown in Table 1.

Table 1: Summary of yes/no to inclusion of HOM

	Frequency	Per cent
Valid .00	30	8.2
No	133	36.2
Yes	204	55.6
Total	367	100

One set included all teachers (N = 337) who responded to the questions of whether they included or did not include HOM. The other two sets consisted of those who included HOM (N = 204; we will refer to this group of teachers as the Yes group) and those who did not include HOM (N = 133; we will refer to this group of teachers as the No group).

Factor analysis of the survey items showed that the perceptions that were most relevant were encapsulated in five fundamental statements, the responses to which were measured on a Likert scale of 1 (strongly disagree) to 5 (strongly agree). These five statements and the mean scores for the entire sample, All Teachers, and for both Yes and No groups are shown in Table 2.

The mean scores for all teachers are significantly greater than 3 (neutral); in fact, they are very high for Likert scale surveys, with the exception of the last statement. Also, the mean score for the first statement is extraordinarily high for a Likert scale survey. There was little disagreement with the statement "All students should be taught some history of math." The fifth statement, on the other hand, suggests a possible incongruity with the first statement. Obviously, many participating teachers believed that HOM has significance and value, yet there was not an agreement with the statement that understanding mathematics would be easier if HOM were taught. Responses were mid-neutral at best. It is likely that teachers could view HOM as a time-consuming burden rather than a valuable teaching tool for capturing students' attention and focusing their learning. More discussion on this issue is below.

Nonetheless, the high mean scores for the entire sample would suggest that if teachers' perceptions of the utility and importance of HOM influence their decision to incorporate HOM in their instruction, the vast majority of our sample should have been employing HOM. Unfortunately, as shown in Table 1, this is

Table 2: Teachers' perceptions of HOM

		All students of mathematics should be taught some history of mathematics.	History of mathematics is worthwhile and necessary to the understanding mathematics.	Knowledge of history of mathematics is valuable to nonscientists or nonmathematicians.	Knowing history of mathematics may assist students in learning mathematical concepts.	Understanding mathematics would be easier if history of mathematics were taught.
All teachers						
N	Valid	347	347	347	346	346
	Missing	20	20	20	21	21
Mean		3.90	3.46	3.48	3.81	3.21
Yes group of teachers						
N	Valid	203	203	203	202	202
	Missing	1	1	1	2	2
Mean		4.14	3.74	3.78	4.03	3.43
No group of teachers						
N	Valid	132	132	132	132	132
	Missing	1	1	1	1	1
Mean		3.54	3.06	3.05	3.48	2.90

not true—only a small majority (55 per cent) were using HOM.

A comparison between Table 1 and Table 2 shows that there are many teachers in the No group with a positive perception of HOM (3.54 mean for statement 1). Interestingly, many teachers in the Yes group think that HOM may not necessarily make understanding mathematics easier (3.43 mean for statement 5). The comparison between groups Yes and No is shown in Table 3.

Perceptions of history of mathematics is a construct that averages all of the statement rankings presented in Table 2. The data ($p < .001$) suggest that teachers who viewed HOM as valuable, necessary or worthwhile would have tended to include it in their classroom lessons.

Teachers in the No group (see Table 2) scored the items lower than the means of the group of all participants, which is consistent with their decision to not include HOM in their instruction. Apparently, among many factors that affect teachers' decisions, there is notable evidence that teachers' beliefs about the importance of HOM and their views on the benefits and values of HOM play an important role.

We also observe a significant difference in the Yes and No groups (see Table 2). Teachers in the Yes group appeared to be more likely to agree that all high school mathematic students should be taught some HOM, that HOM is worthwhile and necessary to the understanding of mathematics, that HOM is valuable to nonscientists or nonmathematicians, that knowing HOM may assist students in learning mathematical concepts, and that understanding mathematics would be easier if HOM were taught. That was anticipated and expected. It is just as important to note that the No group, while not as strongly supportive of these statements, is nonetheless moderately favourably disposed to HOM except for the last statement. "Understanding math would be easier if the history of mathematics were taught."

There is an interesting inconsistency in the ranking of a few very closely related statements. The teachers in the No group do not see that "Understanding mathematics would be easier if the history of mathematics were taught" (mean 2.90). However, the statement "Knowing the history of mathematics may assist students in learning mathematics concepts" was rated at 3.48. But an even more prominent result is that the teachers in the Yes group ranked the statement "All students of mathematics should be taught some history of mathematics" very high (mean of 4.14), but the statement "Knowing the history of mathematics may assist students in learning mathematics concepts" was ranked with a mean of 4.03. What would seem to be one of the major reasons for inclusion of HOM (that is, to make understanding mathematics easier), the statement "Understanding mathematics would be easier if the history of mathematics were taught" was ranked with a mean of 3.43. This would imply that the Yes group also did not view the learning of HOM as an aid to understanding mathematics.

A question can be raised as to why there was such a varied rating for these two statements. Either the teachers in this sample did not associate these statements or they did not think that the teaching of HOM would indeed assist the students to learn mathematics; that is, *make it easier.*

We suggest that teachers would benefit from more guidance and training about the values and use of HOM as a means to help students understand mathematics.

Factors That Affect Teachers' Decision

The factors that affected the Yes group's decision to include HOM are shown in Figure 1. The top two responses indicate that the Yes group enjoyed teaching HOM and believed that their students enjoyed learning it.

Enjoyment seems a very important factor to this group. The ratings of the third and fourth responses revealed that teachers view HOM as a means to help students see the development of and connections between mathematical concepts and that HOM affects their attitude toward mathematics.

However, the responses to the last statement prompt an idea that the teachers in the Yes group yet again separated mathematics from its history. They may not have seen that helping students understand the development of and connections between mathematical concepts and enhancing students' interest in mathematics are closely linked and that they make it easier for students to understand mathematics.

The factors that affect the No group are shown in Figure 2 in descending order.

Table 3: Comparison between groups Yes and No

	Perceptions of History of Mathematics	N	Mean	Std Deviation	Std Error Mean
I include History of Mathematics in my lessons	No	132	3.206	.684	.060
	Yes	201	3.824	.573	.040

Figure 1: Teachers' reasons for including HOM

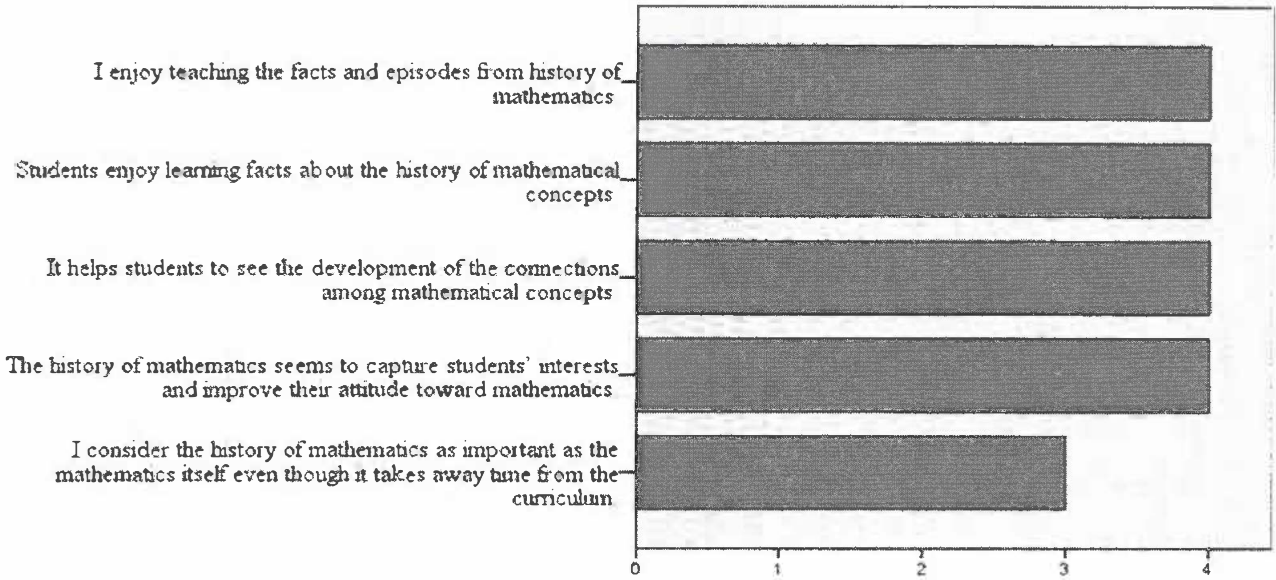
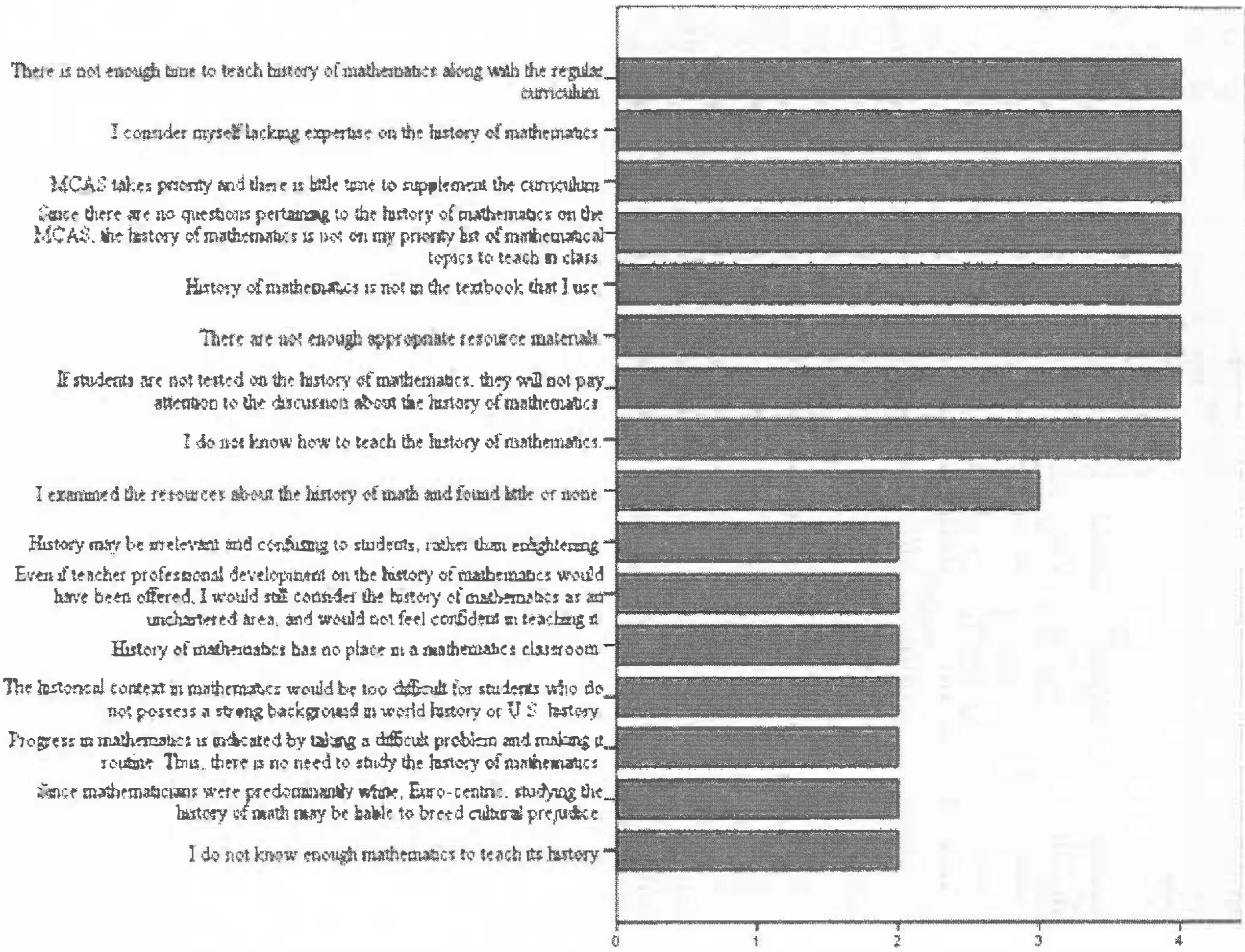


Figure 2: Teachers' reasons for not including HOM



The ranking of each statement indicates the extent to which each factor would deter teachers from including HOM. All top eight factors were notable and are addressed in the next section. However, two of the factors that suggest a lack of confidence (ie, "I consider myself lacking experience in the history of mathematics" and "I do not know how to teach the history of mathematics") prompted us to examine the relationship between a teacher's decision to include HOM and the number of courses on HOM taken by the teacher (see Table 4).

The results of the chi-square test of independence analysis ($p < .005$) indicated significance between the two variables, and thus a possible relationship between the number of HOM courses a teacher took and his or her decision to incorporate HOM into the curriculum. We have presented several factors that may influence a teacher's decision whether or not to include HOM in classroom instruction. Detailed analysis and discussion follow in the next section.

Discussion

After analyzing all constructs and factors examined so far, we offer several speculations and suggestions.

Time and High-Stakes Testing

Mathematics teachers in the United States are expected to teach (not just "cover") and their students are expected to learn (not just "go over") a mile-long curriculum. Thus, the shortage of class time is certainly a serious concern to many teachers when it comes to supplementing the curriculum. This concern is amplified by the advent of the new Common Core Standards for classroom instruction (www.corestandards.org). Adjustment to the new curriculum and accommodation of the new mandate, as it is anticipated, will take a substantial amount of time. And since nowhere in the Common Core Standards is there a mention of students learning HOM, it is unlikely that teachers will be able to consider the inclusion of HOM as a supplement to their curriculum, except perhaps for some true enthusiasts. One participant wrote

> I assign my students a brief presentation on the biography of the mathematicians we talk about in class. Many [students] like it; particularly those who are liberal-arts inclined; they write nice essays and show more interest in math. (David K, seven years of teaching Grades 9–12)

Until education policy makers understand the value of learning HOM at all grade levels and supplement the common core standards with their own requirement that students become familiar with HOM, it is doubtful that it will be widely and consistently incorporated into an already full curriculum.

States' high-stakes testing was also indicated as one of the primary reasons that teachers decide to not teach the history of mathematics. Obviously, high-stakes testing is a priority, and unless HOM is included as an essential practice standard in state frameworks, most teachers would either not include HOM or would struggle with how to sacrifice or alternate the curriculum to include it.

The highly ranked statement (mean of 4, see Figure 2) "If students are not tested on HOM, they will not pay attention to the classroom discussion about the HOM" reveals a pedagogical misconception. Such a pragmatic view is probably based on a teacher belief that all learning experiences must necessarily be directly related to performance and immediately and

Table 4: Summary of the number of courses on HOM for groups Yes and No

			How many courses on history of mathematics have you taken?			
			0 courses on history of mathematics	1 course on history of mathematics	2+ courses on history of mathematics	Total
I include the history of mathematics in my lessons	No	Count	80 (80/331)	43 (43/331)	7 (7/331)	130
		% of Total	24.2%	13.0%	2.1%	
	Yes	Count	90 (90/331)	79 (79/331)	32 (32/331)	201
		% of Total	27.2%	23.9%	9.7%	
Total		Count	170	122	39	331
		% of Total	51.4%	36.9%	11.8%	100%

unswervingly measurable. However, one cannot expect a straightforward and instantaneous relationship between learning HOM and performance in mathematics. Factors such as motivation, appreciation and enjoyment are critical to facilitate engagement and interest in mathematics, which are, in turn, prerequisites for better performance.

> When I teach distributive law in algebra, I always tell the students about Euclid's geometric algebra and the ways to multiply binomials with a diagram, and other diagrams that show algebraic identities, which I found in Euclid's *Elements, Book II*. It is amazing how many useful diagrams that help students visualize math can be found in Euclid. (Linda M, five years of teaching Grades 9 and 10).

Teacher Knowledge of History of Mathematics

"Confidence in knowledge and teaching history of mathematics" is another reason why teachers chose to not include HOM. The statement "I consider myself lacking expertise on the history of mathematics" received a mean rating of 4 (Agree), and presents an important issue worth addressing. We offer two accounts that may be associated with the issue. First, teaching a subject requires content knowledge of the subject (Shulman 1986). HOM content knowledge does not need to be saturated with complex mathematics concepts; yet, to effectively include HOM into their instruction, teachers must have pedagogical content knowledge of how to make it understandable to others, including what makes it simple or difficult to learn. Second, research (eg, Ball 1988; Cooney, Shealy and Arvold 1998; Dutton 1962; Furinghetti, 2007; Philippou and Christou 1998) suggests that teachers teach in the manner in which they were taught. They could have subconsciously built in images of pedagogy from their instructors. Thus, the teachers' presentations of mathematical concepts are largely affected by their past experience as students. This implies that teachers who have had relatively little or no exposure to HOM as high school and higher education students may perceive themselves as lacking expertise and, consequently, do not expose their students to HOM.

We support the requirement of at least one course on HOM during mathematics teachers' preparation program. It would serve two important purposes to the teacher: (1) a course on HOM would contribute to the expansion of the repertoire of mathematical content knowledge and would increase teachers' confidence in their ability to teach HOM, and (2) a teacher who is aware of at least some mathematical history will likely be inclined to integrate stories about the twists, turns, trials and tribulations of math into the curriculum. Students will probably be excited to learn about the development of our numeration system, development of geometry, inventions of major constants in mathematics and so on. Thousands of years of HOM provide a massive number of fascinating tales that can be discussed in class to enrich learning:

> I like to tell my students stories about numbers and the origin of their names. For example, it was during the 1700s that π and e both got their names … π is an abbreviation of the Greek word for perimeter (περιμετρος), or an abbreviation for periphery/diameter ... the first appearance of π began with William Oughtred in 1647 when he used π for circumference ... Then, in 1697, David Gregory made π represent the ratio of circumference to radius. In 1706, William Jones defined π as the ratio of circumference to diameter as is done today. The story of e getting its name is equally interesting. Leibniz originally named it b in 1690, but the name did not catch on. The name e was granted by Euler in 1731 and managed to spread. In this case, Leibniz usually goes unnoticed since the name b was never commonly recognized. However, some of his notation did last, especially in calculus. When I was planning lessons on slope, I learned that the earliest known use of m [slope] appears in 1757. It is not exactly known why the letter m was chosen for slope. However, it has been suggested that m could stand for *modulus of slope*. There are remarks that that the French word for *to climb* is *monter* ... Mathematicians like to use the first or the last letter of the words for symbols ... for example, x is the last letter in the word *radix*, which is *root* in Latin. (John B, novice high school teacher)

One can also assume that with increased knowledge of mathematics and its history, teachers will be able to present mathematics as a coherent system of knowledge, rather than a collection of disjointed concepts.

According to Victor Katz (1997), the majority of the certification requirements for teachers at secondary schools in the United States include a course in HOM. However, the evidence presented in Table 4 indicates that over half (51.4 per cent) of the 331 teachers who responded to the survey question "How many courses on the history of mathematics have you taken?" had never taken a course on HOM. This prompts the question, How and why were the majority of the participating teachers certified with no courses on HOM? We offer one possible answer to this question. In an open-response question, a participating teacher commented, "… even though one of the requirements for my graduate degree included the history of math [*sic*]. That was conveniently wived [*sic*] for expediency." Allowing a preservice teacher to opt out of a course on HOM for the sake of expediency suggests that HOM is not important and is not going to be used in the classroom. We have no intention to generalize such a case; yet, the ramifications of such a decision need to be examined. No one can ascertain the extent of the benefits of teachers and students learning HOM. However, to date, there exists no empirical research that indicates that HOM is *not* beneficial to teachers' and students' construction of mathematical content knowledge. This returns us to the point made in the introduction that the history of sciences has become increasingly important to the teaching of the basic sciences.

From another point of view, it is interesting to note that 90 teachers (27 per cent of the total $N = 331$ who responded) who indicated that they had never taken a course on HOM included it in their classroom instruction. This group of teachers is representative of a point discussed previously. We speculate that teachers who believe that HOM is important and beneficial to students' construction of mathematical knowledge will likely include HOM regardless of their formal education.

> I have never taken any courses on the history of math but always include it in my classes. My students love it! I tell them how math has evolved through thousands of years from Babylonian to modern times, about the struggle and success of people who made math. They [students] like to hear about Pythagoras, how and why irrational numbers were discovered and not accepted at first. I think they learn irrationals better when [the material is] presented in a historical context. (Patrick S, 12 years of teaching in K–12)

Resources

The availability of resources is another factor influencing the teachers' decision to teach HOM. Two of the most highly ranked reasons for not including HOM were "History of mathematics is not in the textbook that I use" and "There are not enough appropriate resource materials." Time is a precious commodity for a teacher, and not having readily available grade-appropriate resources may explain why teachers do not include HOM. However, current technological advances provide amazingly easy access to an abundance of information related to practically any historical overview on any mathematics topic studied in the K–12 curriculum.

> I encourage my students to ask where a theorem or a rule came from. We talk about the observations that initiated the theorem and the mathematician who associated with it. (Tapasya D, five years of teaching Grades 9–11)

> I established a Mathematician of the Month to highlight accomplishments in math. Students like it. (Richard H, three years of teaching Grades 9 and 10)

> I use some of the online lectures by Judith Grabiner [professor of math at Pitzer College; eg, www.infocobuild.com/education/learn-through-videos/mathematics/math-philosophy/lecture-16.html] … lectures about Euclid and non-Euclidean geometry are great. (Jenn S, eight years of teaching Grades 9–11)

We argue that professional development on the history of mathematics may serve as an effective learning experience by which mathematics teachers at all levels can be introduced to the teaching of HOM. Seventy-seven per cent of the participating teachers responded positively to the question about whether they would participate in professional development on HOM. Offering practical information on methods of integrating HOM will likely help answer the question of whether all students of mathematics should be taught some history of mathematics.

Conclusion

Teaching and learning HOM may benefit both teachers and students, and there are significant reasons for a requirement that teachers learn the history and nature of mathematics. The impact of the factors mentioned above may be far reaching. It is hard to argue against the humanizing benefit of HOM and its potential to strike a chord with the affective domain of learning (Krathwohl, Bloom and Masia 1973). Our data analysis shows that teachers who enjoyed teaching HOM believed their students enjoyed learning it. Many researchers (Fauvel 1992; Furinghetti 1997; Siu 2004; Smestad 2009) believe that HOM provides students with the opportunity to construct a personal, visual and emotional connection to the development of concepts. Following are recommendations that may lead to systematic integration of HOM in the classroom.

Teaching mathematics with no knowledge of its history is tantamount to a lawyer being allowed to practise law with no knowledge of the history of the judicial system, or scientists immersed in science without knowing its history. Higher education must not waver on the importance of knowing HOM—to do so would suggest that awareness of the evolution of mathematics concepts is not essential to understanding mathematics itself, and to teaching and learning of mathematics in particular.

There is strong need for students to view mathematics as a human creation that began thousands of years ago as an ever-changing body of knowledge. Students who view mathematics as a set of discrete, disconnected topics may have difficulty in understanding the relational worth of each mathematical concept or its attachment and value to human life.

In addition to undergraduate and graduate education in HOM, we suggest that systematic professional development focused on HOM would be an excellent way to introduce teachers at all levels to the resources for teaching HOM and a great opportunity for them to acquire the necessary pedagogical content knowledge and to build upon subject content knowledge. The National Council of Teachers of Mathematics (NCTM) has initiated a professional development scholarship emphasizing the history of mathematics (NCTM 2011); its goal is to

> provide financial support for (1) completing credited course work in the history of mathematics, (2) creating and field-testing appropriate classroom activities incorporating the history of mathematics and (3) preparing and delivering a professional development presentation to colleagues. (NCTM 2011)

In summary, including HOM as part of classroom instruction requires community effort. It requires teachers to reconceptualize the importance of learning the evolution of the mathematics concepts. It requires higher education instructors, administrators and parents to expect that teachers are entering the classroom with strong background and confidence in their knowledge of HOM. For HOM to be included in the classroom, publishers will need to provide teachers with resources that allow for the inclusion of HOM. Finally, it will require the inclusion of HOM as a standard of mathematical practice for state frameworks. If teachers were supported by the states' mandates or recommendations, they would not be alone in their efforts to integrate the history of mathematics into their classrooms.

Note

1 Available at www.corestandards.org (accessed January 11, 2012).

References

Alken, L R. 1974. "Two Scales of Attitude Toward Mathematics." *Journal for Research in Mathematics Education* 5, no 2: 67–71.

Ball, D. 1988. "Unlearning to Teach Mathematics." *For the Learning of Mathematics* 8, no 1: 40–48.

Barbin, E, G T Bagni, L Grugnetti, M Kronfellner, E Lakoma and M Menghini. 2000. "Integrating History: Research Perspectives." In *History in Mathematics Education: The ICMI Study* vol 6, ed J Fauvel and J Van Maanen, 63–90. Dordrecht, Netherlands: Kluwer.

Charalambous, C Y, A Panaoura and G Philippou. 2009. "Using the History of Mathematics to Induce Changes in Preservice Teachers' Beliefs and Attitudes: Insights from Evaluating a Teacher Education Program." *Educational Studies in Mathematics* 71, no 2: 161–80.

Cooney, T J, B E Shealy and B Arvold. 1998. "Conceptualizing Belief Structures of Preservice Secondary Mathematics Teachers." *Journal for Research in Mathematics Education* 29, no 3: 306–33.

Dutton, W H. 1962. "Attitude Change of Prospective Elementary School Teachers Toward Arithmetic." *Arithmetic Teacher* 9, no 8: 418–24.

Fasanelli, F, A Arcavi, O Bekken, J Carvalho e Silva, C Daniel, F Furinghetti et al. 2000. "The Political Context." In *History in Mathematics Education: The ICMI Study* vol 6, ed J Fauvel and J Van Maanen, 1–38. Dordrecht, Netherlands: Kluwer.

Fauvel, J. 1992. "Using History in Mathematics Education." *For the Learning of Mathematics* 11, no 2: 3–6.

Furinghetti, F. 1997. "History of Mathematics, Mathematics Education, School Practice: Case Studies in Linking Different Domains." *For the Learning of Mathematics* 17, no 1: 55–61.

———. 2007. "Teacher Education Through the History of Mathematics." *Educational Studies in Mathematics* 66, no 2: 131–43.

Glaisher, J W L. 1890. "Opening Address in Section A (Mathematics and Physics) at the British Association." *Nature* 42, 464–68.

Ho, W K. 2008. "Using History of Mathematics in the Teaching and Learning of Mathematics in Singapore." In *Proceedings of 1st Raffles International Conference on Education*, 1–38. Available at http://math.nie.edu.sg/wkho/Research/My%20publications/Math%20Education/hom.pdf (accessed January 12, 2012).

Katz, V J. 1997. "Some Ideas on the Use of History in the Teaching of Mathematics." *For the Learning of Mathematics* 17, no 1: 62–63.

Kline, M. 1972. *Mathematical Thought from Ancient to Modern Times*. Vol 1. New York: Oxford University Press.

Krathwohl, D R, B S Bloom and B B Masia. 1973. *Taxonomy of Educational Objectives: The Classification of Educational Goals. Handbook II: Affective Domain*. New York: McKay.

Kuhn, T S. 1996. *The Structure of Scientific Revolutions*. 3rd ed. Chicago: University of Chicago Press.

National Center for Education Statistics. 2009. National Assessment of Educational Progress Mathematics Teacher Background Questionnaire. Available at http://nces.ed.gov/nationsreportcard/pdf/bgq/teacher/BQ09_teach_math_g08.pdf (accessed January 12, 2012).

National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics*. Reston, Va: NCTM.

———. 2011. "Professional Development Scholarship Emphasizing the History of Mathematics for Grades 6-12 Teachers." Available at www.nctm.org/resources/content.aspx?id=23623 (accessed January 12, 2012).

Philippou, G N. and C Christou. 1998. "The Effects of a Preparatory Mathematics Program in Changing Prospective Teachers' Attitudes Towards Mathematics." *Educational Studies in Mathematics* 35, no 2: 189–206.

Schram, P. S Wilcox, G Lappan and P Lanier. 1988. "Changing Preservice Teachers' Beliefs About Mathematics Education." In *Proceedings of PME-NA 11* vol 1, ed C A Maher, G A Goldin and R B Davis, 296–302. New Brunswick, NY: Rutgers University.

Shulman, L S. 1986. "Those Who Understand: Knowledge Growth in Teaching." *Educational Researcher* 15, no 2: 4–14.

Siu, M-K. 2004. "No, I Do Not Use History of Mathematics in My Class. Why?" In *History and Pedagogy of Mathematics: Proceedings of HPM 2004*, ed S Kaijser, 375–76. Uppsala, Sweden: Uppsala University.

Siu, M-K, and C Tzanakis. 2004. "History of Mathematics in Classroom Teaching—Appetizer? Main Course? Or Dessert? *Mediterranean Journal for Research in Mathematics Education* 3, no 1–2: v–x.

Smestad, B. 2009. "Teachers' Conceptions of History of Mathematics." Available at http://home.hio.no/~bjorsme/HPM2008paper.pdf (accessed January 12, 2012).

Stander, D. 1989. "The Use of the History of Mathematics in Teaching." In *Mathematics Teaching: The State of the Art*, ed P Ernest. 241–46. Philadelphia, Pa: Falmer.

Swetz, F J. 1994. *Learning Activities from the History of Mathematics*. Portland, Maine: Walch.

Swetz, F, J Fauvel, O Bekken, B Johansson and V Katz. eds. 1995. *Learn from the Masters*. Washington, DC: Mathematical Association of America.

Tapia, M, and G E Marsh. 2004. "An Instrument to Measure Mathematics Attitudes." *Academic Exchange Quarterly* 8. no 2. Available at www.rapidintellect.com/AEQweb/cho25344l (accessed January 12, 2012).

Tzanakis, C, A Arcavi, C Correia de Sa, M Isoda, C-K Lit, M Niss, J Pitombeira de Carvalho et al. 2000. "Integrating History of Mathematics in the Classroom: An Analytic Survey." In *History in Mathematics Education: The ICMI Study* vol 6, ed J Fauvel and J Van Maanen, 201–40. Dordrecht, Netherlands: Kluwer.

Whitrow, G J. 1932. "The Importance of the History of Mathematics in Relation to the Study of Mathematics Technique." *Mathematical Gazette* 15, no 220: 225–27.

Wilder, R L. 1968. *Evolution of Mathematical Concepts: An Elementary Study*. New York: Wiley.

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