Explanation and Discourse in 9th-Grade Mathematics Classes[1]

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The NCTM's *Professional Standards for Teaching Mathematics* suggests that "the nature of classroom discourse is a major influence on what students learn about mathematics" (1991, 45); but what does discourse look like in a mathematics classroom, what are its features and how does it come about? Of particular interest to us is the development of mathematical understanding through student interaction in discourse (Cobb and Bauersfeld 1995). At minimum, classroom discourse implies talk among teachers and students in mathematics classes. A more elaborate understanding might suggest that it is social interaction in language that supports the construction of mathematics in the microculture of the classroom. This article offers our interpretations of classroom discourse that specifically involves mathematical explanations.

Explaining and Explanations

It is not surprising that the NCTM (1989) asserts that communication and mathematical discourse are primary in mathematics teaching and learning. Various studies have demonstrated the importance of mathematical explanations; for example, clear explanations (Westbury et al 1994), conceptually oriented explanations (Fuchs et al 1996) and demanding explanations (Fennema et al 1996) have been found to be positively correlated with student achievement. While an explanation may be the "heart of any teaching episode" (Leinhardt 1988), some research suggests that discourse is at the heart of learning (see, for example, Cobb and Bauersfeld 1995). The connection between explanation and discourse is described in the NCTM's (1991) *Professional Standards for Teaching Mathematics*. In that document it is suggested that teachers should "consistently expect students to explain their ideas ... [and should] help students learn to expect and ask for justifications and explanations from one another" (p 58). It further suggests that teachers' own explanations must focus on underlying conceptual meanings.

What is it about an explanation that makes it a key element in the processes of teaching and learning? From our work we learned that students think good teachers explain things well, offer good explanations and explain things until everyone understands. Although the students' perceptions appear to imply that an explanation is a gift or something that is given to them by the teacher, we think an explanation is more than that. Might it be that explanations are important to students and key to their mathematical understanding because, as Reid (1995) suggests, "explaining provides connections between what is known in a way that clarifies why a statement is true" (p 24)? Could it be that asking a student to explain himself or herself encourages reflection on one's own understanding? Does the demand for an explanation serve the student by requiring that they talk about (Mason 1996) their own explanations and the explanations of others? In this article we investigate the source of the mathematics explanations and how classroom discourse is used to facilitate the learning of those explanations.

The Teaching Practices Project

The Teaching Practices Project (Simmt et al 1998) was a study to identify features of the teaching practices in schools whose students have a history of performing well on the Grade 9 achievement test in mathematics. The schools selected to participate in the study had a three-year history of meeting provincial expectations at the acceptable standard and exceeding expectations at the standard of excellence. Eight case researchers observed and interviewed 15 Grade 9 mathematics teachers in 13 schools across the province. In this study, we were interested not only in learning what teachers did in their classrooms but also about the nature of the discourse in their classrooms.

Explanations in Discourse and a Discourse of Explaining

One of the common observations that case researchers made was that the teachers' classes observed were highly interactive. This was evident from the quantity and quality of the talk between the participants in the class. Specifically, the researchers reported the extensive use of explanations in the classes they observed. Two things stand out for us: the first is that explanations and classroom discourse were characteristics that co-emerged in the mathematics lessons we observed; the second is that the source of the explanations differed from teacher to teacher. In some classes the teacher was the source of the explanation, while in others the students were the source of explanations.

We would like to offer vignettes[2] taken from two classes in which each teacher happened to be teaching the same topic. These two vignettes illustrate the ways in which these teachers promoted classroom discourse and prompted explanations. In each class it was not good enough to be able to do something; the students also had to be able to explain how it was done. In the first vignette we see the interaction as the teacher helps students to make sense of his [teacher] explanation of how to find the area of a regular polygon. The teacher encourages his students to talk to help them come to an understanding of a statement he made. In the second vignette, the interaction among students and between the students and their teacher is the source of a number of student explanations for finding the area of a regular polygon.

Vignette 1: Bellcroft School[3]

Ron Flynn, the teacher featured in the case, has been teaching mathematics for more than 30 years. Most of those years have been at Bellcroft, a large urban junior high school. Mr Flynn is well prepared to teach his Grade 9 mathematics classes; his course overview reads like a textbook page and lessons unfold like clockwork. His students and their parents talk about Mr Flynn's availability for extra help outside of class, his clear explanations using practical and relevant examples and his use of manipulatives to help students understand mathematics.

The illustration begins with Mr Flynn asking the students to consider a common formula for the area of a regular polygon. Through a series of questions he helps the students make sense of the formula he has suggested. In lines 7 through 38, notice how Mr Flynn "funnels" (Bauersfeld 1988) the discussion toward his "standard" formula. There are both advantages and disadvantages to funnelling the discussion. On one hand, the students are kept focused on the ideas that Mr Flynn believes are needed for the explanation. On the other hand, such funnelling may prevent the students from making decisions for themselves about what is important about the explanation. It is interesting to note that when Kevin (in line 36) asks a question connected to the area of a circle, Mr Flynn does not attempt to explain relationship between the two ideas; rather, he keeps the class discussion moving toward the target—the teacher's formula for area of a regular polygon.

Vignette 2: Gilhooly Junior/Senior High School

This case features the teaching and learning in Bill Wilchuk's classroom, where mathematical conversation is both commonplace and awe-inspiring. Students in Mr Wilchuk's class appreciate the way he begins class by talking. One student said, "We talk about something different every day. He asks the class to give answers. It makes me listen more." Mr Wilchuk credits this kind of relaxed conversation to the extra time that he gets to teach math; the school schedules more time for core subjects than is mandated by Alberta Learning. He says that, before he had the extra time, he used to take up homework at the beginning of class. But now that he has more time, he chooses to begin each class with something new.

In this vignette we see the interaction between students and their teacher as he tries to help them make sense of the area of a regular polygon. In this case we note that Mr Wilchuk encourages his students to offer their explanations of how to find the area of a regular polygon. Rather than have the students work toward understanding the teacher's explanation, the teacher solicits many explanations, each of which is treated as a valid possibility. These explanations are illustrated in lines 2 to 5, 6 to 41, 42 to 51, 52 to 86, 87 to 94, and 95 to 105. In this discourse pattern, the teacher does not funnel the conversation but instead allows the conversation to remain open, thus providing the possibility that students might make sense of multiple explanations of a single concept. The vignette begins with Mr Wilchuk asking the students how they are going to find the area of a regular hexagon.

Vignette 1

"Does anybody know what the formula is for the area of a regular polygon?" asked Mr Flynn.

"Susan?"

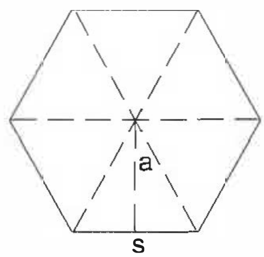
"No," she shook her head.

"Okay, this is probably one that you haven't seen before. So let's just take a moment to look at that."

Mr Flynn drew a hexagon on the board. "If we take our regular hexagon and divide it up. What do we have to divide it into?"

Susan answered, "Six triangles."

Figure 1. Board Diagram for Area of a Regular Polygon



"Six congruent triangles. That's right. This distance here ..." he points to the apothem, "is the altitude of the triangle. And a side of the hexagon would be the base of each triangle. How do we find the area of triangle?"

"Base times height divided by two," Kevin suggested.

"Right." Mr Flynn gestured to the parts of the diagram that corresponded to the base and the height of the triangle. "This times that, divided by two. I'm just going to call this *a* for 'altitude' right now. There is another word that I'll tell you in a minute .... So we would have *a* times *s* divided by two. And how many of those triangles would we have?"

Many students called out, "Six."

"How many would we have if we had a regular pentagon?"

"Five."

"An octagon?"

"Eight."

"Right, so it's the number of sides." Mr Flynn concluded.

Mr Flynn returned to his board formula *as*/2 and added, "So, I'm just going to multiply that by *n*, standing for the number of sides that we have. Now, I'll write it in this form: 1/2 *ans* ... or ... *ans* over 2."

He continued, "*a* is the altitude of the triangle, but when you get a question like this, you're not going to have it divided up for you." He draws a diagram of the way a question might appear and puts a dotted line for the height of the apothem. "And the *a* will stand for, and write this word down so that you remember it, 'apothem.'"

Then he printed the word on the board and continued, "So for any regular polygon, that's how we find the area. The apothem times the number of sides times the length of each side divided by two."

Kevin asked, "Is that the same for a circle? Is that thing in a circle, the radius, called an apothem too?"

"No. This only works for regular polygons. The distance from the centre of the polygon to the midpoint of the side—that's called the apothem."

Vignette 2

"I'm curious as to what you're going to do for area."

Trevor blurted out, "Radius. The diameter."

"Okay, careful Trevor. Radius refers to basically circles as a subject."

Many students interjected with comments about Trevor's idea and tried to offer other possibilities. "Only one at a time," Mr Wilchuk reminded his class. He then called on Mike who suggested splitting the polygon into smaller shapes.

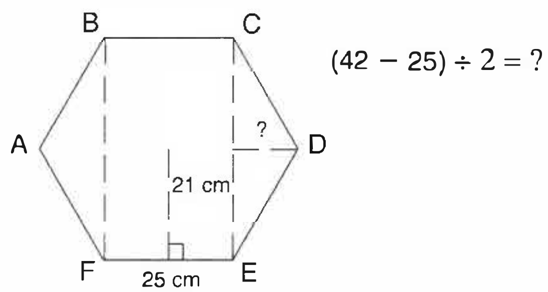
"Can you be more specific?"

Mike answered, "Two triangles and a rectangle."

A student sitting near Mike grasped the idea and said, "Oh, I see what he's saying."

Mr Wilchuk began to sketch Mike's idea on the overhead. "I'm going to try to copy what he's indicating. I don't know if Mike wanted me to split it this way, but at least this is one way I could do it." He sketched Mike's idea on the overhead (Figure 2). Then before making an attempt to solve using Mike's method, Mr Wilchuk asked the class, "Can we check to see whether or not his method will work?"

Figure 2. Mike's Method: Two Triangles and a Rectangle



Andrew responded, "I think it will."

And there was a chorus of, "It will."

Mr Wilchuk began to calculate, "I can take 25 and I can multiply it by ..."

He overheard a student say, "42."

"Excellent," Mr Wilchuk commended, "42. That will give me the area of the rectangle. How about these triangles?"

A student volunteered, "25 and 25." She was talking about the two short sides of the triangles in the sketch (Figure 2).

"This is 25, that's 25 ..." Mr Wilchuk confirmed. "What's the distance from here to there? In other words, the height of the triangles in Mike's method."

Mike called out, "42 take away 25 divided by 2."

"Now you're going to have to slow down for us. Say that again?"

"42 ... take away 25 ... divided by 2."

Mr Wilchuk thought out loud, "42 ... take away 25 ..." With further explanation from Mike, Mr Wilchuk discovered that Mike thought that the distance from vertex A to vertex D was the same as twice the apothem. In this case, 42 centimeters. The clarifying took about 45 seconds. Several students got confused and began to murmur.

Mr Wilchuk did not correct Mike. Instead he deferred Mike's idea for a moment and said, "It may or may not work, but I'm with one of your colleagues here—it's sort of confusing. 'It hurts the head.' I'm not going to say right now that your method will not work, but I think that some of your fellow students are saying that it's a tough one."

Mr Wilchuk encouraged more discussion. "Other suggestions, one at a time. Okay, Allen?"

Allen suggested dividing the hexagon into six triangles.

Mr Wilchuk wondered aloud, "Six triangles..." Without being prompted students turned to their neighbors and chatted about whether Allen's idea would work. Mr Wilchuk began to divide his hexagon into six triangles and listened to the student discussion.

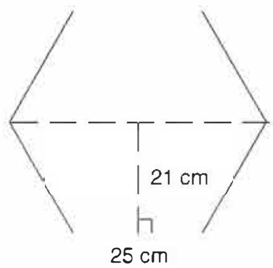
The conversation continued for about 10 seconds until Dean said, "Stop right here!"

So Mr Wilchuk interrupted the class discussion, "Hold it. Allen said 'six triangles' and in my attempt to start to divide it, Dean said, 'Stop right here!' So, Allen, I'm just going to put you on hold for a second—"

Some students giggled at the idea of Allen being put on hold.

"Now, Dean is suggesting that this will work." Based on Dean's suggestion, Mr Wilchuk divided the hexagon in half making two trapezoids (Figure 3).

Figure 3. Dean's Method: Two Trapezoids



"Do you know how to find the area of a trapezoid?"

Several students answered, "Yeah."

The teacher continued, "So, I could find the area of one, double it, and I've got the area of this thing. Now, do I have all my numbers? Do I know this distance?" He pointed to the base of the trapezoid and the distance that Mike had earlier thought was 42 cm.

"42," offered someone.

"It's not 42," said Mr Wilchuk.

"45," suggested Cynthia.

And Jeff echoed, "45."

"46?"

Then, "Uh-oh."

And another, "Uh-oh."

And, "Uh-oh."

Mr Wilchuk interrupted the confusion. "Actually, Dean the neat thing about a hexagon is that it is made up of six equilateral triangles. Or regular triangles. And, if this is 25," as he pointed to the side of the hexagon, "this side must be 25, from here to here." He gestured toward the distance from a vertex to the centre of the hexagon. "Believe it or not, I could find the base of this trapezoid."

"50," someone shouted.

"That is 50," Mr Wilchuk confirmed. Then he went back to the original question. "Do I have all my numbers?" And discovering that indeed he did, he proclaimed, "Hey, I can do this!"

A student wrinkling her nose caught Mr Wilchuk's attention.

"Okay, you may not like this method," Mr Wilchuk said to the student, "but, nonetheless, I can do what Dean suggested. Dean, can I get you to focus on *this* shape?" Mr Wilchuk indicated the pentagon on the overhead. "Does your method still work when I change the shape?"

"No," several students concluded.

"Okay, so even though his procedure will work for one, it's not universal in that it will work for all the regular polygons? Now, Mike's method. He's maybe saying, 'You didn't give my method a chance.' His method may have worked had I gone through it completely with him. Would his method work down here with the pentagon?"

"Yes," said Mike.

Mr Wilchuk questioned, "You can break it up into a rectangle and two triangles?" With that, Mike agreed that his procedure would not work for the pentagon and played at looking deflated.

Allen laughed. So did Mr Wilchuk.

"Okay, Allen, going back to yours." Then he teased, "We'll criticize *yours* in a minute. So, we'll see."

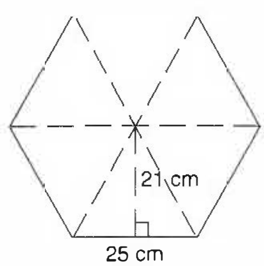
Others laughed, anticipating that Allen might be proven wrong.

"There's nothing wrong with it," Allen asserted, smiling.

"He says 'nothing's wrong with it.' Well, we'll see. You'll have to excuse my rough drawings here but I'm supposed to have six regular triangles. Before *you* draw, we better make sure that everybody's comfortable with this material. Okay, Allen, would you explain your procedure."

Allen explained, "You divide the hexagon into six triangles and use the area of a triangle formula to calculate the area of each triangle (Figure 4). Then, you multiply by 6."

Figure 4. Allen's Method



Mr Wilchuk asked Allen, "Dean's method worked for the first question. Is your method going to work for this guy, the pentagon?"

"Nope." Allen quickly accepted defeat for comic effect.

The students laughed.

"Why don't you show us the right method?" Jason asked.

"Slow down," said Mr Wilchuk, responding to Jason. "Okay, Allen, why didn't you say your method worked for the second one? I'm confused. Your method worked so nicely here."

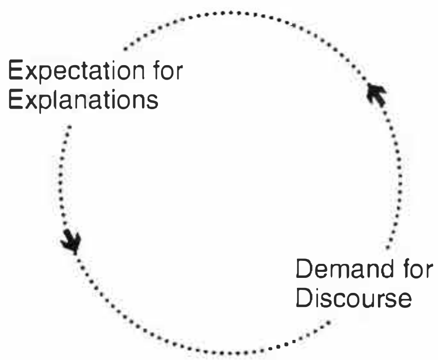
"Yes, but the second one ..." And so the conversation continued.

Prompting Explanations and Promoting Discourse

In both of the classes observed, students were required to participate actively in the lessons, and teachers were very good at making sure students did so. Also common was that the discourse both between the students themselves and between students and the teacher was not simply around the answer to a mathematics question or problem (for example, calling out the answer to homework questions) but also consisted of explanations for the answers or explications of how a problem could be solved.

From our observations and analyses, we have come to understand the relationship between discourse and explanations as reflexive (see Figure 5). On one hand, the classroom discourse was fostered by the teacher's requests for explanations from the students; in complying, the students participated in the discourse. On the other hand, the expectation that the class be highly interactive, especially in terms of discourse, meant that as part of their teaching of mathematics teachers requested students to participate in the explaining.

Figure 5.
Reflexive Nature of Explanation and Discourse



Of particular interest to us was our observation that the explanations had two distinct sources depending on the teacher observed (and sometimes depending on what mathematics was being taught). Further, the discourse was intended to serve one of two purposes depending on the source of the explanation. In the first discourse pattern, we observed the teacher Mr Flynn to be the source of the explanation. That is, the teacher offered a mathematical explanation to the students and the discourse around this explanation was directed toward students making sense of the teacher's explanation. We will call this form of discourse teacher source/student matching discourse. Notice that this form of discourse assumes a preferred explanation and the need for students to construct or acquire the teacher's explanation. This was a common form of discourse in the classes we studied in the Teaching Practices Project.

The second discourse pattern is different from the first in that the students are the source of the explanations. This discourse pattern involves the teacher facilitating a discussion about a particular topic and soliciting student explanations. In the second vignette, we note how Mr Wilchuk encourages the students to offer their explanations in the discussion and for discussion. Here the students' explanations are treated as legitimate possibilities, and there is an acceptance of multiple explanations. Students are not expected to come to an understanding of a single explanation offered by the teacher; rather, they are expected to actively listen and participate in the explanations of others, formulate their own explanations and offer those explanations as contributions to the class. In this discourse pattern, which we will call student source/student fitting discourse, there is construction of mathematics knowledge in community with others. We witnessed this discourse pattern in a few of the classes we observed.

Based on our observations, the teacher's demand for talk and interaction around an explanation did not seem to depend on who was the initial source of the explanation. In both classes students were expected to actively participate in the large group discussion. In Mr Flynn's class, the students' role in discourse was to match the teacher's explanation, and in Mr Wilchuk's class, the students were expected to talk out their explanations to see how they fit with the explanations of others in the class. Any distinctions between the discourse patterns with respect to the meaning students made of the explanations was beyond the scope of this study. However, we think this is an important question and we would like to explore (in another study perhaps) the qualities and growth of student mathematical understanding in the two situations: when the students are themselves the source of the explanations and when the teacher is the source of the explanation.

From observing these teachers in action, we have been prompted to think about the importance of highly interactive classes where explanations are the focus of discussions. In classes where students actively participate in discussions, there is plenty of opportunity to learn *why* and *how* mathematics works—not just that it works. We invite teachers to reflect on their own teaching practices and ask themselves about the role explanation plays in their mathematics classes and to what extent they foster discourse around mathematical explanations. Based on our observations these appear to be very important questions for mathematics teachers to consider.

Notes

1. This research was supported by a grant from Alberta Learning, Curriculum Standards Branch.

2. The cases are presented in full in *The Teaching Practices Project: Research into Teaching Practices in Alberta Schools that Have a History of Students Exceeding Expectations on Grade 9 Provincial Achievement Tests in Mathematics*. A copy of this report has been sent to all junior high schools in Alberta.

3. The names of the schools and teachers have been changed to protect anonymity.

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