Introducing Contemporary Concepts in Traditional Arithmetic

James M Grasley

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Although most elementary schools are still using traditional arithmetic texts, it is possible for teachers to introduce phases of the "new mathematics" without special fanfare or radical measures. Many teachers hesitate to stray too far from the text and its traditional approaches. If you were to try some of the ideas that follow, you might introduce the flavour of new mathematics into the present course. The content material is clearly traditional but its vocabulary and presentation are that found in contemporary approaches. Once you get used to the unfamiliar vocabulary of the contemporary material, you will see that most ideas presented in new mathematics are quite familiar. I am sure that as the new terms become familiar you will find yourself introducing them where they seem appropriate to your traditional teaching. Maybe there is some comfort in the fact that your pupils have less trouble with the new vocabulary than you will.

As you study the contemporary ideas being advanced, perhaps you can suggest procedures which will allow children to work with familiar operations and interpret them in the light of the new mathematics. By so doing we need not wait years to have the best of the new approach incorporated into our school program. It can be done now.

Number Line

Some people find that the number line is a good way to introduce new material. Most traditional texts do very little with the number line, and its use to supplement the arithmetic program is excellent.

Careful study of the use of the number line by teachers, from primary to high school, will reveal much material and many approaches for classroom use. Many teachers find use of the number line helps pupils clarify their thinking. Why not have a number line painted on the floor in the room? Children could step off addition and subtraction facts on it. They could use the line to discover the commutative property of addition—from 2 you step off 3 additional spaces and you are at 5: $2 + 3 = 5$. Also from 3, by adding 2, you are at 5: $3 + 2 = 5$.

The associative property of addition could also be discovered by the pupils themselves. After doing the operations physically by stepping them off on the line, they could generalize that $(2 + 3) + 4$ is exactly the same as $2 + (3 + 4)$ and might even relate it to the previous idea of equality with $4 + (3 + 2)$. Whether or not the terminology, associative property and commutative property are used depends on the teacher and class. However, what is important is the idea.

Perhaps a bright pupil might ask why just one side of zero is used. What about numbers to the left of zero? I am sure that such an opportunity would arise long before high school, and negative numbers could be introduced. Using the number line on the floor, children could soon be doing examples like $3 + (-4) = -1$ and $(-4) + (-5) = -9$. With a little imagination, some reading of current articles on new mathematics and a few carefully planned lessons, most teachers could supplement their traditional programs with ideas from the new, modern mathematics.

More Than One Name for a Number

Have a set of objects ready for your class to see. Suppose you use five objects in a set—the four fingers and thumb on the left hand, the five pussy willows on a twig, or the five girls in the row of desks next to the windows. Have different pupils write on the chalkboard ways of representing the number in the set. You could get 11111, 5, V, *five* and so on. You could encourage discussion of different ways of naming 5 such as $(4 + 1)$, $(3 + 2)$, (5×1), $(7 - 2)$. You could set up a table.

$5 = 4 + 1$	$1 + 4 = 5$
$5 = 3 + 2$	$2 + 3 = 5$
$5 = 5 \times 1$	$1 \times 5 = 5$
$5 = 7 - 2$	$6 - 1 = 5$

Emphasize that the numerals on either side of the equal sign are ways of naming the same number, 5. That these are *numerals*, not numbers, should be stressed, for this is basic to the new approach.

This idea of renaming numbers can be used profitably in practice and review exercises. For example, pupils may express 14 in different ways as the sum of two whole numbers:

14 = 7 + 7	14 = 13 + 1
14 = 2 + 12	14 = 0 + 14
14 = 3 + 11	14 = 5 + 9
14 = 4 + 10	14 = 8 + 6
14 = 6 + 8	14 = 11 + 3

Or express 14 as the difference of whole numbers three, five or a specific number of ways.

The exercises suggested above can be used by pupils to discover properties of numbers. For example, if they have written 17 as the sum of two whole numbers, they may be asked, "Did you use odd numbers to name 17 as the sum of two whole numbers? Did you use an odd and an even number to name 17 as the sum of two whole numbers?"

Some examples of how the idea that numbers have more than one name may be used in traditional arithmetic:

1. Write 6 as the sum of two addends where
   a) One addend is a proper fraction and the other is a mixed number, both fractions having the denominator 6.
   b) Both addends are mixed numbers, the fractions, having the denominator 3.
   c) One addend is the whole number 4 and the other is an improper fraction having the denominator 2.
2. Name each number as tens and ones in two ways.
   a) 27 (2 tens and 7 ones, 1 ten and 17 ones)
   b) 109 (10 tens and 9 ones, 9 tens and 19 ones)
3. Complete the following sentences.
   a) 286 = 2 hundreds + ____ tens + 6 ones
   b) 286 = 2 hundreds + 5 tens + ____ ones
   c) 286 = 1 hundred + ____ tens + 6 ones
4. Name each of the following in three different ways using hundreds, tens and ones.
   a) 405
   b) 312, etc
5. Name the following using thousands, hundreds, tens and ones.
   a) 9,125
   b) 3,047, etc

In addition to the ideas presented above on the use of the number line and on number–numeral distinctions, operations and their opposites and the properties of operations can also be used to blend some contemporary mathematics into the traditional approach.

Editor's note [from original publication date]: Max Grasley of Saskatchewan has been a tireless worker in that province, in training elementary teachers in modern concepts. He has played a large part in preliminary work for the new course for their new course for Grade IX and is very active in the development of training courses at the University of Saskatchewan for junior high school teachers. This article deals with methods any elementary teacher may use in his interpretation of the "new mathematics."