Discovery or Programming

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Is it possible that one sees two different trends in mathematics education as he surveys current publications from afar? On the one hand we have the experimental developments by such groups as the University of Illinois Committee on School Mathematics, the School Mathematics Study Group, the Ball State Project and the Madison Project. Upon closer inspection, one finds the discovery approach built into the material. Many present-day authors are trying to imitate their style and their approach to the material.

Each of the groups mentioned previously and most of the authors currently producing material for mathematics courses recognize and make use of pupil discovery of relationships and structure. Mathematics is recognized as a subject area that has a structure that is meaningful to the student. Thus, the students are able to discover relationships and to build one concept upon another. Mathematics does not become a series of isolated bits of fact and computations to be mastered until they become habit.

Taking the teacher and the teaching method into consideration, this approach is designed to give the student an opportunity to think for himself. The teacher must know a great deal about the material. He must know where he has been and where he is going and must know how to guide the pupils to develop the structure for themselves. Questioning techniques take on greater importance. Rules must not be presented to the student to be applied blindly to a multitude of simple examples until the mechanics become habit and these rules can be recalled upon the receipt of the proper stimulus.

It has been argued that the statement of the generalization by the student is not necessary. Some even feel that this is a hindrance. The spoken or written language gets in the way of the mathematical ideas. When a student is able to apply the generalization in an unfamiliar situation, then he knows what it tells him.

Others argue that it is necessary for the student to express the generalization. Only then can one be certain that the concept is known and understood.

On the other hand we have the type of curriculum exemplified by programmed materials. This development is not new but in recent years it has gained impetus, especially in mathematics.

One of the most significant points involved in programming is that the student is led down a very definite path. The material to be mastered is presented in very small steps to ensure understanding and correctness. The student is able to proceed very slowly and along a path determined by the author of the program. At no point is he permitted to meditate upon a related topic. His attention is always directed toward the mastery of one specific concept.

If one sticks to the traditional definition of mathematics, one thinks of it more as a tool subject. Is this all that mathematics is in this day and age? Many very prominent people do not accept this view alone. Mathematics has become more than mere subject matter to be mastered because it is useful in some other field of endeavour. It is thought of more as a way of thinking, as an academic discipline to be studied for itself. A great many mathematicians study mathematics just for the sake of the mathematics involved and not because of its utilitarian value.

Looking for a moment at the mathematics curriculum or at mathematics education, can we note any relationship in the trends? How do they appear to be affecting the curricula in mathematics? What is the effect of each of them on the teaching of mathematics?

It would appear that the two ideas are not very closely related. They would seem to be worlds apart. The discovery approach adopted by the UICSM and the SMSG would seem to give the pupil credit for being able to think for himself, for being able to recognize meanings, for being able to direct his attention toward a series of related learning tasks.

The approach adopted by those who advocate programmed instruction would appear to deny the ability of the pupil to do independent thinking. A stimulus is presented to the student to which he must make one and only one response. Since this response is right 90 per cent or more of the time, he has little

or no opportunity to analyze. His attention is directed toward a rather narrow, limited topic.

The effect on the curriculum in the one instance seems to be a freeing one. Pupils are given an opportunity to act as mature mathematicians. Observations are made. These are accepted or rejected by proof. New observations or relationships are introduced, not necessarily by the teacher or text. These, too, are accepted or rejected by proof from what has gone before. The pupil is an active participant in the development of mathematical concepts.

Programmed instruction tends to do the opposite, as far as the pupil is concerned. Pupils are not given an opportunity to make independent observations. They have little chance to analyze so that they might accept or reject a relationship. The opportunity to act as a mathematician is absent.

As far as the teaching of mathematics is concerned, one of these trends would permit the individual to develop as a skilled craftsman. The teacher would have a vast storehouse of knowledge which he would need to rely upon to keep the class moving in a correct fashion. For example, if a student wanted to solve quadratic equations, the teacher would know immediately whether or not this could be done with the knowledge possessed by the student. The teacher could then guide the student through the discovery of the various processes of solving this particular type of problem. Knowledge of his subject, then, is very important to the teacher who wants to follow the discovery approach used by the SMSG or the UICSM.

Programmed instruction would seem to leave very little for the teacher to do. When a student is unable to understand a specific point, the teacher could assist the student in mastering this concept. One other aspect of programming comes into play when a teacher builds a program of his own. During his labours, he becomes more intimately acquainted with the particular topic, with some of the problems involved in learning this topic, and with some of the problems involved in teaching this topic.

In this paper a brief look at two apparently divergent trends in mathematics education was attempted. Each teacher of mathematics must look more closely at each of these trends to see how they will or will not affect his teaching. It seems obvious that no teacher will remain untouched by these trends. Many people are advocating one or the other of these two approaches, people who are recognized as authorities in mathematics education. Perhaps it will be best for each teacher to conduct a little action research in his own classroom to help him decide. There can be no fence straddlers.