Implementing Manipulatives in Mathematics Teaching

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Manipulative materials have emerged in mathematics instruction as more than just a means to add variety to lessons; they are an essential element for effective mathematics instruction. However, for manipulatives to be used successfully in the classroom, a great deal of thought must precede their implementation. What role do manipulatives play in mathematics instruction? What factors influence the effective implementation of manipulatives in the instructional process? This paper presents alternative answers to these questions.

A Definition of Manipulatives

The purpose of manipulatives is to make mathematics more concrete. Manipulatives enable students to play with, experience and develop for themselves mathematical principles, relationships and ideas. For manipulatives to have any place in the mathematics classroom, they must embody or physically represent specific mathematical concepts (Wiebe 1983). Consider two examples and one counterexample.

A concept that many elementary mathematics students struggle with is why the remainder after division can never exceed the divisor. This concept may be illustrated when teaching division using a balance beam (Knifong and Burton 1985). To model the equation $7 \div 2$, the student would place 1 weight on the balance a distance of 7 units to the left of the fulcrum (see Diagram 1). Because the divisor is 2, weights are hung 2 units from the fulcrum on the right side until the beam is balanced. The situation quickly arises that when 3 weights are hung on the right side, the beam tips to the left, but when another weight is added, the beam tips to the right. Where then should the final weight be hung in order to balance the beam? Through experimentation, it is obvious that hanging the weight any further to the right (eg, a value greater than the divisor) is counterproductive, and thus a position closer to the fulcrum (eg, a value less than the divisor) must be selected. In this case, the weight must be hung 1 unit to the right of the fulcrum to completely balance the beam. The remainder must always be less than the divisor; this mathematical concept is actually embodied within the manipulative materials.

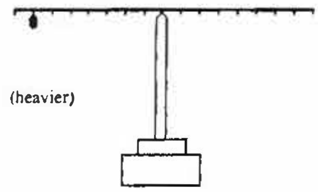
Poker chips may be manipulated to model the subtraction of negative integers. Assume that a blue poker chip represents +1 and a red poker chip represents -1. Thus the subtraction of -4 from 3 in the equation $3 - -4 = ?$ may be modelled as follows. Set out 3 blue poker chips (+3) and then remove 4 red ones (-4). It is obvious that no red chips may be removed because there are only blue chips available. However, note that a blue and a red chip together total zero (ie, $-1 + 1 = 0$). Thus, any number of pairs may be added without changing the value of the expression. Pairs are added until there are enough red chips such that 4 may be removed (add 4 pairs). Now, when 4 red chips are removed, 7 blue chips remain. This process illustrates that $3 - -4 = 7$. This model makes it clear why the difference is greater than the minuend when subtracting a negative subtrahend.

As a counterexample, consider the common exercise in which students pair numbered cards with corresponding word cards (see Diagram 2). This activity, and others like it, may be called manipulative only in that students are given some object (cards) that they may touch and move. The cards do not embody any mathematical concept, however, and this exercise only serves to help students develop correspondence between names and symbols (a vocabulary exercise). For the student, no greater understanding of "oneness," "twoness," or "fiveness" is developed simply by matching symbols to words; memory skills are drilled.

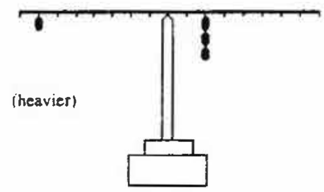
Diagram 1

Using a balance beam to illustrate division. Solve 7 ÷ 2 = ?

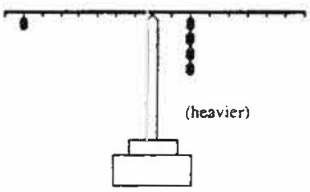
A. Begin with one weight seven units to the left of the fulcrum.



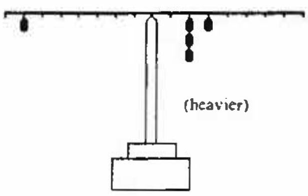
B. Add weights 2 units to the right of the fulcrum to balance the beam. The left side is still too heavy.



C. The right side is now too heavy, so the whole number quotient must be 3. Experiment with one weight to find the remainder.



D. The right side is still heavier, so the remainder must be less than 2, which is the divisor.



E. The beam is now balanced. The remainder must always be less than the divisor.

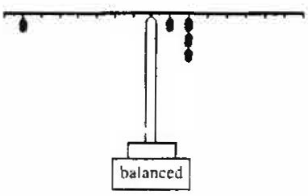
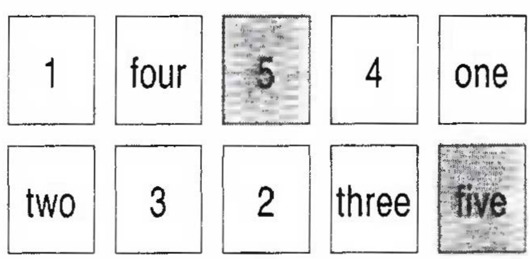


Diagram 2

Matching numbered cards with name cards.



The shaded pair is a match and may be removed. This activity is sometimes played as a game which begins with all cards face down. Two cards, one at a time, are turned over by a player. If a match is found then those cards are removed from the game. If no match is found, the cards are turned back face down and the other player takes a turn. The winning player is the one having made the greatest number of matches once all the cards have been used.

Three Implementation Models

What role do manipulatives play in mathematics instruction? Where do manipulatives fit into the typical instructional sequence: introduce, develop, review and evaluate? The following three general models for implementing manipulatives offer some alternative answers to these questions. These models may be applied to either individual lessons or to complete units.

The first implementation model is called the *introductory model* (see Diagram 3) because the manipulatives are used only in the beginning stages of instruction. The purpose of the manipulative in this model is to introduce the mathematics concept to be learned and to provide a body of concrete experiences that can be drawn upon or synthesized during later formal instruction. The manipulative also fulfills the purpose of increasing student interest and motivation. In some cases the manipulative also provides a sense of relevance to the later formal instruction delivered by the teacher. The learning sequence flows from the concrete to the abstract. In this model, knowledge is organized from the general to the specific; general concrete experiences are provided first and followed by highly structured formal sessions in which specific concepts are revealed through an oral exposition delivered by the teacher. The major assumptions of this model are that students require a context for effective formal instruction, and that general concrete experiences facilitate the learning of specific abstract concepts.

The second implementation model is called the *tertiary model* (see Diagram 3) because the manipulatives are not introduced until the latter stages of instruction. Early instruction is teacher controlled, but later experimentation is less closely monitored.

In the initial stages of this model, the teacher provides formal focused instruction on specific abstract concepts; the focus is not on understanding but on the awareness of principles. These specific principles are later linked to create more general knowledge through informal experimentation with manipulatives; knowledge is organized from the specific to the general, while experiences are organized from the abstract to the concrete. The manipulative serves a synthesis role and functions as a context in which learned concepts may be applied. In this model, the manipulative may also serve as a means for the teacher to evaluate student progress and understanding, as well as a means to undertake review of specified concepts. The second model is built upon the assumption that students require basic skills and knowledge before they can fully benefit (eg, draw conclusions and formalize mental structures) from the experiences and environment manipulatives provide.

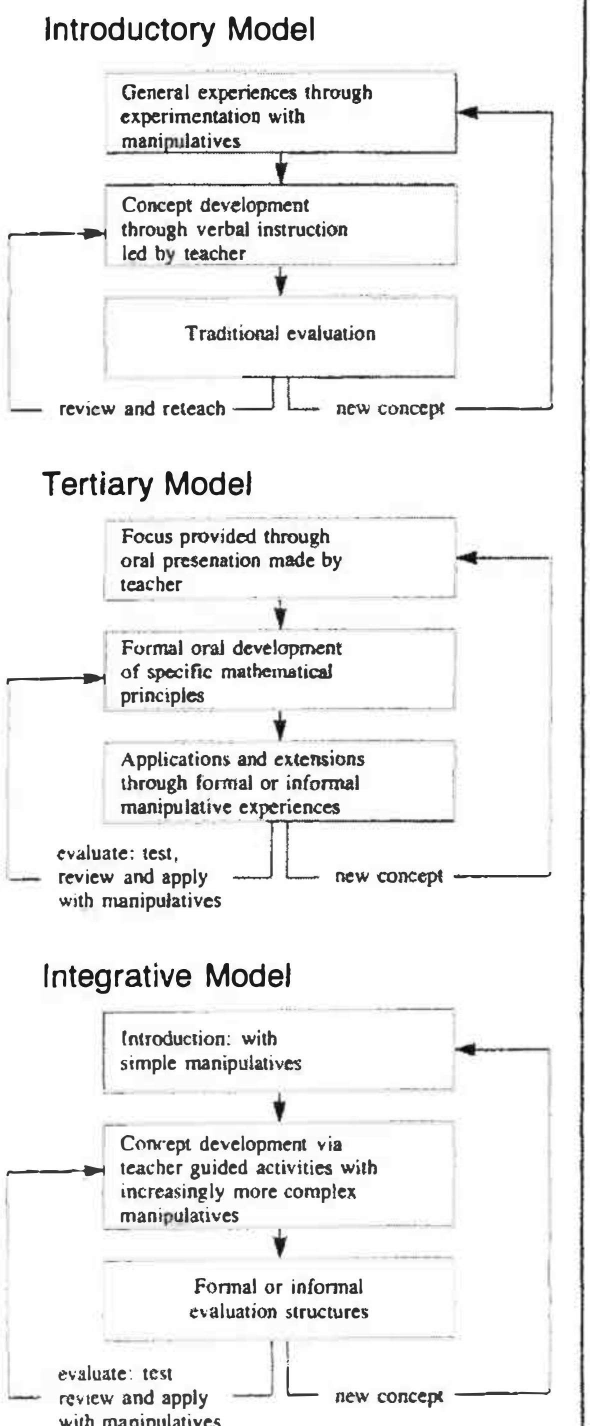
The final model is the *integrative model* (see Diagram 3). In this model, manipulatives are used continually throughout instruction; knowledge and skills are introduced, developed, reviewed and evaluated through concrete experiences with physical representations of mathematical concepts. By using manipulatives at all points during instruction it is hoped that high motivation and interest levels will be maintained throughout the entire instructional cycle. Using a manipulative for all phases of instruction eliminates the need to introduce more than one set of materials. The major assumptions of this model are that students learn better and retain longer what is learned in a familiar context, and that all phases of instructional cycle may be delivered easily using manipulatives.

No one model is correct or better than another. Instead, the teacher should use the model that best suits the material to be taught, the needs of the students and his or her own instructional style. The teacher may wish to consider the mathematics skills and motivation levels of the students, the students' learning styles, the synthesis and generalization skills of the students, the ease with which students master and apply learned concepts, and the complexity of the mathematics concepts to be taught. Each model possesses its own advantages, disadvantages and assumptions. The teacher must select the model in which the disadvantages are minimized, the assumptions appear realistic and the advantages are exploited. When these conditions exist, the purpose of the manipulative is maximized and effective implementation is achieved and measured by improved student learning.

Some Factors Influencing Effective Implementation

Manipulatives may be evaluated according to a variety of criteria. Hynes (1986) has suggested that manipulatives may be evaluated according to both their pedagogical and physical attributes. With respect to pedagogical attributes, manipulatives must provide a clear representation of a mathematical concept, be appropriate for the student level, interest the students, be versatile, contribute to the building of a mathematical concept, assist in developing vocabulary, improve spatial visualization, promote problem solving, provide a sense of proof and promote creativity. With respect to physical attributes, the manipulative must be durable, simple, attractive, manageable, cost effective and reasonable in terms of the quantity required. Not all manipulatives exemplify all of these

Diagram 3
Implementing Models



attributes, but generally, the better the manipulative, the more conditions it will satisfy.

The attributes that Hynes describes are valuable when discussing the relative differences between manipulatives, but the true value of manipulatives lies in how effectively they may be employed in teaching and learning situations. The most versatile, motivating and attractive manipulative will not be effective unless properly employed. Therefore, the manner in which the activity is conducted may be just as important as the materials themselves.

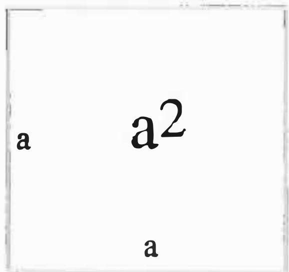
The first implementational consideration is the degree to which the student has control over concept development. If given time to simply experiment and play with the objects, will students develop the desired concept on their own? To what extent must the students' interaction with the manipulatives be guided by the teacher? To allow students to discover and develop concepts independently is often too time consuming, and there is no guarantee that the concept will ever be clearly or correctly formalized; however, concepts that are developed independently are more likely to be retained and treasured. The teacher must decide which form of manipulative is preferable based upon such considerations as students' past experiences with discovery learning, students' learning styles, the time available for the development of a concept, the motivation level of the students and the ease with which the concept may be summarized from the play experience.

The second implementational consideration pertains to the degree to which the student may control the manipulative. Is it desirable that each student has his or her own set of manipulatives, or is it sufficient that the teacher manipulate one set for the benefit of all? When the teacher manipulates the materials for the students, then the visual experience is substituted for the tactile. When working with individuals or small groups, the discovery learning approach is possible, and this approach necessitates a tactile experience. When working with a large group, a visual learning experience is more practical. In essence, the teacher defines his or her own role as that of consultant or group leader. The choice of role dictates the degree to which the teacher intervenes in and controls the concept development process.

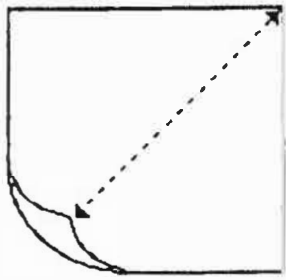
The third implementational consideration is the degree to which the mathematical concept embodied within the manipulative is obvious to the students. When the concept is obvious, then the materials are appropriate for developing mathematical relationships or facts. When the concept is less obvious, then the manipulative serves as a data-keeping tool. When the manipulative is used as a data-keeping tool, the

Diagram 4

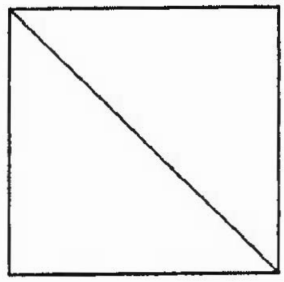
Illustrating the difference of squares: $a^2 - b^2 = (a-b)(a+b)$



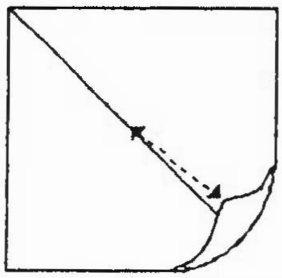
1. Begin with a square piece of paper.



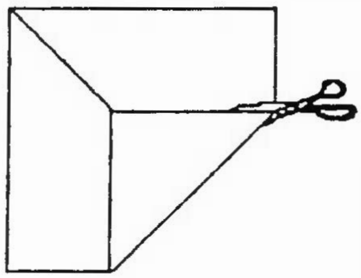
2. Fold diagonally.



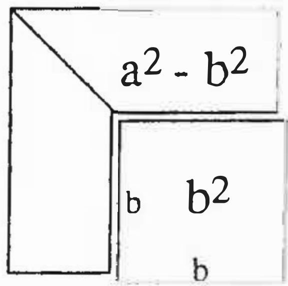
3. Crease.



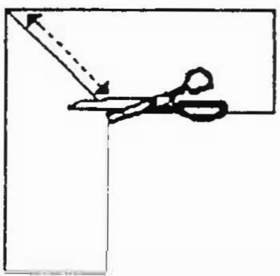
4. Fold one corner on the diagonal crease to any point part way along the crease.



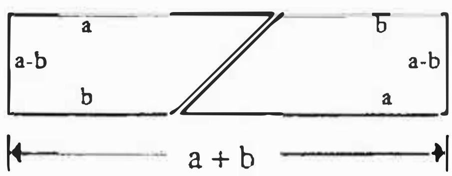
5. Cut along the paper's edge.



6. Remove b^2.



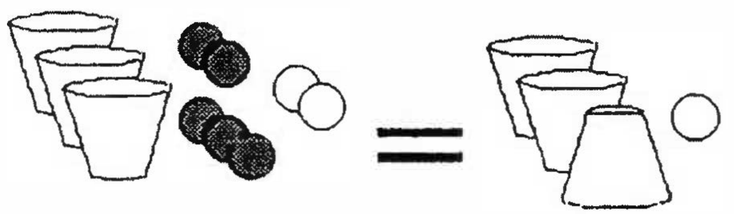
7. Cut along the remaining diagonal fold.



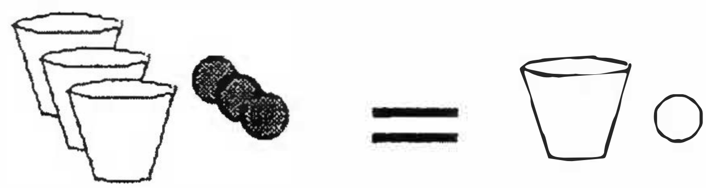
8. Rearrange.

Diagram 5

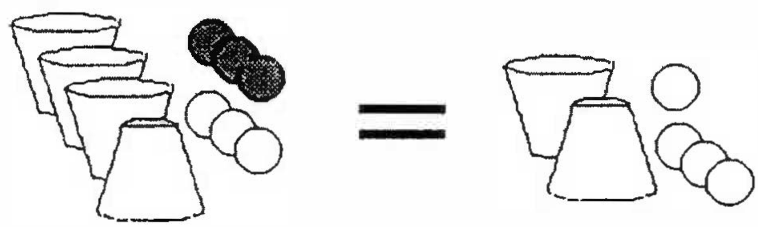
Modeling the process of solving algebraic equations with paper cups and colored paper chips.



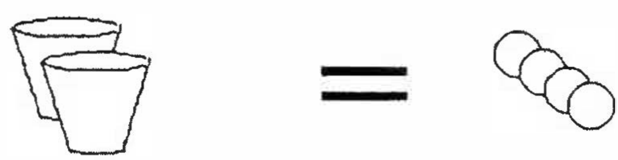
$3x - 5 + 2 = 2x - x + 1$
Remove opposites to simplify.



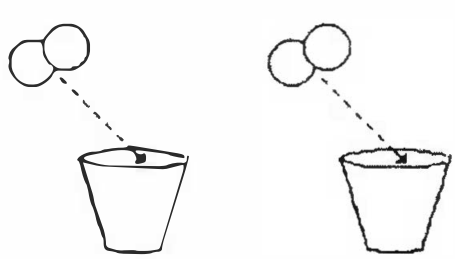
$3x - 3 = x + 1$
Add opposites to get like expressions on the same side of the equation.



$3x - x + 3 - 3 = x - x + 1 + 3$
Remove opposites to simplify.



$2x = 4$



Distribute.
$x = 2$

student is relieved of data-keeping functions and may concentrate more fully on process skills. The following examples clarify the data-keeping and concept-development natures of manipulatives.

The first example is illustrated in Diagram 4. In this example, paper is folded and cut (Bober and Percevault 1987) in such a way as to illustrate why $a^2 - b^2 = (a - b)(a + b)$. Once the activity is completed, an inherent sense of proof or obviousness makes it difficult to contest that the relationship is true.

The second example is illustrated in Diagram 5. In this example, the students model the process of solving simple algebraic equations through the manipulation of ordinary objects such as paper cups and circles cut from coloured construction paper. In this exercise, the process of solving equations is emphasized, and the materials serve the purpose of keeping track of the various symbols and quantities found on each side of an algebraic equation. Certain key relationships, such as $-x + x = 0$ and $-1 + 1 = 0$, are not made more obvious through this exercise.

Experience with these manipulatives simply provides the students with an alternate way of conceptualizing and remembering a process; it does not necessarily impart a greater understanding as to why the inherent relationships within the process are true.

The manipulatives are useful in learning the process because it is easier to remember how to distribute paper chips equally to paper cups than it is to correctly divide both sides of an equation by a constant value. The teacher must decide under what circumstances it is preferable to select manipulatives that facilitate a deeper or more complete understanding of a concept as opposed to manipulatives that simply promote an alternate conceptualization of a process. Both alternatives have value depending upon the students with whom the teacher is working and the objectives to be met.

The fourth implementational consideration is the number of concepts a manipulative supports within an instructional unit. If many concepts within an instructional unit are to be taught using manipulatives, then it is desirable to use similar materials for each topic within the unit. Using similar materials helps students link and relate these topics, relieves the need to constantly introduce and familiarize students with new materials, and provides a sense of continuity and coherence to the unit. However, a manipulative can be effective even if it supports only one concept, especially when used to review a concept or provide a brief extension to a previously developed concept. The manipulative must fit the instructional purposes and processes that the teacher has designed.

The fifth implementational consideration is the degree of familiarity students need with the materials in order to use them properly. How much time must be spent introducing the materials to the students and developing necessary vocabulary? If students are not properly familiarized with the materials, they will spend less time focusing on mathematical principles and more time trying to remember the manipulative procedure. Furthermore, if students are not familiar with the materials, they will not possess the vocabulary or language necessary to ask questions of themselves and others or to summarize their new knowledge. Certain materials require a longer introduction time, and generally, materials that require more introduction are less desirable. In order to justify a longer introduction time, the teacher must consider how well the manipulative embodies the mathematical concept, the number of concepts that may be taught using the materials, the required degree and extent of teacher–student interaction and whether students will work with their own sets of materials.

Well-constructed manipulative materials do not guarantee effective instruction. Even good manipulative materials will only be as effective as the process through which they are employed, and this process requires careful thought and reflection by those who understand the mathematics curriculum as well as children's thinking processes, capabilities, needs and interests.

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