

# A Collection of Connections for Junior High Western Canadian Protocol Mathematics

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*Note: This article first appeared in delta-K volume 35, number 1, pages 29–37 (1998). Minor changes have been made in accordance with current ATA style.*

We have put together “A Collection of Connections” that consists of 12 uses of junior high school mathematics. These activities support the communication and connections strands of the Western Canadian Protocol mathematics curriculum. In using them, teachers may adapt them extensively. They can serve as a basis of one to three mathematics periods. We have also found that teachers need to plan if they intend to incorporate them into their teaching units. They can be used as end-of-unit activities or as focal activities in the development of a unit. We would encourage teachers to use the activities as a means of bringing mathematics to life. These contexts provide an opportunity to enhance a student’s view of mathematics. As one Grade 9 girl said at the conclusion of one activity, “That just proves that mathematics is everywhere.”

The following are samples from the measurement and algebra strand:

## *Measurement*

Why Do Sled Dogs Curl Up?

Why Do Sled Dogs Curl Up? Student Activities

## *Algebra*

Managing an Elk Herd

Managing an Elk Herd Student Activities

## Why Do Sled Dogs Curl Up?

### Intent of the Lesson

An important mathematical idea developed here is that surface area and volume are independent. The sled dog is able to change its surface area although its volume remains constant. Formulas for surface area (and volume), estimating and visualizing are used in this lesson.

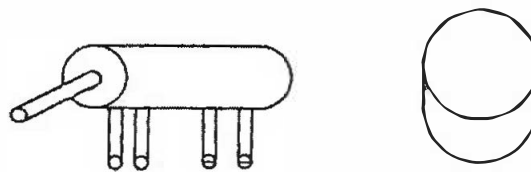
### General Question

Zoologists and veterinarians are interested in studying the behaviour of animals. They found that on cold winter nights sled dogs curled up. In fact, all dogs curl up when they are cold. However, dogs sleeping in the house in front of an open fire usually sleep stretched out.

What is the reason for such behaviour? It is a known fact that heat loss from an animal is related to the amount of surface area of the animal that is exposed to the surroundings. A dog would, therefore, want to decrease its surface area exposed to the cold. We should note that a dog cannot change its volume, so it must change its shape. Can we calculate the surface area of a dog when it is stretched out and when it is curled up?

### Discussion Questions

A discussion of the problem at this point should raise several points. A point of mathematical interest is that a dog stretched out can be thought of as five small cylinders (legs and tail) and one large cylinder (body and head). However, when it is curled up it represents one very large but flat cylinder.



stretched-out dog

curled-up dog

Answering this question is going to require approximations. In the first place, the parts of the dog’s body mentioned above are only approximately cylinders. When a dog curls up, the curled-up shape is only approximately a cylinder.

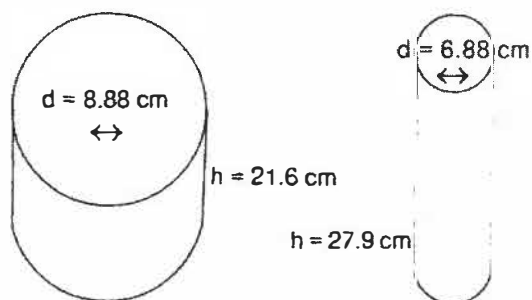
- What shape(s) does a dog represent when it is stretched out? (Long narrow cylinder.)
- What shape does a dog represent when it is curled up? (Flat wide cylinder.)

- What kind of approximations would we need to make when measuring the dog? (Ignore legs and tail.)
- What happens to the legs and tail when it is curled up? (They are tucked inside.)
- Does snow have insulating properties? (Yes.) How does this make a difference to a curled up dog? (It wants to have as large a surface area as possible in contact with the snow.)

## Preliminary Activities

### The Paper Cylinder

Take a sheet of paper (say  $8\frac{1}{2} \times 11$ ). Make a cylinder out of it lengthwise and find its volume. Then make a cylinder out of it widthwise. (See diagram.) What are the surface areas of the two cylinders? What is the volume of each of the two cylinders? The surface areas stay the same (because they are the same sheet of paper) but the volumes differ. Students may do this as a problem to solve. We should notice that when the  $8\frac{1}{2}$ -inch side is made into a cylinder its diameter is  $8\frac{1}{2}$  divided by  $\pi$ . The volume can then be calculated with this diameter and the known height (11 inches). Students can make good use of a calculator in solving this problem.



large flat cylinder

long narrow cylinder

Sample calculations (using metric measurements of 27.9 cm by 21.6 cm):

For the large flat cylinder:

circumference ( $c$ ) = 27.9 cm, height ( $h$ ) = 21.6 cm

$$\text{diameter } (d) = \frac{c}{\pi} = \frac{27.9 \text{ cm}}{\pi} = 8.88 \text{ cm}$$

$$\text{radius } (r) = \frac{d}{2} = \frac{8.88 \text{ cm}}{2} = 4.44 \text{ cm}$$

$$\text{Volume } (V) = \pi r^2 h = \pi (4.44 \text{ cm})^2 (21.6 \text{ cm}) = 1337.7 \text{ cm}^3$$

For the long narrow cylinder:

circumference ( $c$ ) = 21.6 cm, height ( $h$ ) = 27.9 cm

$$\text{diameter } (d) = \frac{c}{\pi} = \frac{21.6 \text{ cm}}{\pi} = 6.88 \text{ cm}$$

$$\text{radius } (r) = \frac{d}{2} = \frac{6.88 \text{ cm}}{2} = 3.44 \text{ cm}$$

$$\text{Volume } (V) = \pi r^2 h = \pi (3.44 \text{ cm})^2 (27.9 \text{ cm}) = 1037.2 \text{ cm}^3$$

We know from this investigation that the volume is independent of the surface area. That is, these two cylinders have the same surface area but their volumes differ. The taller cylinder has less volume. We might also note that, in general, a long skinny object has less volume than a short fat one. In fact, the volume of a cylinder of fixed surface area is greatest when the diameter is approximately equal to the height of the cylinder.

### Discussion Questions

- How can we determine the diameter of these cylinders in two ways? (Measurement and dividing the circumference by  $\pi$ .)
- Why are the surface areas of these two cylinders not exactly equal? (If we add the area of the top and bottom, the shorter cylinder will have slightly greater surface area.)
- How does this demonstration relate to a dog curling up? What remains constant when a dog curls up? (Volume remains constant.)
- Would we expect the [surface area] SA of a curled-up dog to be greater or less than that of a stretched-out dog? (We are going from five relatively long and skinny cylinders to one short and fat one.)

### The Snake

A snake curls up for much the same reason as the dog—to conserve its heat in the cold desert nights. In fact, a snake has to warm up every morning before it can begin to move. Because of this, it is important to the snake not to lose too much heat. Mathematically, the snake is much nicer than the dog. The snake is an obvious cylinder, both stretched out and curled up. Stretched out it is a very long cylinder and curled up it is a large flat cylinder. (For this experiment, a snake can be simulated by a piece of rope with one-inch diameter or a child's play snake or [use] a real snake if you happen to have one.)

In addition to finding the surface areas we should find the volumes of the stretched out shape and the curled-up one. These volumes should be approximately the same. This calculation is an important check on our approximate measurements. Note: because the snake is curled up we cannot find its diameter by dividing its length by  $\pi$ .

### Teaching Suggestion

Although the lesson is centred on the behaviour of sled dogs, the preliminary activities lend themselves to much better calculations. In this lesson, the preliminary activities are more important than "answering the general question." This latter section, because of the difficulty in dealing with a real dog, is more a matter of estimation and discussion. In some ways

the teacher can treat the sled dog question as the motivational device while the preliminary activities become the mathematical aspects of the lesson. With respect to the approximations in the next section of the lesson, we have found that students are eager to participate in these approximations. They do not treat the lesson as less significant just because it contains a lot of approximations.

### Answering the General Question

Having a real husky dog and taking actual measurements would be ideal. A student in the class might have a suitable pet and could be encouraged to provide measurements for the class. Approximation is fundamental to this activity. We need realistic measurements to begin with but approximation is still necessary. One approximation we might make is simply to measure the body and head of the dog, ignoring the legs and the tail. The outside of the fur should be the surface area rather than the actual skin surface because it is the outside that is losing the heat. A rolled-up blanket, a cylindrical pillow, a flexible toy dog or a slinky toy could be used in making approximate measurements.

### Discussion Questions

- Why is the diameter of the dog's body when it is stretched out equal to the height of the flat cylinder of the curled-up dog?
- By ignoring the legs and tail in our calculations, on which side of the comparison of surface areas are we erring? (The legs and tail contribute a large surface area to the stretched out dog and essentially no surface area to the curled-up dog.)
- What happens to the legs and tail of the curled-up dog? Do they lose more heat in the stretched out position or in a curled-up position?
- What is the effect of the snow insulating the dog in both positions? In other words, which position makes best use of the insulating properties of snow?
- Which shape (long or short) of dog benefits most from the curling up to conserve heat?
- Does your knowledge of surface area increase your understanding of the behaviour of dogs?

We might consider the curled-up dog to be a sphere. A discussion among students could settle this issue and calculations for the sphere could be made for comparison with a cylinder. That is, what happens to surface area if we consider it a sphere?

$$\begin{aligned} \text{Volume of cylinder} &= \pi r^2 h \\ \text{Surface area of cylinder} &= 2\pi r h + 2\pi r^2 \\ \text{Volume of sphere} &= (4/3)\pi r^3 \\ \text{Surface area of sphere} &= 4\pi r^2 \end{aligned}$$

### Materials

Coloured sheets of  $8\frac{1}{2} \times 11$  paper, transparent tape, toy snake or 1-inch rope (4 feet long), long cushion (to simulate a dog), stuffed toys and a slinky toy. Although we have not tried it, a piece of dryer venting hose could be used to simulate a dog.

### Modifications

#### *Other Animals*

Once the situation of the dog has been discussed, students can bring stuffed toys from home that could be measured in the stretched-out position and curled-up position. As mentioned earlier, a Slinky toy has possibilities.

When a fat teddy bear is curled up, its shape becomes a sphere rather than a cylinder. Some stuffed toys have very large legs that cannot be ignored in the calculation of the surface area before the toy is curled up. The legs can be treated as cylinders with one end.

The idea of the proportion of the surface in contact with the snow being much larger in the curled-up position than the stretched-out position is worth discussing. In general, the snow is a better insulator than the air, especially because of the wind blowing (wind chill) on the exposed side. In fact, the dog loses only about one-tenth as much heat through the snow as through the air. This makes the curling-up behaviour even more understandable. If the dog is considered to be a flat cylinder, using your measurements, what per cent of the surface is next to the snow? In curling up, not only does a dog reduce his or her surface area but also a good percentage of this smaller surface area is better insulated.

A survival suggestion for humans in cold water is to curl up into the fetal position. Students can approximate the surface area of a classmate in stretched-out and curled-up positions.

### Why Do Sled Dogs Curl Up? Student Activities

#### *General Question*

Zoologists and veterinarians are interested in studying the behaviour of animals. They found that on cold winter nights sled dogs curled up. In fact, all dogs curl up when they are cold. However, dogs sleeping in the house in front of an open fire usually sleep stretched out.

What is the reason for such behaviour? It is a known fact that heat loss from an animal is related to the amount of surface area of the animal that is exposed to the surroundings. A dog would, therefore,

want to decrease its surface area exposed to the cold. We should note that a dog cannot change its volume, so it must change its shape. How much more is the surface area of a dog when it is stretched out than when it is curled up? [See illustration on page 71.]

## Activities

1. (a) With an  $8\frac{1}{2} \times 11$  sheet of paper, we can make a cylinder in two ways. Once the cylinder is made, we can find the diameter of the cylinder in two ways. What are they?
  - (b) How do you convert inches to centimetres? What is  $8\frac{1}{2}$  inches in centimetres?
2. (a) Why are the curved surfaces of both cylinders that you made with the paper (question 1) equal?
  - (b) To what other surface area are they both equal?
  - (c) Why are the *total* surface areas of both cylinders not the same? By how much are they different?
3. (a) A tall cylinder has less volume than a short cylinder of the same surface area. However, if a tall cylinder and a short cylinder have the same volume, which will have the greater surface area?
  - (b) How does this conclusion relate to the stretched-out dog when he curls up?
4. (a) Using your measurements for a “snake,” show the calculations for the snake stretched out and the snake curled up.
  - (b) How much surface area does the snake lose by curling up?
  - (c) What percentage is this loss of the stretched-out surface area?
  - (d) What per cent surface area does the snake gain when he goes from being curled up to stretching out?
  - (e) Why are these percentages not the same? If you wanted to impress someone with the importance of the reduction of surface area, which of these percentages would you use?
5. (a) Take approximate measurements from a real dog, a toy animal or a piece of tubing and determine the surface area of the “animal” in both positions.
  - (b) Using the same measurements, make the calculation to determine the volume of the “animal” in both positions. Are the volumes nearly equal? Why?
  - (c) Again with approximate measurement, determine the surface areas of the legs and tail. Go back to question 5(a) and add these surface areas to the surface of the stretched-out animal. How much does this increase the differences in surface areas?
    - (d) Why do we not add the surface area of the legs and tail to the curled-up surface area? What assumptions are we making?
6. (a) What shape do you think polar bears become when they curl up?
  - (b) Why do you think the polar bear’s fur is much thicker on the back and sides than on the belly?

## Managing an Elk Herd

### Intent of the Lesson

The linear equation is used to simulate the growth of a population, a common problem in wildlife management. The mathematics includes linear equations, percentages, probability and graphing. It is possible to use a spreadsheet to illustrate the solution.

### General Question

All wildlife areas, even in a country as large as Canada, are limited in size. Because of this limitation, the wildlife that the area will support is limited. This is especially true in an area such as Jasper National Park. Even in larger areas where hunting is allowed, wildlife managers need information about herd sizes. Each year, decisions are made about how many animals may be taken by hunters. In making this decision, managers need to be able to predict the growth of herds.

Consider a typical problem for a wildlife manager—a herd of animals is 10,000. How large will that herd be in 5 years or 10 years? The answer to the problem can be found with the help of mathematics. First, information about survival (death) rates of the animals from year to year must be known, as well as birth rates. Very simply, if the death rate is 20 per cent and the birth rate is 40 per cent, the herd size will grow. Under such conditions, the herd will soon become too large for the area. Animals will need to be moved to new areas and/or hunted. The question is can we predict the size of the herd from year to year?

### Teaching Suggestions

The teacher should be prepared to fully discuss the context of this problem. This may include discussion of biological processes, ethical considerations of managing any wildlife population or hunting.

### Discussion Questions

- What do you think *survival rate* means?
- Will the survival rate be different for males and females? For fawns? (Will be higher for males than females and higher for adults than for fawns.)
- If the herd is growing too fast and a harvest is needed, what options do wildlife managers have?

(Allowing hunting, or capturing and moving animals to other areas.)

- What kind of herd growth is desirable in most wildlife areas? (No population growth.)

## Preliminary Activity

### Exam Marks

Interestingly, the mathematics needed for the elk herd problem is the same mathematics that students need to figure out final marks:

If the midterm mark (M) counts for 25 per cent of the course mark (C) and the final exam (F) counts for 75 per cent, what will the course mark be if your midterm mark was 72 per cent and your final exam mark was 92 per cent?

To solve this we take 25 per cent of 72, which is 18, and 75 per cent of 92, which is 69, and add them together for a final score of 87. We should observe that the 87 is much closer to 92 than to 72. Why is that? The reason is that the final exam mark is much more important than the midterm mark and is worth more. Now, what is the mathematical equation to determine the course mark?

$$0.25M + 0.75F = C$$

If we think about these as fractions,  $\frac{1}{4}$  of the midterm and  $\frac{3}{4}$  of the final exam are added together.

In another class, the course mark (C) is determined by taking 15 per cent of assignments (A), 25 per cent of the midterm (M) and 60 per cent of the final exam (F). What is the equation to determine the mark in this class?

$$0.15A + 0.25M + 0.60F = C$$

In this class the final exam is four times more important than the mark in assignments.

Ask students to do the calculation for given A, M and F.

## Answering the General Question

### The Management of an Elk Herd

A wildlife officer has the following information on an elk herd in her region:

Survival rates for males	95%
females	90%
male fawns	50%
female fawns	45%
Birth rate for	
male fawns	48% of adult females
female fawns	42% of adult females

### Discussion Questions

- How do we get these numbers? (Biologists and wildlife managers.)

- Why are survival rates for male and female fawns different? (Male elk are bigger.)
- Why is birthrate only dependent on adult females? (In a herd of 100 males and 10 females, what is the maximum number of fawns that can be born?)
- What does the male fawn survival rate of 50 per cent mean? (One half live until next year.)

Using these rates, we can predict from year to year how large a herd will be. Very simply, a herd in year one with 100 males and 100 females will have 95 males and 95 females surviving in year two. The number of fawns will be 48 male fawns and 42 female fawns. The total herd will be  $95 + 95 + 48 + 42 = 280$ . This number is based on a birth rate determined from the number of females in the previous year.

The answer will be different if only the surviving females give birth, that is, if the birthrate is based on the number of females surviving from the year before. Initially, let's consider that the birth rate is based on the number in the herd in year one. Now we should write the equation, just as we did for the course mark:

$$0.95M \text{ (males)} + 0.90F \text{ (females)} + 0.48F + 0.42F \\ = \text{Herd in year two}$$

(Using "year one" and "year two" terminology leads to less confusion in the lesson.)

Now we introduce a harvest of the elk herd. The harvest may be gathering them up and moving them or it may mean hunting them. If, in the case above, we wanted to keep the herd at 200, we would need to harvest about 80 animals in year two. (In this equation we are assuming that the count is made after the harvest. In other words, the count is the last thing that happens in the year.) The equation would look like this:

$$0.95M + 0.90F + 0.48F + 0.42F - H \text{ (harvest)} = \\ \text{Herd in year two}$$

Now we can proceed to figure out the herd size in year three. Notice that we have four types of elk:

M(ale) adults	-	95% survival rate
F(emale) adults	-	90% survival rate
m(ale) fawns	-	50% survival rate
f(emale) fawns	-	45% survival rate

To do our calculations for year two, we really need to know not the total size of the herd, but how many of each type of animal we have. Also, because females are more important to the herd survival, we will want a system that harvests more males than females. Therefore, we will have a male harvest (Hm) and a female harvest (Hf). Of course, only adults are harvested. Why is this? We need several equations, one for each type of animal. What, for example, is the equation to determine the number of males in any year? Since the adult male population is made up of

surviving adults plus surviving fawns less the harvest, the equation for males in any year is

$$0.95M + 0.50m - H_m = \text{Number of males}$$

Because we have so many elements to attend to, a table is convenient.

### Using a Table

The use of a table will simplify our calculations.

Column	A	B	C	D	E	F	G
Year	Males (adult)	Females (adult)	males (fawns)	females (fawns)	Total	Hm =	Hf =
1							
2							
3							
4							
5							

We need equations to go from year to year:

Male:  $0.95M + 0.5m - H_m$  where M and m are males adults: from previous year

Female:  $0.90F + 0.45f - H_f$  where F and f are adults: females from previous year

males:  $0.48F$  where F is female adults from previous year

females:  $0.42F$

Harvests (Hm and Hf) are constants from year to year which are subtracted. We should note the assumptions that are being made about the how the herd functions.

### Discussion Questions

- Why does 0.5m become part of the adult males? (We are assuming that fawns become adults after one year.)
- What does 0.48F mean? (For every 100 females that were alive the year before, 48 male calves were born.)
- If we assume that only surviving females give birth, what does the 0.48F become? (It becomes 0.48 (0.90F) because 90 per cent of the females survive and 48 per cent of those have male fawns.)
- What does the 0.48F become if first we count our F, then a harvest occurs (Hf), then 90 per cent survive and then 48 per cent of these have male calves? (The 0.48F becomes  $(F - H_f) \times 0.90 \times 0.45$ .)

The point of these questions is that our model is a simplified version of what probably happens in an elk herd. Once we understand how the simplified version works, we can make it more complicated. A student assignment can be to determine how equations differ under different assumptions about herd growth.

Although we can begin the herd with any numbers, let us start with 400 male and 400 female fawns, 1,000 each of males and females and a harvest of 300 males and 300 females. Note: our equations determine the number of adult males, for example, in any year by calculating survivors (both adults and fawns) subtracting the harvest. So, in the example below the 1,000 males and females is a count after the harvest. Realistically, the count could have been taken around Christmas, after the harvest.

1	A	B	C	D	E	F	G	H	J	K
2	Year	Adult males	Adult females	Newborn males	Newborn females	Total herd	Harvest males	Harvest females		
3	1	1,000	1,000	400	400	2,800	300	300	Male survival rate	95%
4	2	850	780	480	420	2,530	300	300	Female survival rate	90%
5	3	748	591	374	328	2,041	300	300	Male birthrate	48%
6	4	597	379	284	248	1,509	300	300	Female birthrate	42%
7	5	409	153	182	159	904	300	300	Newborn male survival rate	50%
8	6	180	-91	73	64	227	300	300	Newborn female survival rate	45%
9									Harvest males	300
10									Harvest females	300

Year 2 Males =  $0.95(1000) + 0.50(400) - 300 = 850$   
 Year 2 Females =  $0.90(1000) + 0.45(400) - 300 = 780$   
 Year 2 male fawns =  $0.48(1000) = 480$   
 Year 2 female fawns =  $0.42(1000) = 420$



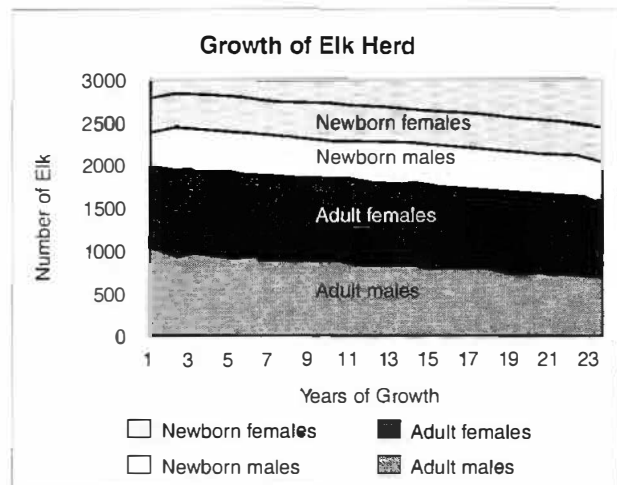
1	A	B	C	D	E	F	G	H	J	K
2	Year	Adult males	Adult females	Newborn males	Newborn females	Total herd	Harvest males	Harvest females		
3	1	1,000	1,000	400	400	2,800	200	90	Male survival rate	95%
4	2	950	990	480	420	2,840	200	90	Female survival rate	90%
5	3	943	990	475	416	2,824	200	90	Male birthrate	48%
6	4	933	988	475	416	2,812	200	90	Female birthrate	42%
7	5	924	986	474	415	2,800	200	90	Newborn male survival rate	50%
8	6	915	985	473	414	2,787	200	90	Newborn female survival rate	45%
9	7	906	983	473	413	2,774	200	90	Harvest males	200
10	8	897	980	472	413	2,761	200	90	Harvest females	90
11	9	888	978	471	412	2,748	200	90		
12	10	879	975	469	411	2,734	200	90		
13	11	869	973	468	410	2,720	200	90		
14	12	860	970	467	409	2,705	200	90		
15	13	851	967	466	407	2,690	200	90		
16	14	841	963	464	406	2,674	200	90		
17	15	831	960	462	405	2,658	200	90		
18	16	820	956	461	403	2,640	200	90		
19	17	810	952	459	401	2,622	200	90		
20	18	799	947	457	400	2,602	200	90		
21	19	787	942	455	398	2,582	200	90		
22	20	775	937	452	396	2,560	200	90		
23	21	762	931	450	394	2,537	200	90		
24	22	749	925	447	391	2,513	200	90		

death rate is 20 per cent and the birth rate is 40 per cent, the herd size will grow. Under such conditions, the herd will soon become too large for the area. Animals will need to be moved to new areas and/or hunted. How can we predict the size of the herd from year to year?

### Activities

- (a) A teacher wants to determine your final grade by giving equal weight to your assignments and midterm test and double that to your final. What would be the equation that she could use?
- (b) What would your mark be if you got 80 per cent on both the assignments and the midterm and 60 per cent on your final?
- (a) When we use the equation  $0.95F + 0.45f - Hf$  to determine the number of adult females in a certain year, what sequence of events are we assuming? (Remember that we have four events: the count, the survival, the birth and the harvest.)

- (b) What would this equation be if adult female survival was only 80 per cent and the survival from fawn to adult for females was 30 per cent?





3. (a) If the survival rate for the adult elk is applied after the yearly harvest, what does the equation for adult males become? In this scenario, the young are born, then the count is made giving the M and F values for the year, then the adults are harvested and then 95 per cent of them survive the winter.
  - (b) In the scenario in question 3(a) what would be the equation for the number of females able to have calves during the next summer?
4. (a) In our mathematical system, the number of fawns depends only on the number of females. Suppose that a biologist tells you that a herd cannot grow normally unless at least 20 of the adults are male. How would you change the mathematical system to take this into consideration?
  - (b) Another biologist tells you that the rule is 20 per cent of the adults are male. How would you account for that?
5. (a) Make up what you think is the most likely sequence of events in the growth of a herd. Assume the year goes from January to January; in other words, the *count* is made in January. Remember four things happen: births, survival, harvest and count.
  - (b) Make the equation for adult males for your sequence from question 5(a).
  - (c) Make the equation for female birthrates for your sequence from question 5(a).
6. If you know the survival rates and birth rate of a herd, how could you estimate what the harvest should be to keep the herd size stable?
7. How would the equations have to be changed for a population of wolves who pair with a mate for life? Which equation(s) would have to change?
8. Find a spreadsheet and write up the first four lines of it for the elk herd.

#### **Authors' Note**

Those readers interested in the entire volume of "A Collection of Connections" may contact Sol E Sigurdson, Faculty of Education, Department of Secondary Education, University of Alberta, Edmonton T6G 2G5; phone 492-0753.

*The authors would like to acknowledge the financial support for this project provided by the Alberta Advisory Committee for Educational Studies (AACES) and the Central Research Fund of the University of Alberta.*