Problem Solving

Srinivasa Swaminathan

The teaching of undergraduate and graduate courses in mathematics involves routine exposition of standard topics illustrated by solved problems from the texts. Weekly assignments are generally based on exercises from textbooks. Generally, mathematics is studied not for its own sake, but because the ultimate object is merely to pass an examination or to acquire the minimum knowledge necessary for dealing with some other subject of study. In such a situation, how much of problem-solving ability is acquired by students is doubtful; just propose a problem outside the normal curriculum—one would find that most students are unable to solve it. However, there are gifted students in almost every class. Problem-solving sessions are held to train such students so that they can compete in the annual Putnam and similar exams; they learn to apply previously acquired knowledge to new and unfamiliar situations.

Problems can be classified under different headings: mechanical or drill problems; those that require understanding of the concepts; those that require problem-solving skills or original thinking; those that require research or library work; and, finally, those that are group projects, requiring group participation.

The importance of problem solving in the learning process and also in the growth and development of mathematics has been recognized and emphasized by many prominent authors (for example, George Pólya's [1945] *How to Solve It*). New branches of mathematics have arisen from the search for solutions of challenging problems. Noteworthy examples are the successful attack on the brachistochrone problem by the Bernouilli brothers and the role played by their solution in the evolution of the calculus of variations. Mathematical theory of probability arose from the investigations by Pacioli, Cardan, Tartaglia, Pascal and Fermat. Topology and graph theory had their origin in Euler's analysis of a problem about crossing bridges. The fact that in some fields (algebraic) the resolution into prime factors is not unique as it is in common arithmetic led Dedekind to restore this highly desirable uniqueness by the invention of *ideals,* an important concept in algebraic geometry.

Many mathematical journals contain problem sections inviting readers to submit solutions. From these solutions, the editors select what they consider to be the "best" solution, which they publish along with other interesting solutions, if any. Solutions of difficult and challenging problems may lead to interesting further investigations of the devices employed. Selecting proposals poses a more challenging task to the editors than the selection of solutions; the editors seek to have a diversity of high-quality proposals in geometry, analysis, number theory, etc, rising above the level of unimaginative textbook exercises. Elegant proposals attract a wide range of would-be solvers. The criteria for elegance can be summarized in the ABCDs of elegance as follows: *A* for *accuracy, B* for *brevity, C* for *clarity* and *D* for *display* of insight, ingenuity, originality and generalization, if possible.

Periodically, collections of proposed and solved problems from well-known journals are published. Thus, *The Otto Dunkel Memorial Problem Book* (Eves and Starke 1957) was published by the Mathematical Association of America on the occasion of the 50th anniversary of the *American Mathematical Monthly,* which contains a popular section on problems. The most recent such collection is *A Mathematical Orchard* (Krusemeyer, Gilbert and Larson 2012), from the Mathematical Association of America, which contains 208 challenging, original problems with carefully worked, detailed solutions. One can spend hours browsing through this book, thinking about and trying to solve problems before looking at the solutions. As I was thinking about problem 62 of the book—which is to find the fifth digit (the ten thousands digit) from the end of the number 5 raised to the power of 5, which is raised to the power of 5, . . . up to five times!—the idea for writing this editorial occurred to me! [Answer: 0.]

References

Eves, H, and E P Starke. 1957. *The Otto Dunkel Memorial Problem Book.* Supplement to *American Mathematical Monthly* 64, no 7. Menasha, Wisc: Mathematical Association of America.

Krusemeyer, M I, G T Gilbert and L C Larson. 2012. *A Mathematical Orchard: Problems and Solutions.* Washington, DC: Mathematical Association of America.

Pólya, G. 1945. *How to Solve It.* Princeton, NJ: Princeton University Press.

Reprinted with permission from CMS Notes *45, no 2 (March/April 2013), p 2, http://cms.math.ca/notes/v45/n2/Notesv45n2.pdf. Minor changes have been made to fit ATA style.*

Srinivasa Swaminathan is a professor emeritus in the Department of Mathematics and Statistics at Dalhousie University, in Halifax.