Understanding Studying and Studying Understanding

Jennifer Hyndman

Every mathematician recognizes that adding fractions of polynomials is the same as adding integer fractions—one first factors each fraction as much as possible, finds a least common denominator, rewrites the fractions with that denominator, and then adds the numerators. What is actually happening in our understanding of this? We are able to see the pattern for adding and are able to move this pattern from a simple situation to a more complex situation.

My first-year calculus students make mistakes in adding fractions of polynomials. When I show them a pattern of adding fractions of integers that mimics their incorrect polynomial addition rule, their body language response is one of total understanding of the integer situation.

What is it that they actually understand about addition of fractions of integers? They can find a prime factorization of an integer. They can find common factors of two integers. They can find a least common multiple of two integers. They can multiply integers. They can add integers. However, I doubt that the students could articulate this list of actions as part of their understanding of how to add fractions. What I am not sure of is whether this negatively influences their ability to add fractions of integers or whether this affects their ability to transfer their understanding to adding fractions of polynomials.

I continually observe how people collect knowledge and compare it to how my students learn mathematics. Gary, a member of my family, has over a thousand CDs and listens to music 18 hours a day, yet he cannot always identify the time signature of a song. He will happily ask me what the time signature is. With a little thought, I can tell him. He knows the history of every artist on every CD while I might not be able to name the band. Which one of us has a better understanding of music? Which student has a better understanding of mathematics? The student who can describe the process of adding fractions or the student who can do the process? What about the student who can both add fractions and describe the process? Most of us would agree that this latter student has the best understanding of the three students. What about the music listener who can identify the artist and the musical structure? Does that person have a better understanding of music than Gary or I? Or just a different understanding? Does this person enjoy music more or less because they instinctively hear (and cannot ignore) the underlying structure of every piece of music they hear?

As I believe (at least I think I do) that more knowledge increases understanding and enjoyment, I have been giving one-on-one sessions and running study skills workshops on how to learn mathematics for several years. The workshops were developed and initially run with my colleague, Vivian Fayowski, the coordinator of the University of Northern British Columbia's (UNBC) Academic Success Centre. For one year, they were also part of a UNBC Early Alert research project with Dan Ryan, Kerry Reimer and Peter MacMillan. The focus of these workshops has been, in essence, to explore how to organize mathematical information and how to internalize and articulate mathematics.

The most common response to the question of how a student studies is that they "do" problems. Initially, they are unable to articulate what they mean by "doing" a problem. Several minutes of prompting eventually yields words like *write, read, copy, draw, type, speak, hear, listen* and *rewrite.* Is their inability to describe their own actions relevant to their difficulties in learning mathematics? I think so. However, I also believe one needs to be able to articulate what one is doing and then internalize it so it is nonverbal.

I am part of my own observations of learning. As a student in dance classes, I am continually being challenged by learning new styles of dance and new choreography. Not long ago I suffered the misfortune of not being able to figure out exactly where I was supposed to be while on stage in a group number. This was unusual for me and, to prevent it from happening again, I thought long and hard about what had happened. I certainly knew the choreography thoroughly as I had been talking the group through the steps to help us practise. This talking turned out to be the problem. I had "learned" the choreography as if it included speaking. When I walked on stage and had to smile instead of talking, I was literally lost.

On the verbal, visual and kinesthetic scales of learning I am highly kinesthetic, very visual and almost nonverbal. Speaking the steps had interfered with my own ability to reproduce the steps without speaking. What are our students actually learning when they study mathematics or when they study any subject? What do we actually test for in a midterm or an exam? Are our students self-aware enough to realize which study methods work for them and which don't? Do they even know more than one study method?

While thinking about this article, I asked my family how they studied in university and how they learned to study. The actions they described all fit under my umbrella of things to do to study. What was more interesting were their comments on how they learned to study. David C's first reaction was that he had no memory of ever receiving specific instruction, and then he said he might have had some in high school English. David H's comment was that it was like learning to be a parent; you just do it. David H's son thinks his college course on time management is a waste of time as he is learning nothing new. When I work with students who are failing courses, they frequently and proudly admit they spent very little time on the courses and think that they will fix their grades by "spending more time studying." However, when asked what they will do in this additional time, they say, "Do problems," which brings us back to the earlier-mentioned inability of students to describe what this means.

As instructors, what should we or can we do to assist students to be more self-aware in their studying (without giving time-management courses that are a waste of time)? The lucky students, like the Davids and me, figured out effective study techniques that fit in the time we had available. Other students do less well than they are capable of. I think we should be teaching study skills as part of the ongoing education of our students.

Here are some of the things I would do if I were to teach the perfect course as part of the perfect university degree. The course objectives provided to the students would have components of both mathematical content and study skills. The lectures would have study techniques embedded in the content development. The content to be examined would include techniques for studying the material. Here is an illustration of the second idea: discuss the definition of continuity and then discuss techniques for memorizing a definition, such as writing it out several times, reading it out loud, reading it silently, and reciting it from memory with your eyes closed versus with your eyes open. An exam question could be as simple as "List three techniques for memorizing a definition" or as self-reflective as "What study technique works best for you when you try to memorize the definition of *continuity?*"

Of course, at least in my opinion, learning is an activity that spirals. One initially learns a very rudimentary approximation of a concept and then rethinks and refines the approximation until, with focused attention, one understands the same thing as others do. Where does studying study skills fit in this spiral? It cannot be too early, but it must be early enough to be useful. When I work with students, I often come to the conclusion that they have to be ready to hear what I have to say about study skills (or any subject) before they can actually take in the knowledge.

Returning to learning how to add fractions of polynomials, the spiral of knowledge for this starts with the spiral for adding fractions of integers, layers on the language of polynomials, and then repeats the original spiral another time. How could we help our students learn to add fractions? Test questions like "Explain the steps in adding the following fractions of integers (polynomials)" would be preceded by homework questions like "Build mind maps for integers and for polynomials that illustrate the concepts of adding two fractions. Discuss the similarities and differences." The intrinsic patterns that mathematicians see can be brought into the light for our students.

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Jennifer Hyndman is professor and chair of the Department of Mathematics and Statistics at the University of Northern British Columbia, in Prince George.