Content Knowledge for Teaching Mathematics: How Much Is Needed and Are Saskatchewan Teacher Candidates Getting Enough?

Nico Higgs and Egan J Chernoff

What makes a good math teacher? A dichotomy often emerges when this question is asked—one that pits two hypothetical math teachers against each other. Is the teacher who is an expert in math but not very skilled in pedagogy better than the teacher who knows very little about math but is highly skilled in pedagogy? Various arguments are thrown around, usually with both sides eventually conceding that an effective math teacher needs at least a decent understanding of both math and pedagogy.

Yet the debate continues. In this article, we consider the research on the topic of how much math knowledge teachers and teacher candidates need in order to effectively teach math. We begin with a review of the research and theories on the importance of mathematical knowledge for teacher candidates. Then, we analyze how they fit with the current education that teacher candidates are receiving (with a special focus on the University of Saskatchewan and local school divisions). We conclude with a discussion of the implications for aspiring math teachers.

A Brief Look at the Literature

It may seem that if teachers have greater mathematical knowledge, they will be more effective math teachers and their students will be more successful in mathematics; however, research has shown that this is not the case. Begle (1979) measured teachers' mathematical knowledge (determined by the number of postsecondary courses taken) against student achievement, and found no positive correlation. The Third International Mathematics and Science Study (TIMSS) reported that American students in Grade 4 were adequate and those in Grade 12 were poor at mathematics (US Department of Education 1997, 1998). To see if teacher training had an effect on these results, the US Department of Education (1996, 1997, 1998) conducted a study that found that the training and knowledge of teachers in the United States were comparable to those of their counterparts in other countries, which seems to suggest that teachers' mathematical knowledge has no effect on student achievement (Howe 1999).

In institutions of higher learning, teachers learn increasingly abstract concepts and are able to pass math courses without learning how to increase understanding of more elementary material (Fi 2003). Researchers have investigated teachers' mathematical knowledge and have found evidence that mathematical knowledge does, in fact, play a vital role in students' learning of math (Ball 1990, 1991; Ball, Hill and Bass 2005; Ball, Lubienski and Mewborn 2001; Ball, Thames and Phelps 2008; Conference Board of the Mathematical Sciences 2001; Hill, Rowan and Ball 2005; Ma 1999). However, teachers require a different type of understanding—an understanding that they do not necessarily obtain in postsecondary math courses.

Much of Ball's research, and that of Hill and Ma, is built upon Shulman's (1986, 9) definition of *pedagogical content knowledge:*

> The most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations—in a word, the most useful ways of representing and formulating the subject that make it comprehensible to others. . . . Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons.

Shulman defines, in essence, a discipline of study for teachers different from their subject area and general

pedagogy—one that sees teaching as professional work with its own unique professional knowledge base for each subject area, math especially (Ball, Thames and Phelps 2008). It is not only knowledge of content or knowledge of pedagogy but, rather, an amalgam of the two that is central to the knowledge needed for teaching.

From this perspective, recent experience with advanced math courses should make teacher candidates experts, but it does not (Fi 2003). Research into this new understanding of the mathematical knowledge that teachers need shows that mathematical knowledge correlates with student success, which leads to the conclusion that teacher candidates need a profound understanding of fundamental mathematics in order to teach effectively (Ma 1999).

Further research on the mathematical knowledge teachers need has led to many theories, which have been summarized elsewhere (Dossey 1992; Fennema and Franke 1992; Hiebert and Carpenter 1992; Hiebert et al 1997). These theories help mathematics educators better understand teacher knowledge through the connections between beliefs, the affective domain, pedagogical content knowledge, subjectivity of knowing (von Glasersfeld 1996) and teachers' subject matter (Fi 2003)—as opposed to teachers' subject matter knowledge alone. Reviews of such literature show that teacher candidates ought to be conversant with the subject matter they intend to teach (Ball 1988, 1991; Conference Board of the Mathematical Sciences 2001; Fi 2003; Ma 1999). A more recent study, by Ball, Hill and Bass (2005), shows that student achievement with a teacher who ranks in the top quartile of teacher knowledge is the equivalent of two to three weeks of instruction ahead of students who have a teacher with average teacher knowledge. Another interesting finding is that the size of the effect of teachers' mathematical knowledge for teaching is comparable to the size of the effect of socioeconomic status on student gains. This is significant because it demonstrates that teachers' content knowledge can help create equity for all students and may help combat the frequently reported widening of the achievement gap.

Based on the research and theories discussed, content knowledge and pedagogical content knowledge are significant and necessary components for teacher candidates to become effective math teachers (Kilpatrick, Swafford and Findell 2001). However, mathematical knowledge, both content and pedagogical, among teacher candidates is lacking and needs to be addressed by teacher education programs (Cooney 1999).

The Mathematical Knowledge of Teacher Candidates

A study of teacher candidates' content knowledge, pedagogical content knowledge and envisioned pedagogy of trigonometry found teacher candidates' mathematical knowledge lacking (Fi 2003). Fi's study investigated similar studies that also found teacher candidates' mathematical knowledge inadequate (Ball, Lubienski and Mewborn 2001; Howald 1998; Ma 1999). The study revealed that teacher candidates had not seriously revisited since high school the basic notions and conceptual understandings of the math they were to teach, and that they themselves had never adequately learned the concepts. The teacher candidates also claimed to have not been exposed to foundational ideas in their postsecondary mathematics courses, which is perhaps why teacher education should address these particular issues (Cooney 1999). The research reveals the need to reacquaint teacher candidates with the fundamental mathematical ideas they will be teaching, by re-examining K–12 mathematics content from an advanced perspective (Conference Board of the Mathematical Sciences 2001; Fi 2003; Usiskin et al 2003).

Mathematical Knowledge in the Flatlands

Recognizing the research presented above, we will now critique the education that prospective math teachers are receiving, specific to the University of Saskatchewan and the Saskatoon Public School Division.

As but one example, the College of Education at the University of Saskatchewan is a professional development school that allows teacher candidates to work with nearby school divisions to get classroom experience. This, alongside educational foundations classes, fulfills many areas in which teacher candidates need education, such as curriculum studies, anti-racism and anti-oppression teaching, experiential learning, and differentiated learning. However, there seems to be a serious lack of content knowledge and, more specifically, pedagogical content knowledge.

Secondary teacher candidates take only one methods course devoted to each of their teaching areas (whereas, for example, all primary teachers must take a math methodology course). This methods course, which has a lot to cover, is not able to devote all of its allotted time to pedagogical content knowledge. This equates to under 40 hours of education on content knowledge for math (and other subjects). Yes, teacher candidates do have prerequisite postsecondary

courses in their subject areas prior to gaining entrance to the College of Education. However, as discussed earlier, research demonstrates that these prerequisites do not necessarily translate into the content knowledge teachers require for effective teaching in the future. Thus, it would appear that teacher candidates potentially come up short in their content knowledge of the subjects they are going to teach. Further compounding the issue, once teacher candidates graduate from the college and apply for a teaching certificate, they are considered to be qualified to teach any subject at any grade level. Thus, a teacher candidate could obtain none of the necessary mathematical knowledge during his or her time in the College of Education, but could nevertheless be hired to teach math. Lacking the content knowledge necessary to be an effective math teacher, how well will this teacher teach mathematics?

The local school divisions, Saskatoon Public Schools in particular, seek teacher candidates who have qualifications for "accreditation." In other words, they want teacher candidates who have extra postsecondary courses in mathematics, which are essentially meaningless for teachers, as we have discussed. Teacher interviews are done by subject area and are based on the subject area in which prospective teachers have the most postsecondary courses, not the most pedagogical content knowledge training. The entire process in Saskatoon of teacher education, teacher qualification and even hiring practices appears misguided given the pedagogical content knowledge needs of teachers. Teacher candidates are not adequately supported in learning the required content knowledge, and teachers with inadequate mathematical knowledge are consistently hired to teach math. Even the hiring process that does try to target mathematical knowledge does so inadequately by focusing on postsecondary courses in math that do not provide the knowledge teachers need in order to teach math effectively. Based on (1) the research that shows the importance of proper mathematical content knowledge for teachers and (2) the lack of mathematical content knowledge teachers have in other parts of Canada and in the United States, we contend that teacher candidates in Saskatoon continue to be inadequately served.

Mathematical Knowledge and Student Equity

Those math teachers who do have proper pedagogical content knowledge and mathematical understanding are misplaced in current education systems. For example, Hill (2007) showed that qualified math teachers are unevenly distributed across the United States, resulting in affluent students consistently getting the best math teachers. Similar issues arise in the way qualified math teachers are viewed in both the United States and Canada. Currently, the common practice is to have the most qualified or most senior teachers teaching the higher-level content, such as calculus, while the less experienced and weaker teachers end up in front of students with a history of low achievement in mathematics (Brahier 2013). This discrepancy often results in a downward spiral in which low success, less experienced teachers and fewer supports prevent students from advancing in mathematics. Even with the assumption that all students have the right to equal access to all areas of the curriculum, as well as to high-quality instruction, these practices do not support equity for students (Brahier 2013). By contrast, increased mathematical content knowledge of teachers can facilitate equity for students (Ball, Hill and Bass 2005). In the end, it is the students who suffer the most from these misguided knowledge requirements for teacher candidates.

Conclusion

Mathematical knowledge, especially pedagogical content knowledge, is of vital importance to effective math teaching. However, teacher candidates tend to be inadequately educated in math while also being pressured into taking the wrong math courses in order to get hired. Accounting for how students understand a content domain is a key feature of the work of teaching that content (Ball, Thames and Phelps 2008). The research and theories examined here show the importance of mathematical knowledge in the teaching of mathematics. The distinction between pedagogical content knowledge and subject knowledge itself highlights the importance of teachers mastering the content they are to teach beyond simply passing a course. Perhaps aspiring math teachers should be encouraged (whether in colleges of education or elsewhere) to take their education into their own hands by seeking out opportunities to gain mathematical knowledge, the kind of knowledge that is needed to effectively teach mathematics.

Research shows the positive effects of professional development (Ball, Hill and Bass 2005; Hill and Ball 2004) and, further, that it is important for teachers to be active in their own professional development (Brahier 2013). Given the importance of mathematical knowledge, despite the fact that the current environment does not adequately support education in

pedagogical content knowledge, aspiring teachers—in this age of access to information about modern techniques and theory in math teaching (for example, massive open online courses [MOOCs] by Jo Boaler, Keith Devlin and others)—should adhere to an age-old adage: where there is a will, there is a way. This may become a crucial attitude in future math teacher education.

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Nico Higgs is an undergraduate student in his final year of study in the College of Education at the University of Saskatchewan. He is keenly interested in, and critical of, the teaching and learning of mathematics in Canada. He aspires to improve mathematics education by researching and developing best practice teaching that helps lower math anxiety, effectively engages students and better prepares students for today's mathematical world.

Egan J Chernoff is an associate professor of mathematics education at the University of Saskatchewan. His research uses logical fallacies and particular models from the field of cognitive psychology to account for future math teachers' normatively incorrect, inconsistent and sometimes inexplicable responses to a variety of probabilistic tasks. He is an ardent user of social media for mathematics education, is endlessly fascinated with math wars culture, and one day hopes to help popularize the teaching and learning of mathematics.