An Advisory Exam in Mathematics

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In September 1979, the University of Alberta Mathematics Department administered an advisory exam to students enrolled in their introductory calculus courses. (Similar exams have been given in the previous three years.) The exam problems were based on the algebra, geometry, and trigonometry contained in the high school mathematics program. Since a proficiency in these topics is a great asset in the calculus courses, the purpose of the advisory exam was to locate those students whose background appeared weak. A remedial program was set up for their benefit.

The exam problems are listed below. The time allowed was 50 minutes.

PART I

1. Which of the following is an irrational number?

   (a) $\sqrt[3]{64}$ (b) $\sqrt{\frac{144}{49}}$ (c) $\sqrt[4]{16}$ (d) $\sqrt{65}$ (e) none of these

2. The relation between degrees Fahrenheit (F) and degrees Celsius (C) is given by $F = \frac{9}{5}C + 32$. What is the difference, expressed in degrees Fahrenheit, between a temperature of 20° Celsius and one of 20° Fahrenheit?

   (a) 0 (b) 36 (c) 48 (d) 52 (e) none of these

3. $\frac{1}{\sqrt{5}+1} =$

   (a) $\frac{\sqrt{5}+1}{6}$ (b) $\frac{\sqrt{5}+1}{4}$ (c) $\frac{\sqrt{5}-1}{4}$ (d) $\frac{\sqrt{5}-1}{6}$ (e) none of these

4. Which pair of inequalities represents the shaded region (including its boundary) in the given figure?

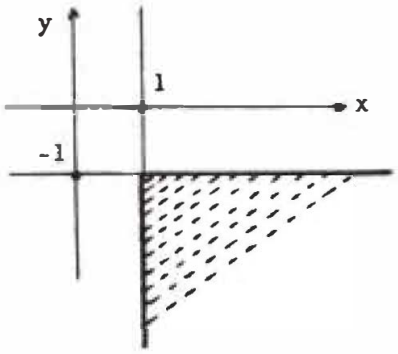
   (a) $x \geq 1$ and $y \leq -1$

   (b) $x \leq -1$ and $y \leq -1$

   (c) $x \leq -1$ and $y \geq -1$

   (d) $x \leq 1$ and $y \geq -1$

   (e) none of these



5. $1 - \frac{7}{2}x < 2 - x$ is equivalent to:

(a) $x < -\frac{5}{2}$ (b) $x < -\frac{2}{5}$ (c) $x < \frac{2}{5}$ (d) $x > -\frac{5}{2}$ (e) $x > -\frac{2}{5}$

PART II: Algebra of Polynomials

6. If $x \neq 2$, then $\frac{x^3-8}{x-2} =$

(a) $x^2 + 2x + 4$ (b) $x^2 + 4$ (c) $x^2 - 2x + 4$ (d) $x^2 - 4$

(e) none of these

7. $\frac{(x-5)x + 6}{(x-2)x - 3} =$

(a) $\frac{x+1}{x-5}$ (b) $\frac{5x-2}{2x+1}$ (c) $\frac{x-6}{x-3}$ (d) $\frac{x-2}{x+1}$ (e) none of these

8. The solution set of $2x^2 - x = 3$ is

(a) $\{-1\}$ (b) $\{-\frac{3}{2}\}$ (c) $\{-1, \frac{3}{2}\}$ (d) $\{-1, -\frac{3}{2}\}$ (e) none of these

PART III: Functions

9. If $f(x) = x^2 + 3mx + 3$ and if $f(2) = 1$, then $m =$

(a) $-\frac{4}{3}$ (b) -1 (c) 1 (d) $\frac{4}{3}$ (e) none of these

10. If $f(x) = x^2 + 1$, then $f(x+h) =$

(a) $x^2 + h^2 + 1$ (b) $x^2 + 1 + h$ (c) $(x+h+1)^2$ (d) $(x^2+1) + (h^2+1)$

(e) none of these

11. By completing the square we see that the minimum value of $f(x) = x^2 + 2x + 2$ is

(a) 0 (b) 2 (c) -1 (d) 1 (e) none of these

PART IV: Logarithms and Exponents

12. $\sqrt[3]{16x^3y^8z^4}$ =

(a) $2xy^2z\sqrt[3]{2y^2z}$ (b) $4xy^4z^2\sqrt[3]{x}$ (c) $(16)^3x^9y^{24}z^{12}$ (d) $\frac{16x^3y^8z^4}{3}$

(e) none of these

13. If $xy \neq 0$, then $\left(\frac{x^3}{2y^{-1}}\right)^{-2}$ =

(a) $4xy^3$ (b) $\frac{xy^3}{4}$ (c) $\frac{1}{4x^6y^2}$ (d) $\frac{x^6}{4y^2}$ (e) none of these

14. 10^{x-y} =

(a) $10^x - 10^y$ (b) $10^{\frac{x}{y}}$ (c) $\frac{10^x}{10^y}$ (d) $\log_{10}\left(\frac{x}{y}\right)$ (e) $\log_{10}(x-y)$

15. If x, y, and z are each positive, then $\log x + \log y - 2\log z$ =

(a) $\frac{\log(xy)}{\log(z^2)}$ (b) $\log\left(\frac{xy}{2z}\right)$ (c) $\log\left(\frac{xy}{z^2}\right)$ (d) $\log(x+y-2z)$

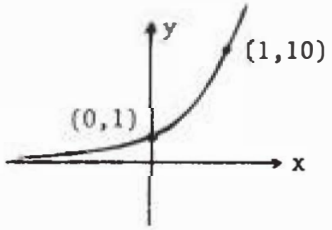
(e) $\log(x+y-z^2)$

16. If $\log_5 x = 1$, then x =

(a) $\frac{1}{5}$ (b) 0 (c) 5 (d) $\log_{10}\frac{1}{5}$ (e) none of these

17. Which of the following represents the graph of $y = \log_{10} x$?

(a)



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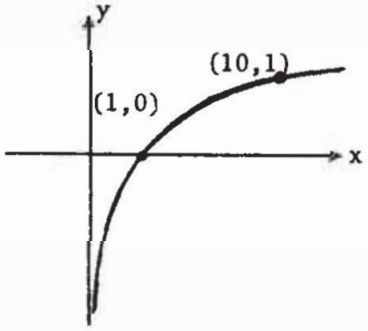
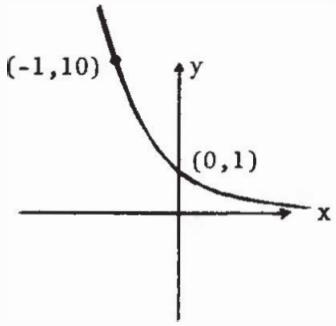
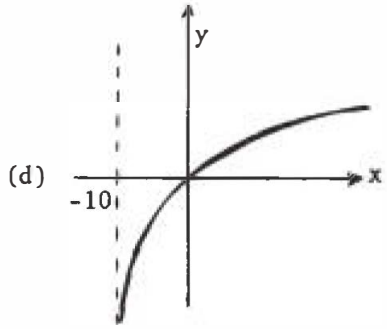
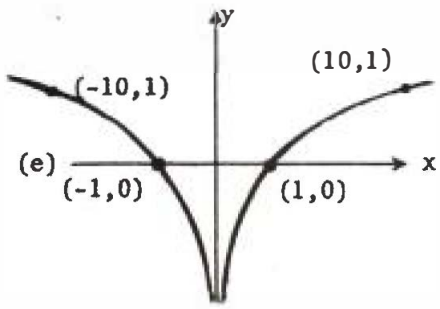


(c)







PART V: Geometry; Lines, Conics

18. A circle with center (-1,3) passes through the point (3,5). The radius of this circle is

(a) $4\sqrt{5}$ (b) $2\sqrt{5}$ **(c) 20** **(d)** $2\sqrt{2}$ (e) none of these

19. Which of the following is an equation of a circle?

(a) $x^2 + 4y^2 - 9 = 0$ (b) $4x^2 - y^2 - 9 = 0$ (c) $x^2 - 4y - 9 = 0$

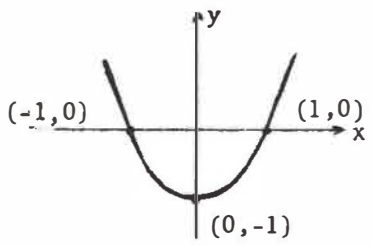
(d) $4x^2 + 4y^2 - 9x = 0$ (e) none of these

20. The parabola shown has the equation

(a) $y = (x-1)^2$ (b) $y = x^2 - 1$

(c) $y^2 = 1 - x^2$ (d) $y = 1 - x^2$

(e) none of these

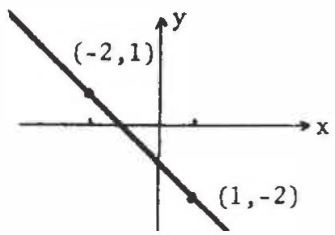


21. The line shown has the equation

(a) $x + y = 1$ (b) $x + y = -1$

(c) $x - y = 3$ (d) $-x + y = 3$

(e) none of these



PART VI: Trigonometry

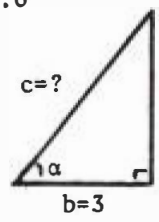
22. $\cos \frac{\pi}{2} - \cos \frac{\pi}{4} =$

(a) $\frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}$ (c) $1 - \frac{1}{\sqrt{2}}$ (d) $\cos(\frac{\pi}{2} - \frac{\pi}{4})$

(e) none of these

23. In the right triangle shown, $\cos \alpha = 0.4$ and $b = 3$. What is c ?

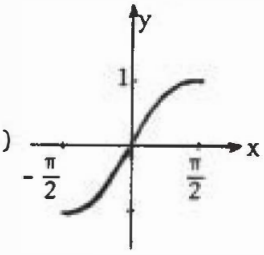
(a) $\frac{3}{0.4}$ (b) $3(0.4)$ (c) $\frac{3}{0.6}$ (d) $3(0.6)$ (e) none of these



24. $\sin^2 x =$

(a) $\frac{1}{\cos^2 x}$ (b) $1 + \cos^2 x$ (c) $1 - \cos^2 x$ (d) $\cos^2 x - 1$ (e) none of these

25. Which one of the following represents the graph of $y = \sin x$ for x between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$?

(a)


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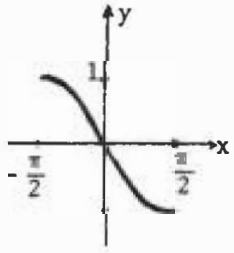

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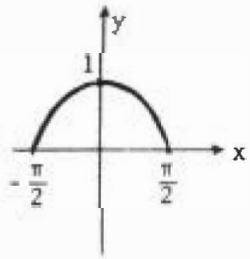
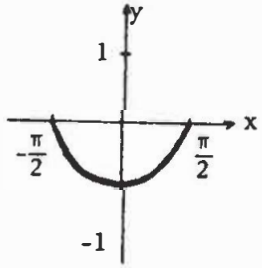


(d)


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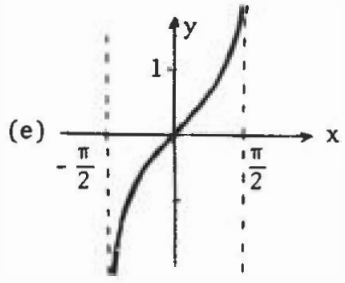

Advisory Exam Statistics

Table I gives the percentage of students answering each of the 25 questions correctly.

Table II gives the relative frequency (R.F.) and cumulative frequency (C.F.) for each of the possible scores (that is, 0 to 25).

TABLE I

Question	*Percentage*
1	73
2	82
3	54
4	89
5	49
6	51
7	52
8	72
9	77
10	63
11	24
12	48
13	31
14	50
15	23
16	38
17	37
18	65
19	20
20	70
21	75
22	32
23	71
24	56
25	45

TABLE II

Score	*R.F.*	*C.F.*
0	0.2	0.2
1	0.2	0.4
2	0.2	0.6
3	0.6	1.2
4	1.2	2.3
5	2.1	4.4
6	2.3	6.7
7	3.6	10.3
8	5.0	15.3
9	5.1	20.4
10	6.9	27.3
11	8.5	35.8
12	7.4	43.2
13	7.3	50.5
14	8.5	59.1
15	7.1	66.2
16	7.1	73.2
17	6.4	79.7
18	5.2	84.8
19	4.0	88.8
20	2.7	91.5
21	3.2	94.7
22	2.4	97.2
23	1.7	98.8
24	0.6	99.5
25	0.5	100.0

Of the 2071 students who wrote the advisory exam, 423 received a score less than 10. These students were advised to take a no-credit remedial program in algebra and geometry along with their calculus course. The attendance in this program was initially about 250, and decreased throughout its four and one-half weeks duration.

The totally voluntary nature of this program, the rate at which the material had to be covered, and the extra workload it placed on the students involved make its success hard to assess. Fortunately the student response was favorable. It is likely that the course was of most benefit to those students who needed it only as a refresher course.