Freddy the Frog and Number Line

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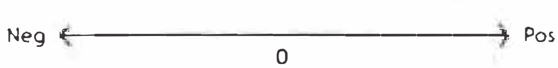
Among the most difficult concepts to teach for understanding from the elementary mathematics curriculum are the operations on negative numbers. A number of mnemonic devices have been developed ("Two like signs yield a positive," "Minus a minus is a plus," "Two unlike signs yield a negative," et cetera) but only help the learner get the right answer mechanically. They do not provide a framework for developing or understanding operations involving negative numbers. In this paper I will describe a model I have developed to help my students accept and understand the use of the basic operations on positive and negative numbers.

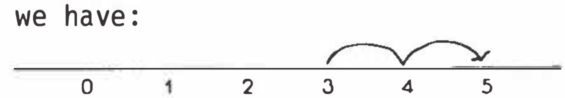
The lessons center around Freddy Frog, who has perfected his jumps in a manner not unlike Jonathan Livingston Seagull's flight. Freddy, also a perfectionist, has developed his jumping skills to the point that every jump, whether forward or backward, is uniform in length. Thus, when Freddy hops on the straight path that goes past his home, he will land on certain predictable spots. Consider this diagram of Fred's home and the path.



"0" marks the location of Freddy's home on the path. The arrows on the path indicate that it does not end, but continues to the right and left toward the nebulous villages of "Pos" and "Neg," respectively. If Freddy starts at home, faces Pos, and takes 3 hops forward, we could represent that action thus:

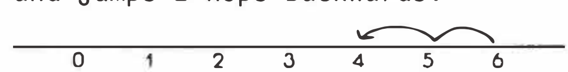
The numerals 1, 2, and 3 indicate where Freddy was at the end of the first, second, and third jumps, respectively.

If Freddy starts at 3, still facing Pos, and jumps two hops forward, we have:



which leaves him the same place he would be if he had merely jumped forward 5 hops. Hence, 3 + 2 = 5.

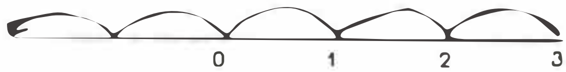
Now consider the instance in which Freddy starts at 6, still facing Pos, and jumps 2 hops backwards:



Upon completion, he will be only 4 hops from home. Thus, hopping backwards is like subtracting (6 - 2 = 4).

It should be noted that the direction Freddy hops determines whether the operation to be performed is addition or subtraction. If he hops forward, we are adding; if backward, subtracting.

Consider the situation in which Freddy starts (still facing Pos) at 3 and jumps backward 5 hops. This can be diagrammed as follows:



Freddy is now 2 hops from home, but on the opposite side as before. We could therefore call this new location "opposite of 2" and write it -2. The points on the path left of home (0) can now be easily established as opposites of those on the Pos side. It is thus possible to handle addition and subtraction of a positive number from any integral location on the path (number line) by merely having Freddy face Pos, start on the first term, and jump the number of hops indicated by the second term, in the direction dictated by the operation sign. For example, 3 + 6 means, "start at 3 facing Pos and jump forward 6," -3 - 6 means, "Start at -3 facing Pos and jump backward 6," and -3 + 6 means, "start at -3 facing Pos and jump forward 6." Before making an in-depth analysis of all that is entailed in this translation process, let us consider one more facet of the system.

Suppose Freddy starts at home, faces Neg and jumps forward four hops. Where will he finally land? Performing this oneself or with a model helps the pupil realize that Freddy will be at -4. Let us now translate Freddy's jumps into our mathematical symbols. He started at home and jumped forward, so we have 0 + , but what did we add? If we added 4, he would have landed at 4 on the Pos side. But he did not; he landed at -4. Therefore, jumping while facing Neg is represented as operating with a Neg number, or in this case, -4. The instance described represents: 0 + (-4) = -4 because Freddy started at home and jumped forward four jumps while facing Neg.

It should now be noted that the first term tells where Freddy starts, the operation sign (+ or -) tells the direction he will jump (forward or backward), the sign of the second term (+ or -) indicates the direction he is facing (Pos or Neg), and the value of the second term determines how many hops he will take. Those are all we need in order to add or subtract any pair of integers.

Answers to Crossword Puzzle
(on page 42)