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delta-k

THE ALBERTA TEACHERS' ASSOCIATION
MATHEMATICS COUNCIL

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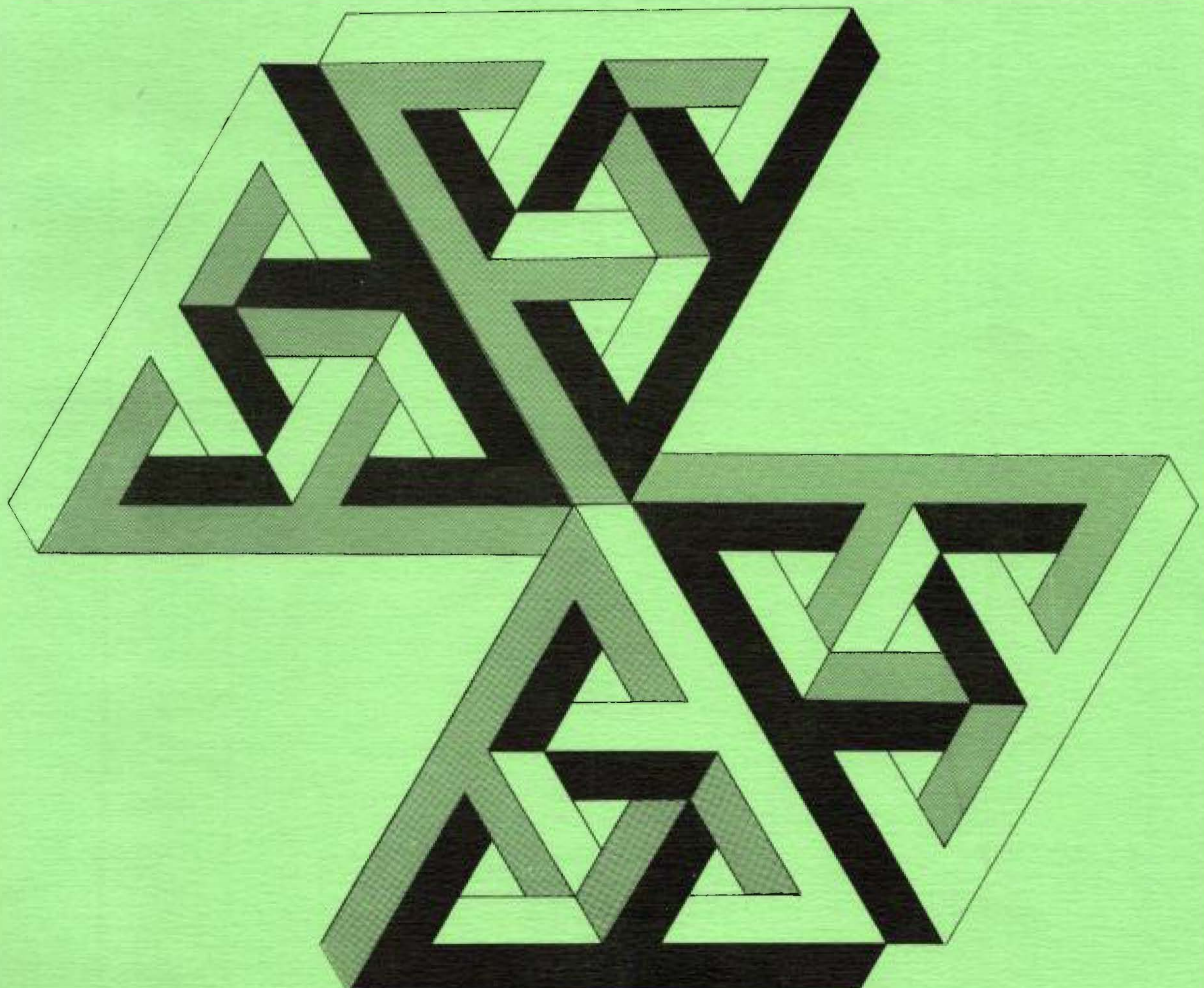

An Agenda for Action

Recommendations for School Mathematics of the 1980s

The National Council of Teachers of Mathematics recommends that—

1. problem solving be the focus of school mathematics in the 1980s;
2. basic skills in mathematics be defined to encompass more than computational facility;
3. mathematics programs take full advantage of the power of calculators and computers at all grade levels;
4. stringent standards of both effectiveness and efficiency be applied to the teaching of mathematics;
5. the success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing;
6. more mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population;
7. mathematics teachers demand of themselves and their colleagues a high level of professionalism;
8. public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society.

The National Council of Teachers of Mathematics, Inc.

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Volume XIX, Number 4 **June 1980**

Contents

EDPRESS

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From the Editor's Desk

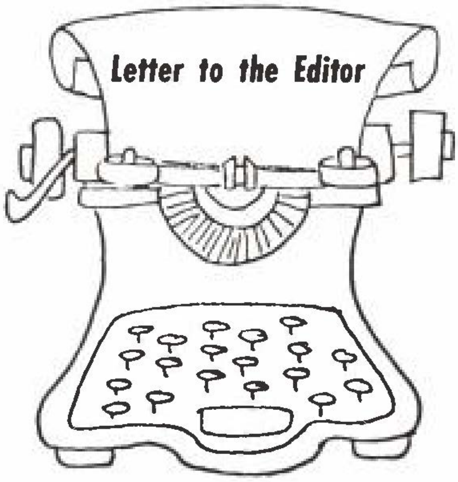
Three new items have come to your editor's attention recently that may be of interest to all.

The first is "An Agenda for Action," a booklet published by the National Council of Teachers of Mathematics. It gives recommendations on objectives in education for the next 10 years which are the expression of mathematics teachers throughout North America and other parts of the world. The summary is printed on the inside front cover of this *journal*. For detailed report, write to NCTM, 1906 Association Drive, Reston, Virginia 22091.

The second item is "statistics." This is a small booklet published by the Mathematics Department of the University of Alberta containing information on the basic needs of university-bound students in the area of statistics. This booklet is a valuable item for teachers of senior matriculation students. It provides an insight into the mathematics skills required of students who want to pursue this area of study. For your free copy, write to Mathematics Department, University of Alberta, Edmonton, Alberta T6G 2G1.

The third item is the fact that the calculator is no longer the new "tool of mathematics." It has been replaced by the computer. However, before you run out to buy computers, it would be wise to find out what is taking place in the industry. At the NCTM annual meeting in Seattle I found an overwhelming maze of hardware and software with a variety of capabilities. The difficulty is the incompatibility of different systems. The only drawback to using computers to assist you is the cost of materials; the only limit is the imagination of your staff. (P.S. Don't tell the English and social studies teachers they can use your computer as easily as you - unless you need their support in obtaining the hardware financing.)

Ed Carriger
Editor



Dear Mr. Carriger,

I thoroughly enjoy your *journal*. The spectrum is down my road - you offer something for K-3 people as well as the secondary panel.

I love to share these ideas with the elementary people who don't receive *delta-k*.

"No Copyright; please plagiarize..." - you are being very realistic when you make this statement. I wish more groups follow suit!

If you have other ideas to share, I am always open to them.

- *Emil Dukovac*, Head of Mathematics
Kapuskasing District High School
Kapuskasing, Ontario

1980 AMTNYS Summer Workshop

The Eighteenth Annual Summer Workshop of the Association of Mathematics Teachers of New York State will be held at Niagara University August 10-14, 1980. The Workshop will feature more than 60 sessions for teachers of mathematics in elementary, middle, junior high and senior high schools. "Mathematics - Cultural and Practical," the Workshop theme, will be developed around six strands: Art, Microcomputers, Life Skills, Problem Solving, Enrichment Mathematics, and New Curricula in New York State and Ontario, Canada. The Workshop will feature American and Canadian speakers, thus providing an international flavor.

There will be sessions for teachers of every grade level and every type of student from basic to advanced. Make-and-Take sessions will be provided at every grade level, K-12.

Niagara University is near the Canadian-American border, and four miles from the Niagara Falls, in Rainbow Country. Niagara Falls, U.S.A., and Niagara Falls, Canada, are both major tourist centers with abundant natural, recreational, and cultural attractions.

Come to Niagara University from August 10 to 14 and combine a vacation with an enriching mathematics program.

For further information, contact Sr. John Frances Gilman, Chairman, 1980 AMTNYS Summer Workshop, Niagara University, Department of Mathematics, Niagara University, NY 14109.

South West Regional News

by *Dennis Burton*
Past President, SWMCATA
Lethbridge, Alberta

The inaugural year of the S.W.M.C. ATA has proven to be very successful. The organizational meeting was held last May with the fee set at $2.50 and an initial executive elected:

President	Dennis Burton
Secretary	Mary Jo Maas
Treasurer	Joe Krywolt
Program Chairman	Dennis Kosaka

The major thrust of our regional was to improve the quality of sessions available to math teachers at the teachers' convention. The result for the 1980 convention was excellent. Through use of regional funds combined with convention we were able to present some excellent sessions.

The convention also serves as the time for our annual meeting and election of officers for the coming year. The incoming executive is:

President	Joe Krywolt
Secretary	Mary Jo Maas
Treasurer	Fern Heinen
Program Chairman	Dennis Kosaka
Regional Reps.	Rosemarie Eklund (Pincher Creek) Hans Holstein (County - Lethbridge) Francis Wilson (Taber) Bill Fukami (Crowsnest)

Through our regional the University of Lethbridge offered a seminar on "Orientation to using Computers in the Classroom," on April 19, 26, and May 3. A workshop was also held Saturday, May 10, at Lethbridge Collegiate Institute, with presentations at elementary, junior, and senior high levels.

At the present time the regional seems to be fulfilling a need for the math teachers and appears to have a good future.

Statistics in the High School

by *Dennis G. Haack*

The author is the section leader of the Biostatistics and Epidemiology Section of the Tobacco and Health Research Institute at the University of Kentucky. As a member of the National Council of Teachers of English (NCTE) Committee on Public Doublespeak, Dr. Haack writes and lectures on the misuse of the language of statistics.

The high school mathematics curriculum is continually changing. One of the more recent changes has been the inclusion of a course in statistics (see Pieters, 1976). As to the specific makeup of a high school statistics course, there is not likely to be agreement. As to the primary objective of such a course, there should be agreement. The purpose of this paper will be to look at the objective of a high school statistics course.

The key to the development of any course in statistics is deciding what statistics is. Statistics has, since the publication of R.A. Fisher's *Statistical Methods for the Research Worker* in 1925, been thought of as a set of research tools. In this regard statistics is the investigation of a population. The population of interest may exist or may be created by the researcher.

The study of an existing population is by a sample survey. A part of, or sample from a population is selected and studied. Examples of sample surveys include opinion polls, marketing research surveys, TV-viewing and radio-listening surveys, and pre-election polls. A 100 percent sample is referred to as a census. Of interest in the study of sample survey techniques is how a survey is designed as well as how to analyze and interpret survey data.

On the other hand, a researcher may wish to study a population which he creates. For example, an agricultural researcher might test a fertilizer on a crop which he has planted on a test plot. The researcher is stimulating the use of the fertilizer by farmers, that is, he tries to create a population which would exist if farmers used the fertilizer on their crops.

Another example of the investigation of a created population involves research on the effects of a drug. A population is created in the laboratory which would simulate use of the drug if the drug were put on the market.

As with the study of an existing population, the study of a created population involves a researcher with the design of his experiment as well as with the analysis and interpretation of experimental data. So we see that statistics is the study of a population which exists or is created. Statistics provide a set of tools which are required by an investigator for the design of a population study and the analysis and interpretation of the data generated by the study.

Traditionally, statistics courses at all levels have been an attempt to teach statistics as the study of a population. Distinction between experimental and sample survey investigations may or may not be made.

But statistics has become more than a research tool. Listen to the news this evening, or read a newspaper or a news magazine. Listen to public officials and advertisers. Statistics has become a language in its own right. This language pervades the media making it nearly impossible to understand a newscast without being quite familiar with the language of statistics. What are these words we hear - "estimates," "significance," "projections," "averages," et cetera? We are bombarded by numbers. But what do the numbers mean?

This is what statistics is to most Americans: a language which is very often used and too often misused. Statisticians have, for the most part, not taught about the language of statistics. Even students who have completed a traditional course in statistics cannot usually understand this language.

Statistics can be thought of as the tools required for the study of a population or statistics can be thought of as a language. We must decide which type of statistics we are to teach our students.

A first course in statistics should not try to teach statistics as a research tool. There are two main reasons for this. First, the study of statistics, a research tool, requires students to memorize the use of formulas, if not to memorize the formulas themselves. Students become so involved with learning to calculate statistics that they fail to learn what the statistics mean. Retention of the manipulative skills is minimal, causing students to have little, if any knowledge of statistics after a course of this type is completed.

A second reason why a first course in statistics should not teach statistics as a research tool is that students, after taking a traditional statistics course, are no better able to understand the statistics they'll encounter in the media than they were before the course started. The better students might be able to run a t-test, but they are not likely to have a feeling for what is involved with a determination that, say, significantly more animals in a treatment group developed cancer than did animals in a control group. Some of the students might be able to calculate the probability of selecting a red ball from an urn, but they may not know how to interpret the statement, "The probability of rain is 20 percent today." That is, the most we can hope of a student is that he or she will become a manipulative "whiz." A student might become quite good at "plugging and chugging": plugging numbers into a formula and chugging until a number results. Yet our students are not likely to be able to interpret the statistics they might have learned to calculate.

Statistics should be taught as a language rather than as a research tool. Students should first be taught how to interpret statistics. A student will be much better off being able to understand statistics than only able to calculate them.

Statistics can be taught as a language. It is being done at the University of Kentucky (see Haack, 1976). The idea behind the course is to downplay the calculation of statistics while concentrating on how to interpret statistics. In fact, students do not calculate any statistics in the course. There is, therefore, no need for mathematical formulas. The course is conducted in a strictly verbal, nonsymbolic manner. Examples used in the course come from the media. Ideally, students will be able to apply the principles they learn to statistics they will encounter, or have encountered in other areas.

One of the major drawbacks with a nonsymbolic statistics course has been the lack of a text, requiring a large amount of work by the teacher. Texts are now becoming available (see, for example, Haack, 1979).

One of the more interesting aspects of teaching statistics as a language is that students become genuinely excited about being able to detect misuses of statistics. When I started this experiment in teaching a few years ago, I did not look forward to trying to find examples of the misuse of statistics. Such examples are, of course, very instructive. As I began looking for cases of the misuse of statistics, I became awed by how easy examples were to find. I became more and more convinced that a course of this type was needed. Students also relish catching advertisers and public officials misusing statistics, that is, detecting doublespeak.

Doublespeak is the "involved, inflated, and often deliberately ambiguous use of language" (*Webster's New Collegiate Dictionary*) (see Rank, 1974, and Dieterich, 1976). The misuse of the language of statistics is statistical doublespeak. Statistical doublespeak can be avoided if statistics are properly understood (see Haack, 1977). This is the objective of the course I propose.

It is possible to teach statistics as a language. It is a challenging, yet rewarding undertaking. As you contemplate offering a course of this type, you might want to look at some of the books which can be used as reference material. There are a few good, readable books which may help you teach about statistics, the language.

With emphasis on sample surveys there are:

1. Gallup, G. *The Sophisticated Poll-Watchers Guide*. Princeton Opinion Press, 1972.
2. Roll, C.W., Jr. and A.H. Cantril. *Polls: Their Use and Misuse in Politics*. Basic Books, 1972.
3. Wheeler, M. *Lies, Damn Lies, and Statistics*. Liveright, 1976.

These books lack adequate discussion of the science of studying an existing population but do give a good discussion of the "art" of sample surveying.

On the general topic of statistics and statistical doublespeak consider:

1. Bross, I.D.J. *Scientific Strategies in Human Affairs: To Tell the Truth*. Exposition Press, 1957.
2. Campbell, S. *Flaws and Fallacies in Statistical Thinking*. Prentice Hall, 1974.
3. Federer, W.T. *Statistics and Society*. Dekker, 1973.
4. Hauser, P.M. *Social Statistics in Use*. Russell-Sage Foundation, 1975.
5. Huff, D. *How to Lie with Statistics*. Norton, 1954.
6. Messick, B.M. *Mathematical Thinking in Behavioral Sciences*. Readings from *Scientific American*. Freeman, 1968.
7. Mosteller, F. (editor) *Statistics By Example*. Addison-Wesley, 1973.
8. Reichard, R. *The Figure Finaglers*. McGraw-Hill, 1974.
9. Taner, J. (editor) *Statistics: A Guide to the Unknown*. Holden-Day, 1972.

You will find these books to be very interesting. Taner's collection of essays is an excellent source for the statistics course I propose. The essays are on the application of statistics in just about any area that students might have an interest.

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Dieterich, D. (editor) *Teaching Public Doublespeak.* NCTE/Citation Press: Urbana, IL. 61801, 1976.

Haack, D.G. "A Nonsymbolic Statistics Course." *Communications in Statistics*, A(5) 10:943-47, 1976.

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Pieters, R.S. "Statistics in the High School Curriculum." *American Statistician*, 30: 134-39, 1976.

Rank, H. (editor) *Language and Public Policy.* NCTE/Citation Press: Urbana, IL. 61801, 1974.

CREATIVE MATH EXPERIENCES FOR THE YOUNG CHILD
(Grades K-1)

- Forte, MacKenzie

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The mastery of foundational concepts and understandings is developed in nature and should be presented to the child in proper sequence at the strategic time in keeping with his readiness for them in terms of his past experiences and presently recognized needs. This phase of the young child's intellectual development should never be left to chance, but planned and carefully monitored by an adult. It is important to remember that learning is continuous and developmental, and that each stage of mathematical growth is dependent upon success in the stage that preceded it. In keeping with this belief, the activities in *Creative Math Experiences for the Young Child* are sequentially planned and should be introduced to the child in the order of their presentation in the book. A test is included at the end of each section. Cost is $7.25; over 200 pages.

This book is also developed in three duplicating master books - Learning about Shapes and Sizes; Learning to Read and Write Numbers; Learning about Sets, Fractions, Measuring and Money. Each book costs $7.25; 36 masters in each.

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Publisher: Incentive Publications.

A Geometrical Study on...

Is Seeing Believing?

by *Bill Jenkins* and *Ved Madan*
Red Deer College, Red Deer, Alberta

In this note, we would like to present some ideas on the question: Is Seeing Believing? - posing it as a geometrical problem! "Seeing is believing" as well as the saying "I'll believe it when I see it" are often-used terms which hold for most situations. However, there are some exceptions. In this article, we are concerned with a few of these exceptions - exceptions that crop up by way of the geometrical illusions. Important factors causing geometrical illusions relate essentially to the influence of location in the visual field, the influence of angles, extent, contrast and perspectivity. To elaborate further on these points, we consider the following examples:

1. *The location in the visual field* has an effect on illusions. An example would be figure 1, where the height and width of the diagram of the hat are equal, yet the height appears to be greater than the width. Likewise, when a tree is in a vertical position, it seems longer than when it is in a horizontal position.

2. *Angles.* When dealing with the influence of angles, it has been found that angles play an important role, either directly or indirectly, in the production of illusions. For a long time many geometrical illusions were accounted for by "overestimation" or "underestimation" of angles. A good example of the influence of angles on illusions would be figure 2.

3. *Extent.* In dealing with the illusions of "interrupted extent," the distance and area tend to change in extent depending upon the fullness or emptiness of the particular object. For instance, in figure 3, b seems to be longer as well as having more area than c, and a is even larger. Hence, the more light which can be seen, the longer objects appear, even though they are equal. Second, the illusions of contour are also related to distance as seen in figure 4. When concerning ourselves with this type of illusion, we can notice from the three squares (two incomplete and one empty square) that the outside squares are extensive. Besides this example, there is the well-known Müller-Lyer illusion (figure 5) which makes the left side appear longer than the right side, but in actual fact they are of equal length.

4. *Contrast.* This illusion refers to the lines, angles, and areas of different sizes. Contrast plays an important part in most of the geometrical-optical illusions. The illustration in figure 6 gives an effect that the middle segment of a seems to be longer than the middle segment of b. However, both segments are of equal length.

5. *Perspectivity.* Last, there is the illusion of perspectivity which deals with the influence of numerous factors such as lines,

angles, and occasionally contour and contrast. For instance, the square formations in figure 7 are of the same size, but the most remote formation looks much larger than the other two. Apparently, converging lines influence these equal figures in proportion as they suggest perspective.

Furthermore, there are other illusions such as "after-images" that are caused by continuous visual contact and they depend upon certain conditions. For instance, when looking at the sun for a moment and then looking at a plain colored wall you will notice there is an after-image or spot that will change in color frequently. Another example is when a spoked bicycle wheel is revolving so rapidly that the spokes become invisible, but occasionally, when there is a rapid eye movement in the direction of the wheel, the spokes may be seen for a brief moment.

One of the most remarkable illusions is that when the sun or moon rise, they appear to be closer. This is untrue. In fact, the distance of the moon or sun from the earth at the horizon is the same as when they are at the zenith or center of the sky.

To conclude, although we have not covered all the areas of geometrical and optical illusions, the facts that we have supplied support the view that seeing is not necessarily believing.

The following references will shed more light on the subject.

References

Campbell, Donald T., Marshall H. Segall, and Melville J. Herskovits. *The Influence of Culture on Visual Perception.* Indianapolis: Bobbs-Merrill Company, Inc., 1966.

Gregory, R.T. *Eye and Brain.* New York: McGraw Hill Book Company, 1969.

Luckiesh, M. *Visual Illusions.* New York: Dover Publications Inc., 1965. (Note: All illustrations are from this book.)

Illustrations

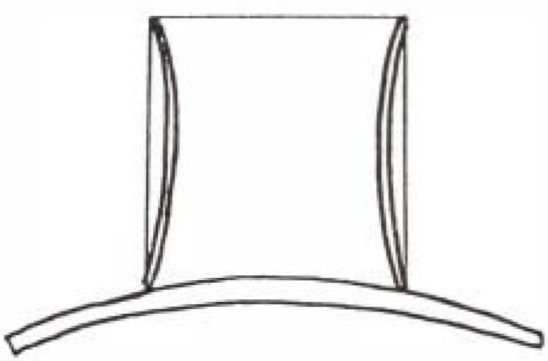


Figure 1.

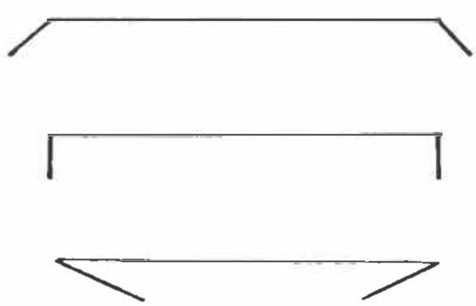


Figure 2.

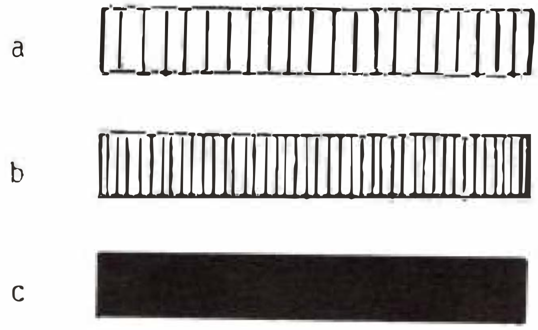


Figure 3.

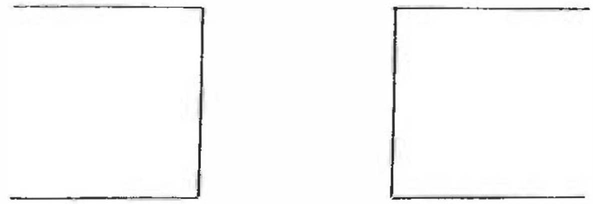


Figure 4.

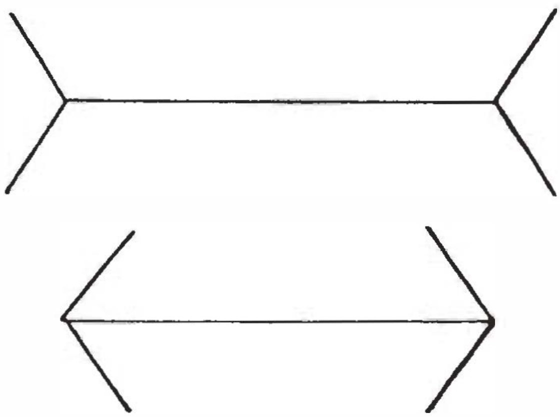


Figure 5.

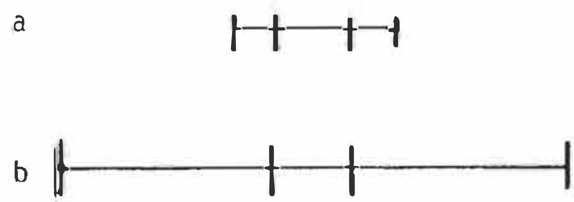


Figure 6.

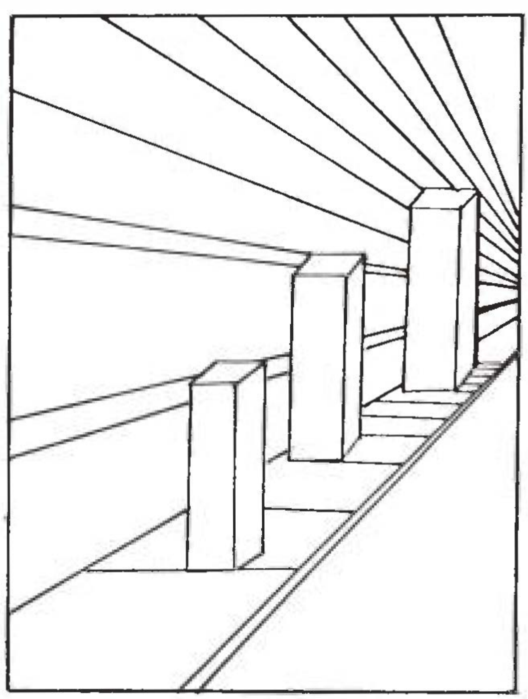


Figure 7.

??? PROBLEM CORNER ???

edited by *William J. Bruce* and *Roy Sinclair*
University of Alberta
Edmonton, Alberta

Problems suggested here are aimed at students of both the junior and senior high schools of Alberta. Solutions are solicited and a selection will be made for publication in the next issue of *delta-k*. Names of participants will be included. All solutions must be received (preferably in typewritten form) within 30 days of publication of the problem in *delta-k*. *Mail solutions to:*

Dr. Roy Sinclair
Department of Mathematics
University of Alberta
Edmonton, Alberta T6G 2G1

Problem 1 was first published in the March 1980 issue and is reproduced here along with its solution.

Problem 1:

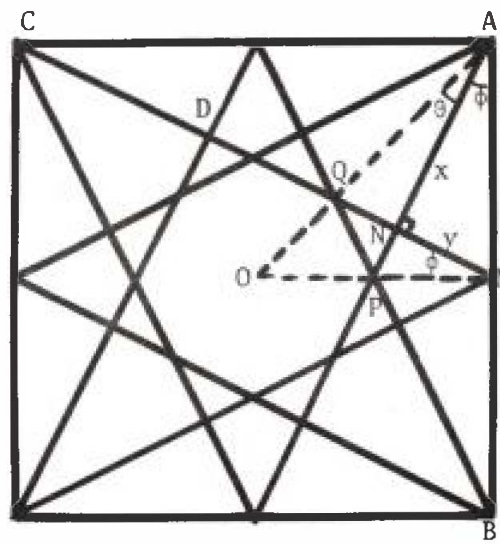
An 8-point star is formed in a square region of side S units by drawing two lines from each mid-point of a side of the square to opposite corners of the square. Note that these lines also form two identical smaller squares as well as one octagon.

(a) If the star is cut out of the square region, what fraction of the square is wasted?

(b) In terms of S, what is the area of one of the smaller square regions?

(c) In terms of S, what is the area of the octagonal region?

Solution of Problem 1



The following (edited) solutions to parts (a) and (b) were supplied by Ian Weitz, a Mathematics 30 student at Winston Churchill High School in Lethbridge, Alberta.

Note that the two diagonals and the two perpendicular bisectors of the sides are axes of symmetry. Hence only one corner of the diagram need be considered.

Now if $\angle MAN = \angle NMO = \phi$, then $\tan \phi = \frac{1}{2}$. Thus $\angle MNA = 90°$ and if $AB = s$, $MN = x$ and $AN = y$, then $AM = \frac{1}{2} s$,

$$x = AM \cdot \sin \phi = \frac{s}{2} \cdot \frac{1}{\sqrt{5}}$$

and
$$y = AM \cdot \cos \phi = \frac{s}{2} \cdot \frac{2}{\sqrt{5}} = \frac{s}{\sqrt{5}} .$$

(a) Area (Δ MAN) $= \frac{1}{2} \cdot x \cdot y = \frac{1}{20} s^2$. Hence the wasted area is $8\left(\frac{s^2}{20}\right) = \frac{2}{5} s^2$.

(b) Now $MC = \sqrt{s^2 + \left(\frac{s}{2}\right)^2} = \frac{\sqrt{5}}{2} s$. So $ND = MC - (x+y) = \frac{s}{\sqrt{5}}$. Hence the area of the square with side ND is $\left(\frac{s}{\sqrt{5}}\right)^2 = \frac{1}{5} s^2$.

(c) By symmetry, each side of the octagon has the same length; however, the angles are not all equal. Now $MP = MA \cdot \tan \phi = \frac{s}{4}$. Thus $OP = \frac{s}{4}$. Also $\theta = \angle NAQ = 45° - \phi$. So $AQ = y \cdot \sec \phi = \frac{s}{\sqrt{5}} \cdot \frac{1}{\cos(45° - \phi)} = \frac{\sqrt{2}}{3} s$, since

$\cos(45°-\phi) = \cos 45° \cos \phi + \sin 45° \sin \phi = \frac{3}{\sqrt{10}}$, and $OQ = OA - QA = \frac{s}{\sqrt{2} \cdot 3}$.

Then Area (ΔOPQ) $= \frac{1}{2} \cdot OP \cdot OQ \sin 45° = \frac{1}{2} \cdot \frac{s}{4} \cdot \frac{s}{3\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{s^2}{48}$ so that the area of the octagon is $8\left(\frac{s^2}{48}\right) = \frac{s^2}{6}$.

Problem 2:

(submitted by William J. Bruce, University of Alberta)

Clearly $1 = 1^2$ is a perfect square, but 11 and 111 are not. Consider all numbers $11111 \cdots 1 = S$, in which all digits are unity, and prove or disprove that, except for $s = 1$, no such number is a perfect square.*

*EXTENSION (Proof not to be submitted for publication.)

The theorem is true for any number that can be written in the form $100m + 10 + 1$ (m a positive integer). Also, true for $100m + k + 1$ when k is *not* divisible by 4.

PLUS +++

The following material is reprinted from Issue No. 4 of Plus + + +, *a short magazine informing mathematics educators across Canada about important events, research, curriculum development and items of national interest.*

OAME

The Ontario Association for Mathematics Education held its annual meeting on May 9-11, 1980, in Toronto. The keynote speaker, George Immerzeel, of Cedar Falls, Iowa, spoke on "Problem Solving."

Research Council for Diagnostic and Prescriptive Mathematics Conference

This conference, of particular interest to learning assistance teachers and those dealing with pupils with learning disabilities, took place in Vancouver on April 13-15, 1980. Further information is obtainable from the Conference Chairman, Ian Beattie (604-228-5204), c/o 7th Annual RCDPM Conference, Centre for Continuing Education, University of British Columbia, Vancouver, B.C. V6T 2A4.

University of British Columbia Programs in Mathematics Education

Graduate and undergraduate courses are offered for elementary and secondary teachers. M.Ed. programs can be completed through part-time study during winter sessions or *3 consecutive summer sessions*. M.A. and Ed.D programs may be completed in winter sessions. Programs and courses focus on various areas including Computing Studies, Diagnosis and Remediation, Curriculum Development, Problem Solving, and Assessment. For information, contact: Chairman, Mathematics Education, Faculty of Education, University of British Columbia, Vancouver, B.C. V6T 1Z5.

Concordia University Master in the Teaching of Mathematics

A thesis or nonthesis M.T.M. program in the Department of Mathematics is available in two options: Secondary or Collegial Teachers, Elementary Teachers. Part-time studies or special summer series are available. For information, contact: Professor J. Hillel (514-879-7356), Department of Mathematics, H-939-1, Concordia University, 1455 de Maisonneuve Blvd. W., Montreal, Quebec H3G 1M8.

Canadian Mathematics Education Study Group

The 1980 meeting will be at Laval University, Quebec City, on June 6-10, 1980. Open to mathematics educators and mathematicians with an interest in teaching teachers of mathematics, the conference offers participants a choice of working groups, each meeting for nine hours. Guest lecturers will be Caleb Gattegno and David Hawkins. Costs: $10 registration, $18 per night accommodation. For further information, write to Dr. J. Hillel, HB206, Loyola Campus, Concordia University, Montreal, Quebec H4B 1R6.

NCTM Canadian Meeting

(advance notice)

The 60th annual meeting of the National Council of Teachers of Mathematics will take place in Toronto on April 14-17, 1982, the first time such an event has occurred outside the U.S.A.

Canadian Mathematical Society

1980 Alberta High School Prize Examination Results

Prize	$ Amt.	Student	School
Canadian Mathematical Society Scholarship	400	MOREWOOD, Robert	Medicine Hat High School Medicine Hat
Nickle Foundation Fellowship	400	Not awarded this year.	
First Runner-Up	150	WILTING, Carl	M.E. LaZerte Composite High Edmonton
Second Runner-Up	150	WILLIAMS, Daniel	McCoy High School Medicine Hat

Special Provincial Prizes

Prize	$ Amt.	Student	School
Highest Grade 12 student (below first 3)	75	URSENBACH, Charles	Dr. E.P. Scarlett Senior High Calgary
Highest Grade 10/11 student	75	BARAGAR, Arthur	Old Scona Academic High School Edmonton

District Prizes

District No.	$ Amt.	Student	School
1	50	WONG, Garant	Saint Thomas More Fairview
2	50	HAYASHI, Peter	Harry Collinge School Hinton
3	50	NICOL, Bonnie	Fort Saskatchewan High School Fort Saskatchewan
4	50	KOMISHKE, Bradley	William E. Hay Composite High Stettler
5	50	SHELDON, Joanne	Banff Composite High School Banff

District No.	$ Amt.	Student	School
6	50	NEUFELDT, David	Kate Andrews High School Coaldale
7(1)	50	TRUMPENER, John	Strathcona Composite High School Edmonton
7(2)	50	BOWMAN, John	Old Scona Academic High School Edmonton
8(1)	50	BAMBER, Jim	Queen Elizabeth High School Calgary
8(2)	50	MA, Felix	Dr. E.P. Scarlett Senior High Calgary

367 students from 64 schools in Alberta and the Northwest Territories wrote the 1980 examination. The following students took the first 16 places and were nominated for the Canadian Mathematical Olympiad.

Student	School
BAMBER, Jim	Queen Elizabeth High School, Calgary
BARAGAR, Arthur	Old Scona Academic High School, Edmonton
BONDY, Brad	Old Scona Academic High School, Edmonton
BOWMAN, John	Old Scona Academic High School, Edmonton
DVORKIN, Len	Henry Wise Wood Senior High, Calgary
FILIPCHUK, David	Harry Ainlay Composite High, Edmonton
FRAGA, Werner	St. Mary High School, Edmonton
GREENWAYS, Jeannette	St. Mary High School, Edmonton
KENYON, Jim	M.E. LaZerte Composite High School, Edmonton
LEE, Michael	Old Scona Academic High, Edmonton
MA, Ching-Kwok Felix	Dr. E.P. Scarlett Senior High School, Calgary
MOREWOOD, Robert	Medicine Hat High School, Medicine Hat
TRUMPENER, John	Strathcona Composite High School, Edmonton
URSENBACH, Charles	Dr. E.P. Scarlett Senior High School, Calgary
WILLIAMS, Daniel	McCoy High School, Medicine Hat
WILTING, Carl	M.E. LaZerte Composite High School, Edmonton

The following 11 students placed 17-27th. Credit for an additional two or three part one questions or for an additional 1/2 of a part two question would have put any one of these students into the top 16.

Todd Anderson (Dr. E.P. Scarlett Sr. High, Calgary); Jack Chu (Henry Wise Wood Sr. High, Calgary); Scott Craig (Henry Wise Wood Sr. High, Calgary); Mark Jorgenson (Dr. E.P. Scarlett Sr. High, Calgary); John Marko (Louis St. Laurent Sr. High, Edmonton); Michael Markowski (Strathcona Comp. High School, Edmonton); Steven Leikeim (Bishop Grandin High School, Calgary); Aaron Ng (M.E. LaZerte Comp. High School, Edmonton); Adam Parrish (Strathcona Composite High School, Edmonton); Eleanor Stein (Ross Sheppard Composite High School, Edmonton); Raymond Wong (James Fowler Sr. High, Calgary)

The following 11 students placed 28-38th. These students were approximately one full part two question away from being in the top 16.

John Achilles (Harry Ainlay Comp. High School, Edmonton); Allison Chernowski (Old Scona Academic High School, Edmonton); Mike Jackson (Harry Ainlay Comp. High School, Edmonton); David Leung (Harry Ainlay Composite High School, Edmonton); Craig McLurg (Dr. E.P. Scarlett Sr. High School, Calgary); David Neufeldt (Kate Andrews High School, Coaldale); Joyce Oman (William Aberhart High School, Calgary); Ashok Patel (Old Scona Academic High School, Edmonton); Joanne Sheldon (Banff Comp. High School, Banff); Stuart Thompson (Louis St. Laurent Catholic School, Edmonton); Garant Wong (Saint Thomas More School, Fairview)

The following 17 students also placed in the top 15% of all who wrote.

Stephen Bell (Dr. E.P. Scarlett Sr. High, Calgary); David Berg (William Aberhart High School, Calgary); Marc Berner (William Aberhart High School, Calgary); Gordon Boyes (Louis St. Laurent Catholic School, Edmonton); Jean Chan (Harry Ainlay Composite High School, Edmonton); Bobby Chum (Jasper Place Comp. High School, Edmonton); Richard Garside (Raymond High School, Raymond); William Graham (Queen Elizabeth Sr. High School, Calgary); Peter Hayashi (Harry Collinge High School, Hinton); Todd Herron (Dr. E.P. Scarlett Sr. High School, Calgary); Bradley Komishke (William E. Hay Comp. High School, Stettler); Leanne Koziak (Louis St. Laurent Catholic School, Edmonton); Bonnie Nicol (Fort Saskatchewan High School, Fort Saskatchewan); Dwaine Palmer (Raymond High School, Raymond); Arvid Schultz (Noble Central School, Nobleford); Wenona Urquhart (Sir Winston Churchill High School, Calgary); Derek Woolner (Queen Elizabeth Sr. High School, Calgary).

ANSWER SHEET

To be filled in by the Candidate.

PRINT:

Last Name ______ First Name ______ Initial ______

Candidate's Street Address ______ Town/City ______ POSTAL CODE ______

Name of Candidate's School ______

Grade ______ Male or Female ______

ANSWERS:

1	2	3	4	5	6	7	8	9	10

11	12	13	14	15	16	17	18	19	20

To be completed by the Department of Mathematics, University of Alberta:

	Points	Points Correct	Number Wrong
1-20	5	5 × =	1 × =
TOTALS		C = ______	W = ______

SCORE = Correct - Wrong = ______

PART I **Time: 60 Minutes**

<u>*Instructions to Candidates*</u>

1. *Please do not open this booklet beyond Page 2 until instructed to do so by the supervisor.*

2. *Please turn now to Page 2 (the next page) and fill in the top four lines - <u>Page 2 is your answer sheet.</u>*

3. *This exam is multiple choice. Each question will be followed by 5 possible answers, labelled, A, B, C, D, E. For each question, list your choice of answer in the box on the answer sheet directly above the question number. For example, if you decide the correct answer to question 3 is labelled C, then enter C in the box above 3 on the answer sheet.*

4. *To discourage random guessing, there is a penalty for each incorrect answer; there is no penalty for unanswered questions. Filling in more than one letter in any box counts as a wrong answer for that question.*

5. *At the signal from your supervisor, detach both this page and page 2 (the answer sheet) and begin the examination. Pencil, graph paper, scratch paper, ruler, compass, and eraser are allowed.*

   Calculators are <u>not</u> allowed.

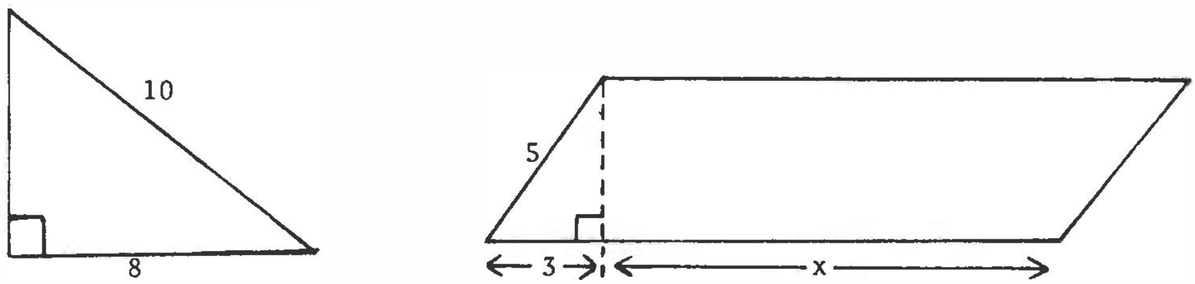
Do all problems. Each problem is worth five points.

1. There are 5 roads between the towns A and B and 4 roads between the towns B and C.

   The number of different ways of driving the roundtrip $A \rightarrow B \rightarrow C \rightarrow B \rightarrow A$ without using the same road more than once is:

   (A) 32 (B) 240 (C) 400 (D) 16 (E) none of these.

2.



In the above diagrams, the area of the triangle is two-fifths of the area of the parallelogram. The value of x is therefore:

(A) 30 (B) 12 (C) 15 (D) 24 (E) none of these.

3. A die is thrown repeatedly until a 6 is obtained. Assuming the die to be fair, the probability that this will happen on the third throw is:

(A) 1/6 (B) 1/216 (C) 5/36 (D) 25/216 (E) none of these.

4. For which real values of k does $kx^2 + kx + 1$ have no real root?

(A) $-2 < k < 6$ (B) $-2 < k < 4$

(C) $0 < k < 2$ (D) $0 < k < 4$

(E) none of these

5. For which value(s) of k are the lines $9x + ky = 7$ and $kx + y = 2$ parallel?

(A) $k = 3$ (B) $k = \pm 3$ (C) $k = 1/3$ (D) $k = \pm 1/3$

(E) none of these.

6. Let $x = \log_{a^2} b$. Then $\log_b a^x$ equals

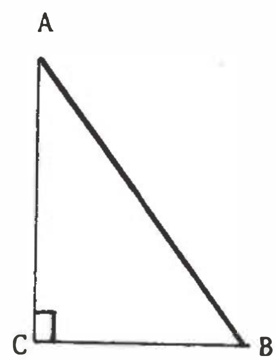
(A) $\sqrt{b}$ (B) b^2 (C) 1/2 (D) 2 (E) none of these.

7. If $\sin 3x = 0$, then $\sin x$ equals

(A) 0 (B) 0 or $\pm 1/2$ (C) 0 or $\pm \sqrt{3}/2$

(D) 0, $\pm 1/2$ or $\pm \sqrt{3}/2$ (E) none of these.

8.



In the right-angled triangle ABC, the lengths of the sides AB and BC are $(x^2 + y^2)$ and $2xy$ units respectively. The length of the side AC is therefore:

(A) (x^2-y^2) (B) $(x-y)^2$

(C) $(x^2-y^2)^2$ (D) $(x+y)^2$

(E) none of these.

9. In a certain examination, the average mark (out of 100) of the 30 boys in a class was 60. The girls did rather better, their average being 65. If the overall average for the class was 62, the number of girls who took the examination was:

(A) 20 (B) 24 (C) 18 (D) 36 (E) none of these.

10. The following is true for all real numbers x,y:

(A) $(x+y)^2 \geq (x-y)^2$

(B) $|x+y| \geq x + y$

(C) $\sqrt{x^2+y^2} \geq x + y$

(D) $xy \geq x + y$

(E) none of the above is true for all real x,y.

11. $\log_{10}\left[(144)^{144}\right]$ equals

(A) $576 \log_{10}2 + 288 \log_{10}3$

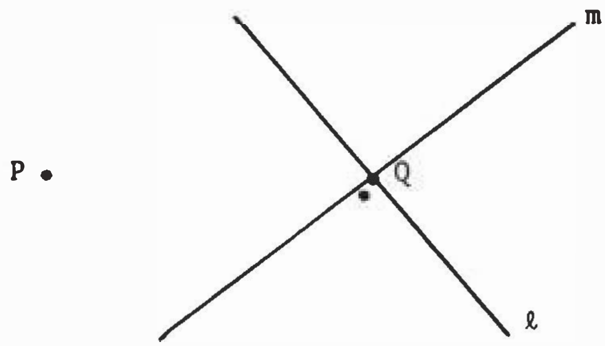
(B) $144 \log_{10}2 + 144 \log_{10}3$

(C) $(\log_{10} 144)^{144}$

(D) $2 \log_{10}144$

(E) none of these.

12.



The line ℓ has the equation $y = -2x - 4$ and the line m has the equation $y = 2x + 4$. If P is the point (0,0) then the equation of the line through P and Q is

(A) $y = x$

(B) $y = -2x$

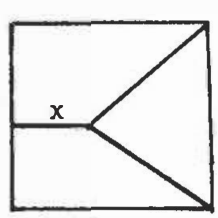
(C) $2x + 3y = 1$

(D) $-y - 2x = 1$

(E) none of these.

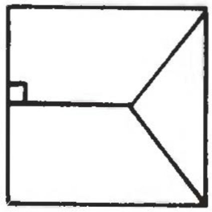
13. The maximum possible value of $3x - 3x^2$ where x is real is
(A) 5/4 (B) 0 (C) -1 (D) 3/4 (D) none of these.

14.



A unit square is divided into three parts of equal area as shown. Then x is

(A) 1/3 (B) $1/\sqrt{2}$ (C) 1/4 (D) $1/\sqrt{3}$ (E) 1/2 .

15. 

A unit square is divided into three parts by three lines of equal length as shown. The length of each line is

(A) $1/2$ (B) $5/8$ (C) $2 - \sqrt{2}$ (D) $1/\sqrt{2}$ (E) $3/5$.

16. $\sqrt{\dfrac{\sqrt{5} - 2}{\sqrt{5} + 2}}$ equals

(A) $\dfrac{1}{\sqrt{5} - \sqrt{2}}$ (B) $\dfrac{1}{\sqrt{2} + 5}$ (C) $\sqrt{5} - 2$

(D) $\sqrt{5} + 2$ (E) $\dfrac{1}{5 - \sqrt{2}}$.

17. If $f(x) = x^2 - 5$ and $f(4+a) = f(4a)$ then a is

(A) $4/3$ or $-4/5$ (B) 4 or $-4/3$ (C) $4/3$ or $4/5$

(D) -4 or $4/3$ (E) -4 or $-4/5$.

18. Two circles and a straight line are drawn in the plane and form exactly N bounded regions. N is <u>not</u>

(A) 3 (B) 4 (C) 5 (D) 6 (E) all these are possible

19. A bowl contains 2 marbles of each of 4 colours. If you randomly remove 3 marbles from the bowl without replacing them, what is the probability that you have removed two of the same colour?

(A) 1/4 (B) 1/7 (C) 3/7 (D) 1/2 (E) none of these

20. If n is a positive integer so that

$$1 + 2 + \ldots + n = (n+1) + (n+2) + \ldots + 118 + 119$$

then n is

(A) 60 (B) 69 (C) 76 (D) 84 (E) 89 .

PART II **Time: 110 Minutes**

INSTRUCTIONS TO CANDIDATES

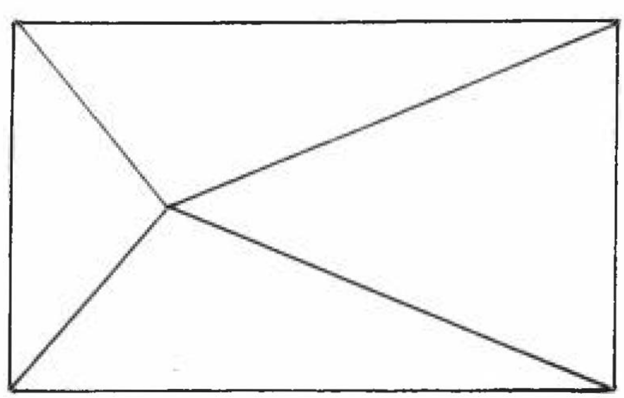
Do all questions. All questions are weighted equally and you may attempt them in any order you wish.

Your paper should be concise and complete and each step of your answer should be clearly justified and presented in a legible and intelligible form. Extra credit will be given for particularly elegant solutions as well as for non-trivial generalizations with proof. If there is any doubt about an interpretation of a problem, make a note of that on your paper and state and solve what you consider to be a valid non-trivial interpretation of the problem.

---

1. Suppose we start with 7 sheets of paper and then some number of them are each cut into 7 smaller pieces. Then some of the smaller pieces are each cut into 7 still smaller pieces and so on. Finally the process is stopped and it turns out that the total number of pieces of paper is some number between 1976 and 1986. Can one determine the exact final number of pieces of paper?

2. A rectangle is divided into 4 triangles as in the diagram.



Three of the triangles have areas of 5, 9, and 11. Find all possible areas for the fourth triangle.

3. A certain sequence of real numbers is defined as follows:

$$x_1 = 2$$

$$x_{n+1} = \frac{2x_n}{3} + \frac{1}{3x_n}; \qquad n = 1,2,3,\ldots.$$

Show that for all values of $n \geq 2$, $1 < x_n < 2$.

4. If all cross-sections of a bounded solid figure are circles, prove that the figure is a sphere.

5. Consider a line segment AB of length $2a$. A point P is chosen at random on this line segment, all points being equally likely.

Show that the probability that the product of the lengths of the segments AP and PB exceeds $a^2/2$ is $1/\sqrt{2}$.

6. Show that one root of the equation

$$x^4 + 5x^2 + 5 = 0$$

is $x = \omega - \omega^4$ where ω is a complex fifth root of unity (that is, $\omega^5 = 1$). Determine the other three roots as polynomials in ω .

Solutions to Part I

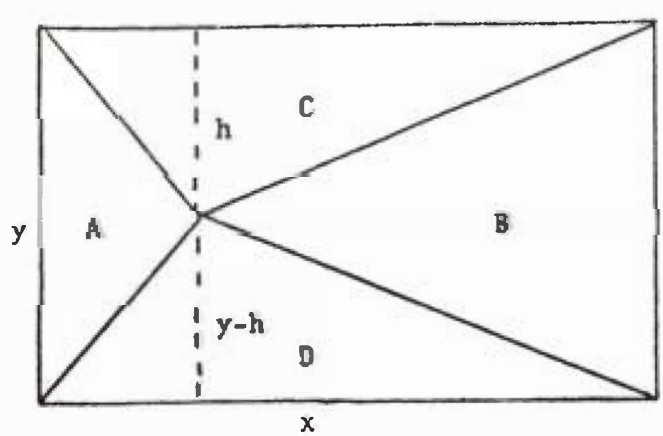
1	2	3	4	5	6	7	8	9	10
B	B	D	D	B	C	C	E	A	B

11	12	13	14	15	16	17	18	19	20
A	E	D	A	B	C	A	E	C	D

Solutions to Part II

1. If x of the pieces are cut into 7, we have $7 - x + 7x = 7 + 6x$ pieces. In fact, at every stage the number of pieces is a number of the form $7 + 6n$ for some integer n. To see this, suppose we have $7 + 6n_1$ pieces and cut n_2 of these into seven. Then we have $7 + 6n_1 - n_2 + 7n_2 =$ $= 7 + 6(n_1+n_2)$ pieces. Now, if we subtract 7 from a number of the form $7 + 6n$, what remains is a multiple of 6. The only number between 1976-7 and 1986-7 which is a multiple of 6 is 1974, so $1974 + 7 = 1981$ is the only possibility for the final number of pieces.

2.



Label the areas of the four triangles A,B,C,D as shown. Suppose the rectangle has length x and width y. If triangle C has height h, then triangle D has height $y - h$. Consequently,

$C + D = \frac{1}{2}hx + \frac{1}{2}x(y-h) = \frac{1}{2}xy$ is half the total area of the rectangle.

But then $A + B$ is also half the area and so $A + B = C + D$. If A and B are 5 and 9, then one of C and D is 11 and the other is 3. If A and B are 5 and 11, then one of C and D is 9 and the other is 7. Finally, if A and B are 9 and 11, then one of C and D is 5 and the other is 15. Thus the possible areas for the fourth triangle are 3, 7 or 15.

3. Solution 1 (Without calculus)

$$x_2 = \frac{2(2)}{3} + \frac{1}{3(2)} = \frac{9}{6} \quad \text{so} \quad 1 < x_2 < 2 .$$ Assume, as an induction hypothesis, that $1 < x_k < 2$.

Then $1/x_k < 1$ and so

$$x_{k+1} = \frac{2x_k}{3} + \frac{1}{3}\left(\frac{1}{x_k}\right) < \frac{2(2)}{3} + \frac{1}{3}(1) = \frac{5}{3} < 2 .$$

Also if $x_k > 1$, then $x_k = 1 + a$ for some $a > 0$. Consequently,

$$x_{k+1} = \frac{2(1+a)}{3} + \frac{1}{3(1+a)} = \frac{2}{3} + \frac{2}{3}a + \frac{1}{3(1+a)} .$$

We need to know that $x_{k+1} > 1$.

Now $$x_{k+1} - 1 = \frac{2}{3} + \frac{2}{3}a + \frac{1}{3(1+a)} - 1$$

$$= \frac{1}{3}\left(2a + \frac{1}{1+a} - 1\right)$$

$$= \frac{1}{3}\left(\frac{2a^2+a}{1+a}\right)$$ which is > 0 since $a > 0$.

Thus $x_{k+1} > 1$ as required.

Solution 2 (Calculus)

Let $f(x) = \frac{2x}{3} + \frac{1}{3x} \qquad 1 \le x \le 2$.

Then $f(1) = \frac{2}{3} + \frac{1}{3} = 1$ and $f(2) = \frac{2(2)}{3} + \frac{1}{3(2)} = \frac{9}{6}$.

Also $f'(x) = \frac{2}{3} - \frac{1}{3x^2} = \frac{2x^2 - 1}{3x^2} > 0$ for all x , $1 \le x \le 2$.

Thus $f(x)$ is a strictly increasing function on the interval $1 \le x \le 2$. Consequently $1 < x < 2$, implies $f(1) < f(x) < f(2)$. That is $1 < x < 2$ implies $1 < f(x) < \frac{3}{2}$. In particular $1 < x_n < 2$ implies $1 < x_{n+1} < 2$. Since $x_2 = 3/2$ we are again done by induction.

4. Any bounded solid figure has a maximum diameter, d . Consider a line segment, L, which connects two points on the figure and whose length is d . Now consider any plane which includes L. This plane intersects the figure in a circle, C. Moreover because L has maximal length, L must be a diameter of C and the midpoint of L is the center of C . Now using L as the axis of rotation, rotate this plane through space. The intersection of the plane with the figure will always be a circle whose center is the midpoint of L . Hence the figure is exactly the sphere obtained by rotating C around L.

5. Suppose AP has length x so that PB has length $2a - x$. We are interested in the probability that

$$x(2a-x) > a^2/2 .$$

But $\quad x(2a-x) > a^2/2$

$\Leftrightarrow \quad x^2 - 2ax + a^2/2 < 0$

$\Leftrightarrow \quad (x-a)^2 < a^2/2$

$$\Leftrightarrow \qquad -a/\sqrt{2} < x - a < a/\sqrt{2}$$

$$\Leftrightarrow \qquad a - a/\sqrt{2} < x < a + a/\sqrt{2} .$$

Thus for the required inequality to be satisfied x must be in the given interval of length $2a/\sqrt{2}$. Since the total length of the interval is $2a$ the probability of this happening is

$$\frac{\frac{2a}{\sqrt{2}}}{2a} = \frac{1}{\sqrt{2}}$$

6. Since $\omega^5 = 1$, $\omega^6 = \omega$, $\omega^7 = \omega^2$ and so on. In particular, $(\omega^2)^5 = \omega^{10} = 1$, $(\omega^3)^5 = \omega^{15} = 1$, and $(\omega^4)^5 = \omega^{20} = 1$ so that if ω is a fifth root of unity so are ω^2, ω^3 and ω^4. Also $0 = \omega^5 - 1 = (\omega-1)(\omega^4+\omega^3+\omega^2+\omega+1)$ and since $\omega \neq 1$, this implies $\omega^4 + \omega^3 + \omega^2 + \omega + 1 = 0$. Now by direct calculation

$$(\omega-\omega^4)^2 = \omega^2 - 2\omega^5 + \omega^8 = \omega^2 - 2 + \omega^3$$

and $$(\omega-\omega^4)^4 = (\omega^2 - 2 + \omega^3)^2 = \omega^4 - 4\omega^3 - 4\omega^2 + \omega + 6 .$$

Then

$$(\omega-\omega^4)^4 + 5(\omega-\omega^4)^2 + 5 = \omega^4 - 4\omega^3 - 4\omega^2 + \omega + 6 + 5\omega^2 - 10 + 5\omega^3 + 5$$
$$= \omega^4 + \omega^3 + \omega^2 + \omega + 1 = 0 .$$

Thus if ω is a fifth root of unity, then $x = \omega - \omega^4$ is a root of the given polynomial. Applying this argument to ω^2, ω^3, and ω^4 which are also fifth roots of unity, we see that $\omega^2 - (\omega^2)^4 = \omega^2 - \omega^3$, $\omega^3 - (\omega^3)^4 = \omega^3 - \omega^2$, and $\omega^4 - (\omega^4)^4 = \omega^4 - \omega$ are the other roots of the polynomial.

An Advisory Exam in Mathematics

by Z.M. *Trollope*
Department of Mathematics
University of Alberta
Edmonton, Alberta

In September 1979, the University of Alberta Mathematics Department administered an advisory exam to students enrolled in their introductory calculus courses. (Similar exams have been given in the previous three years.) The exam problems were based on the algebra, geometry, and trigonometry contained in the high school mathematics program. Since a proficiency in these topics is a great asset in the calculus courses, the purpose of the advisory exam was to locate those students whose background appeared weak. A remedial program was set up for their benefit.

The exam problems are listed below. The time allowed was 50 minutes.

PART I

1. Which of the following is an irrational number?

   (a) $\sqrt[3]{64}$ (b) $\sqrt{\frac{144}{49}}$ (c) $\sqrt[4]{16}$ (d) $\sqrt{65}$ (e) none of these

2. The relation between degrees Fahrenheit (F) and degrees Celsius (C) is given by $F = \frac{9}{5}C + 32$. What is the difference, expressed in degrees Fahrenheit, between a temperature of 20° Celsius and one of 20° Fahrenheit?

   (a) 0 (b) 36 (c) 48 (d) 52 (e) none of these

3. $\frac{1}{\sqrt{5}+1} =$

   (a) $\frac{\sqrt{5}+1}{6}$ (b) $\frac{\sqrt{5}+1}{4}$ (c) $\frac{\sqrt{5}-1}{4}$ (d) $\frac{\sqrt{5}-1}{6}$ (e) none of these

4. Which pair of inequalities represents the shaded region (including its boundary) in the given figure?

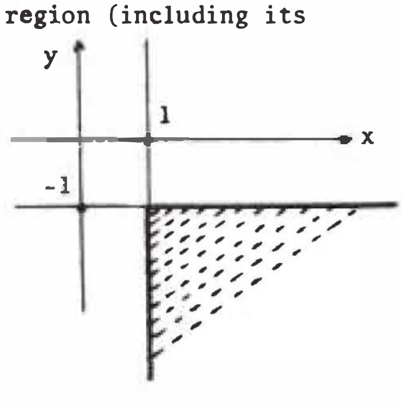
   (a) $x \geq 1$ and $y \leq -1$

   (b) $x \leq -1$ and $y \leq -1$

   (c) $x \leq -1$ and $y \geq -1$

   (d) $x \leq 1$ and $y \geq -1$

   (e) none of these



5. $1 - \frac{7}{2}x < 2 - x$ is equivalent to:

(a) $x < -\frac{5}{2}$ (b) $x < -\frac{2}{5}$ (c) $x < \frac{2}{5}$ (d) $x > -\frac{5}{2}$ (e) $x > -\frac{2}{5}$

PART II: Algebra of Polynomials

6. If $x \neq 2$, then $\frac{x^3-8}{x-2} =$

(a) $x^2 + 2x + 4$ (b) $x^2 + 4$ (c) $x^2 - 2x + 4$ (d) $x^2 - 4$

(e) none of these

7. $\frac{(x-5)x + 6}{(x-2)x - 3} =$

(a) $\frac{x+1}{x-5}$ (b) $\frac{5x-2}{2x+1}$ (c) $\frac{x-6}{x-3}$ (d) $\frac{x-2}{x+1}$ (e) none of these

8. The solution set of $2x^2 - x = 3$ is

(a) $\{-1\}$ (b) $\{-\frac{3}{2}\}$ (c) $\{-1, \frac{3}{2}\}$ (d) $\{-1, -\frac{3}{2}\}$ (e) none of these

PART III: Functions

9. If $f(x) = x^2 + 3mx + 3$ and if $f(2) = 1$, then $m =$

(a) $-\frac{4}{3}$ (b) -1 (c) 1 (d) $\frac{4}{3}$ (e) none of these

10. If $f(x) = x^2 + 1$, then $f(x+h) =$

(a) $x^2 + h^2 + 1$ (b) $x^2 + 1 + h$ (c) $(x+h+1)^2$ (d) $(x^2+1) + (h^2+1)$

(e) none of these

11. By completing the square we see that the minimum value of $f(x) = x^2 + 2x + 2$ is

(a) 0 (b) 2 (c) -1 (d) 1 (e) none of these

PART IV: Logarithms and Exponents

12. $\sqrt[3]{16x^3y^8z^4}$ =

(a) $2xy^2z\sqrt[3]{2y^2z}$ (b) $4xy^4z^2\sqrt[3]{x}$ (c) $(16)^3x^9y^{24}z^{12}$ (d) $\frac{16x^3y^8z^4}{3}$

(e) none of these

13. If $xy \neq 0$, then $\left(\frac{x^3}{2y^{-1}}\right)^{-2}$ =

(a) $4xy^3$ (b) $\frac{xy^3}{4}$ (c) $\frac{1}{4x^6y^2}$ (d) $\frac{x^6}{4y^2}$ (e) none of these

14. 10^{x-y} =

(a) $10^x - 10^y$ (b) $10^{\frac{x}{y}}$ (c) $\frac{10^x}{10^y}$ (d) $\log_{10}\left(\frac{x}{y}\right)$ (e) $\log_{10}(x-y)$

15. If x, y, and z are each positive, then $\log x + \log y - 2\log z$ =

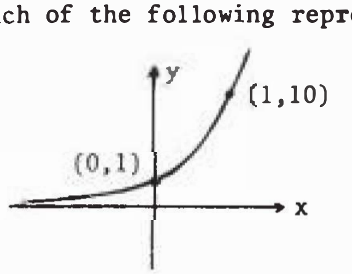
(a) $\frac{\log(xy)}{\log(z^2)}$ (b) $\log\left(\frac{xy}{2z}\right)$ (c) $\log\left(\frac{xy}{z^2}\right)$ (d) $\log(x+y-2z)$

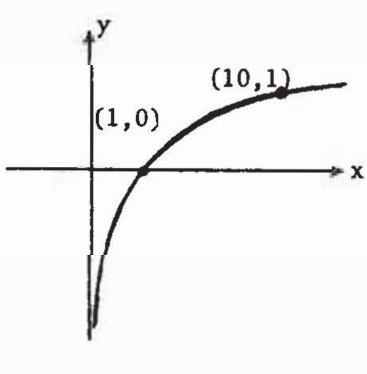
(e) $\log(x+y-z^2)$

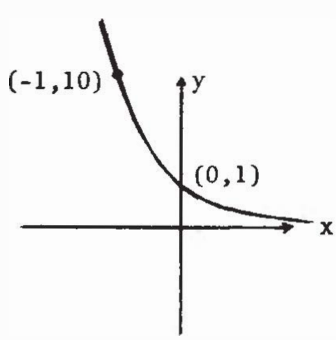
16. If $\log_5 x = 1$, then x =

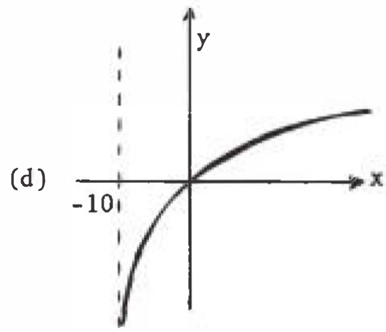
(a) $\frac{1}{5}$ (b) 0 (c) 5 (d) $\log_{10}\frac{1}{5}$ (e) none of these

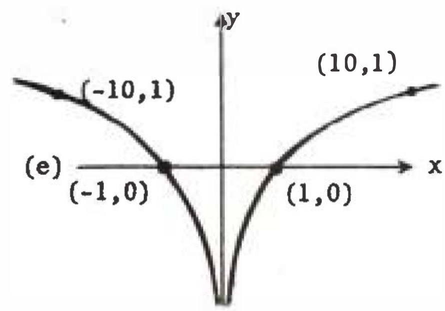
17. Which of the following represents the graph of $y = \log_{10} x$?

(a) 

(b) 

(c) 





PART V: Geometry; Lines, Conics

18. A circle with center (-1,3) passes through the point (3,5). The radius of this circle is

(a) $4\sqrt{5}$ (b) $2\sqrt{5}$ (c) 20 (d) $2\sqrt{2}$ (e) none of these

19. Which of the following is an equation of a circle?

(a) $x^2 + 4y^2 - 9 = 0$ (b) $4x^2 - y^2 - 9 = 0$ (c) $x^2 - 4y - 9 = 0$

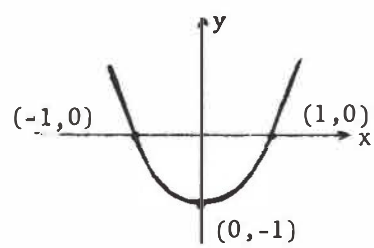
(d) $4x^2 + 4y^2 - 9x = 0$ (e) none of these

20. The parabola shown has the equation

(a) $y = (x-1)^2$ (b) $y = x^2 - 1$

(c) $y^2 = 1 - x^2$ (d) $y = 1 - x^2$

(e) none of these

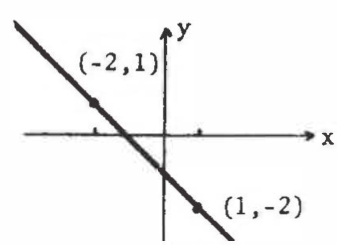


21. The line shown has the equation

(a) $x + y = 1$ (b) $x + y = -1$

(c) $x - y = 3$ (d) $-x + y = 3$

(e) none of these



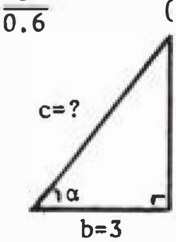
22. $\cos \frac{\pi}{2} - \cos \frac{\pi}{4} =$

(a) $\frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}$ (c) $1 - \frac{1}{\sqrt{2}}$ (d) $\cos(\frac{\pi}{2} - \frac{\pi}{4})$

(e) none of these

23. In the right triangle shown, $\cos \alpha = 0.4$ and $b = 3$. What is c ?

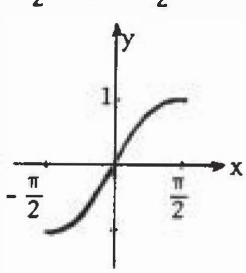
(a) $\frac{3}{0.4}$ (b) $3(0.4)$ (c) $\frac{3}{0.6}$ (d) $3(0.6)$ (e) none of these

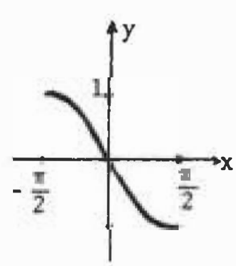


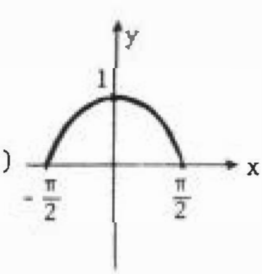
24. $\sin^2 x =$

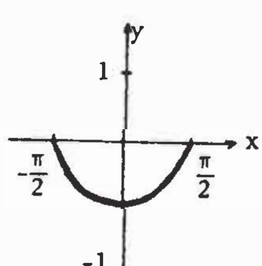
(a) $\frac{1}{\cos^2 x}$ (b) $1 + \cos^2 x$ (c) $1 - \cos^2 x$ (d) $\cos^2 x - 1$ (e) none of these

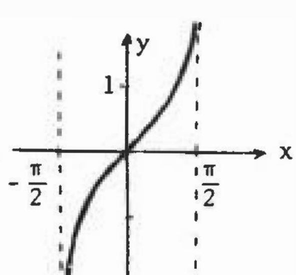
25. Which one of the following represents the graph of $y = \sin x$ for x between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$?

(a)


(b)


(c)


(d)


(e)


Advisory Exam Statistics

Table I gives the percentage of students answering each of the 25 questions correctly.

Table II gives the relative frequency (R.F.) and cumulative frequency (C.F.) for each of the possible scores (that is, 0 to 25).

TABLE I

Question	*Percentage*
1	73
2	82
3	54
4	89
5	49
6	51
7	52
8	72
9	77
10	63
11	24
12	48
13	31
14	50
15	23
16	38
17	37
18	65
19	20
20	70
21	75
22	32
23	71
24	56
25	45

TABLE II

Score	*R.F.*	*C.F.*
0	0.2	0.2
1	0.2	0.4
2	0.2	0.6
3	0.6	1.2
4	1.2	2.3
5	2.1	4.4
6	2.3	6.7
7	3.6	10.3
8	5.0	15.3
9	5.1	20.4
10	6.9	27.3
11	8.5	35.8
12	7.4	43.2
13	7.3	50.5
14	8.5	59.1
15	7.1	66.2
16	7.1	73.2
17	6.4	79.7
18	5.2	84.8
19	4.0	88.8
20	2.7	91.5
21	3.2	94.7
22	2.4	97.2
23	1.7	98.8
24	0.6	99.5
25	0.5	100.0

Of the 2071 students who wrote the advisory exam, 423 received a score less than 10. These students were advised to take a no-credit remedial program in algebra and geometry along with their calculus course. The attendance in this program was initially about 250, and decreased throughout its four and one-half weeks duration.

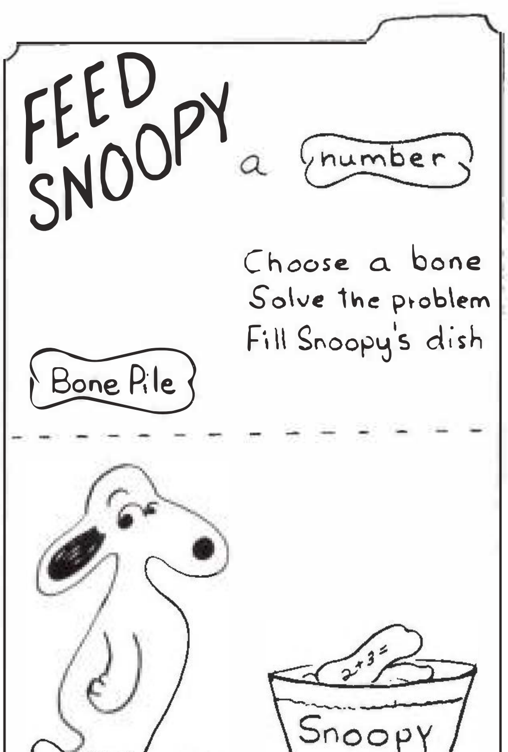
The totally voluntary nature of this program, the rate at which the material had to be covered, and the extra workload it placed on the students involved make its success hard to assess. Fortunately the student response was favorable. It is likely that the course was of most benefit to those students who needed it only as a refresher course.

This remedial course in algebra and geometry is also available in the second term, as is a refresher course in trigonometry.

We welcome any suggestions from teachers on this whole procedure. It should be evident that our advisory exam and its subsequent remedial courses are intended only to increase the chances of success of the students in our calculus courses. In no way do they constitute a criticism of the instruction and preparation of these students in the high schools.

FILE FOLDER GAME

K-3
by
Joan Penewell



Materials
—One red file folder
—White construction paper for Snoopy and bowl
—Oak tag or construction paper for bones

Procedure
—2 Players
—First player selects bone and solves the problem
—Second player checks against the answer sheet

Activities: Bones can be marked with
—Scrambled words
—Addition or subtraction facts
—Multiplication problems
—Recognition of numerals
—Recognition of numeral words

Suggestions
—Laminate or cover with clear contact
—Attach bowl separately. This will allow bones to slip inside.
—Answer list can be coded by marking the back of bones with alphabet letters

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1980 Junior Mathematics Contest Results

sponsored by *The University of Waterloo*
and *Mutual Life Assurance Company*

The following listings are directly reprinted from The Junior Mathematics Contest - 1980 Results *booklet, and are based on a total enrolment of 26,142 competitors representing 1076 schools. There are some competitors whose scores, for various reasons (primarily mail delay), are not included. There are also some names which cannot be interpreted because coding instructions were not followed.*

ENROLMENT BY PROVINCES

Province	Number of Schools	Number of Students
Newfoundland	14	443
Nova Scotia	29	777
New Brunswick	22	632
Quebec	65	1684
Ontario	526	13522
Manitoba	94	1934
Saskatchewan	89	2133
Alberta	80	1302
British Columbia	157	3715
Total	1076	26142

PROVINCIAL TEAM HONOUR ROLL
PROVINCE OF ALBERTA

POSITION	SCHOOL	LOCATION	SCORE
1	HARRY AINLAY COMP. H.S.	EDMONTON	497.75
2	ARCHBISHOP MACDONALD H.S.	EDMONTON	489.00
3	DR. E. P. SCARLETT H.S.	CALGARY	473.00
4	M.E. LAZERTE COMP. H.S.	EDMONTON	469.75
5	JOHN DIEFENBAKER H.S.	CALGARY	450.25
6	LOUIS ST. LAURENT H.S.	EDMONTON	442.00
7	KATE ANDREWS H.S.	COALDALE	440.50
8	LORD BEAVERBROOK H.S.	CALGARY	440.25
9	QUEEN ELIZABETH H.S.	CALGARY	436.50
10	ST. FRANCIS H.S.	CALGARY	435.50
11	SIMON FRASER JR. H.S.	CALGARY	422.25
12	WINSTON CHURCHILL SCHOOL	LETHBRIDGE	414.75
13	PICTURE BUTTE H.S.	PICTURE BUTTE	414.00
14	EASTGLEN COMP. H.S.	EDMONTON	411.00
15	HUGH SUTHERLAND JR.-SR. H.S.	CARSTAIRS	407.50
16	WILLIAM ABERHART H.S.	CALGARY	406.50
17	TEMPO SCHOOL	EDMONTON	405.75
18	FORT MCMURRAY COMP. H.S.	FORT MCMURRAY	403.75
	ERLE RIVERS H.S.	MILK RIVER	403.75
20	STRATHCONA COMP. H.S.	EDMONTON	402.50
21	DIDSBURY HIGH SCHOOL	DIDSBURY	402.00
22	CRESTWOOD SCHOOL	EDMONTON	398.25
23	ST. JEROME'S SEPARATE SCHOOL	VERMILION	396.00
24	VAUXHALL H.S.	VAUXHALL	394.25

POSITION	SCHOOL	LOCATION	SCORE
25	MEDICINE HAT H.S.	MEDICINE HAT	394.00
26	FR. LACOMBE H.S.	CALGARY	389.25
27	EDMONTON CHRISTIAN HIGH SCHOOL	EDMONTON	384.00
28	BOWNESS H.S.	CALGARY	383.75
29	VERNON BARFORD SCHOOL	EDMONTON	381.50
30	ARDROSSAN JR.-SR. H.S.	ARDROSSAN	380.75
31	LINDSAY THURBER COMP. H.S.	RED DEER	375.75
32	WESTERN CANADA H.S.	CALGARY	374.25
33	PARKLAND COMP. H.S.	EDSON	374.00
34	BONNYVILLE CENTRALIZED H.S.	BONNYVILLE	373.50
	H. PANABAKER JR. H.S.	CALGARY	373.50
36	STRATHCONA-TWEEDSMOIR SCHOOL	OKOTOKS	371.25
37	ECKVILLE JUNIOR SR. H.S.	ECKVILLE	371.00
38	BRANTON JR. HIGH SCHOOL	CALGARY	365.00
39	MATTHEW HALTON COMMUNITY SCH.	PINCHER CREEK	362.75
40	JOHN WARE JR. HIGH SCHOOL	CALGARY	362.50

Scores	School Rank
360.50 - 331.50	41 - 50
328.50 - 308.75	51 - 60
307.50 - 276.50	61 - 70

TOP GRADE 9 TEAMS

1	SIMON FRASER JR. H.S.	CALGARY	282.00
2	CRESTWOOD SCHOOL	EDMONTON	245.50
3	VERNON BARFORD SCHOOL	EDMONTON	240.75
4	H. PANABAKER JR. H.S.	CALGARY	240.25
5	HIGHLANDS JR. H.S.	EDMONTON	233.00
6	BRANTON JR. HIGH SCHOOL	CALGARY	228.75
7	JOHN WARE JR. HIGH SCHOOL	CALGARY	225.75
8	LOUIS ST. LAURENT H.S.	EDMONTON	218.00
9	ECKVILLE JUNIOR SR. H.S.	ECKVILLE	213.75
10	NEW HAIRY SCHOOL	HAIRY BILL	213.00

TOP GRADE 10 TEAMS

1	ARCHBISHOP MACDONALD H.S.	EDMONTON	288.50
2	HARRY AINLAY COMP. H.S.	EDMONTON	284.00
3	M.E. LAZERTE COMP. H.S.	EDMONTON	267.50
4	EASTGLEN COMP. H.S.	EDMONTON	259.00
5	KATE ANDREWS H.S.	COALDALE	255.00
6	DR. E. P. SCARLETT H.S.	CALGARY	252.00
7	TEMPO SCHOOL	EDMONTON	249.50
8	WESTERN CANADA H.S.	CALGARY	239.00
9	VAUXHALL H.S.	VAUXHALL	238.25
10	JOHN DIEFENBAKER H.S.	CALGARY	237.75

PROVINCIAL HONOUR ROLL
PROVINCE OF ALBERTA

LAST NAME	FIRST NAME	SCHOOL	LOCATION	GR	AGE	SEX	SCORE
ANGLIN	GREGORY	DR. E. P. SCARLETT H.S.	CALGARY	11	16	M	116.00
LAMOUREUX	NEIL	ARCHBISHOP MACDONALD H.S.	EDMONTON	10	00	M	111.00
ROEHL	DEAN	HARRY AINLAY COMP. H.S.	EDMONTON	11	16	M	109.50
GIBBONS	LAWRENCE	PICTURE BUTTE H.S.	PICTURE BUTTE	11	16	M	104.75
BEATTIE	JAMES	SIMON FRASER JR. H.S.	CALGARY	9	14	M	103.75
OCHOTTA	EMIL	HARRY AINLAY COMP. H.S.	EDMONTON	10	15	M	102.50
KOZIK	DON	LOUIS ST. LAURENT H.S.	EDMONTON	11	16	M	102.00
ABRAHAM	ALAN	JOHN DIEFENBAKER H.S.	CALGARY	11	16	M	101.00
STRANG	LISA	HARRY AINLAY COMP. H.S.	EDMONTON	0	16	F	100.00
DOYLE	STEPHEN	ARCHBISHOP MACDONALD H.S.	EDMONTON	11	16	M	98.50
FOUNTAIN	TOM	DR. E. P. SCARLETT H.S.	CALGARY	11	16	M	98.50

LAST NAME	FIRST NAME	SCHOOL	LOCATION	GR	AGE	SEX	SCORE
WILTING	CARL	M.E. LAZERTE COMP. H.S.	EDMONTON	11	16	M	97.50
ALI	NAVED	ARCHBISHOP MACDONALD H.S.	EDMONTON	11	15	M	97.50
KENYON	JIM	M.E. LAZERTE COMP. H.S.	EDMONTON	0	00	X	96.00
WONG	TERRY	EASTGLEN COMP. H.S.	EDMONTON	10	15	M	96.00
ROHS	CHARLES	ST. FRANCIS H.S.	CALGARY	11	16	M	96.00
BOYLE	SUZANNE	ST. FRANCIS H.S.	CALGARY	11	16	F	95.75
CUDRAK	CONSTANCE	KATE ANDREWS H.S.	COALDALE	11	16	F	95.00
B RG	RANDAL	M.E. LAZERTE COMP. H.S.	EDMONTON	10	15	M	94.75
POON	MARY	HIGHLANDS JR. H.S.	EDMONTON	9	14	F	94.50
RODSETH	LYNN	PARKLAND COMP. H.S.	EDSON	11	15	F	94.50
MIHALCEAN	TINA	NEW HAIRY SCHOOL	HAIRY HILL	9	14	F	94.25
CHENG	EDWARD	HARRY AINLAY COMP. H.S.	EDMONTON	11	16	M	94.00
YUEN	AMY	M.E. LAZERTE COMP. H.S.	EDMONTON	10	15	F	93.25
GRAHAMU	WILLIAM	QUEEN ELIZABETH H.S.	CALGARY	11	17	M	93.25
CHAN	MICHAEL	JOHN DIEFENBAKER H.S.	CALGARY	11	15	M	93.00
WEST	DARRIN	FORESTBURG SCHOOL	FORESTBURG	11	16	M	92.25
TARDIFF	DALE	ST. MARY'S COMMUNITY SCHOOL	CALGARY	10	16	M	92.25
TSUKISHIMA	KENNETH	KATE ANDREWS H.S.	COALDALE	10	15	M	92.25
MARTIN	PATRICIA	MEDICINE HAT H.S.	MEDICINE HAT	11	16	F	92.00
DOUGLASS	DEBORAH	ST. FRANCIS H.S.	CALGARY	11	16	F	92.00
WEISNER	STEVE	LORD BEAVERBROOK H.S.	CALGARY	11	17	M	92.00
STEIN	ANDREAS	LORD BEAVERBROOK H.S.	CALGARY	11	16	M	91.75
LEIPER	THOMAS	HARRY AINLAY COMP. H.S.	EDMONTON	10	15	M	91.75
BARRY	TIMOTHY	ARCHBISHOP MACDONALD H.S.	EDMONTON	11	16	M	91.25
FRASER	KIM	LINDSAY THURBER COMP. H.S.	RED DEER	11	16	F	91.00
MEUNIER	JEANNE	ARCHBISHOP MACDONALD H.S.	EDMONTON	10	15	F	90.75
BARON	LORRENE	QUEEN ELIZABETH H.S.	CALGARY	10	15	F	90.75
OLAND	ARJAY	H. PANABAKER JR. H.S.	CALGARY	9	14	M	90.75
LANG	DARRLL	BOWNESS H.S.	CALGARY	10	15	M	90.75
CHEBIB	SOLOMON	BOWNESS H.S.	CALGARY	11	16	M	90.50
GOSSEN	LINDEN	KATE ANDREWS H.S.	COALDALE	11	16	M	90.50
LEE	PAUL	FR. LACOMBE H.S.	CALGARY	11	16	M	90.25
KIM	DIANA	SIMON FRASER JR. H.S.	CALGARY	9	14	F	90.00
BETT Y	ROBERT	STRATHCONA COMP. H.S.	EDMONTON	11	00	M	89.75
HILL	ROSS	HARRY AINLAY COMP. H.S.	EDMONTON	10	15	M	89.75
WOZNE Y	DAVID	LOUIS ST. LAURENT H.S.	EDMONTON	9	14	M	89.25
LIETZ	TA	DR. E. P. SCARLETT H.S.	CALGARY	11	16	M	89.25
UTH	BRETT	MATTHEW HALTON COMMUNITY SCH.	PINCHER CREEK	11	16	M	89.25
SCHMIDT	GORDON	HUGH SUTHERLAND JR.-SR. H.S.	CARSTAIRS	10	15	M	89.00
MCCREADY	JOHN	QUEEN ELIZABETH H.S.	CALGARY	11	16	M	89.00
CORRAINI	GERRY	JOHN DIEFENBAKER H.S.	CALGARY	11	16	M	88.75
FARNALLS	PAULA	WESTERN CANADA H.S.	CALGARY	10	15	F	88.25
KNOTT	MICHAEL	SIMON FRASER JR. H.S.	CALGARY	9	00	M	88.25
SP DY	CENDY	JASPER PLACE COMP. H.S.	EDMONTON	11	16	F	88.25
TRIKHA	ANIL	M.E. LAZERTE COMP. H.S.	EDMONTON	11	16	M	88.25
OL KSY	BARTON	WINSTON CHURCHILL SCHOOL	LETHBRIDGE	11	16	M	88.00
JORGENSON	MICHAEL	MEDICINE HAT H.S.	MEDICINE HAT	11	16	M	87.50
KONOJACKI	KAREN	ARDROSSAN JR.-SR. H.S.	ARDROSSAN	11	00	F	87.50
GURNETT	STEPHEN	EDMONTON CHRISTIAN HIGH SCHOOL	EDMONTON	11	15	M	87.50
VIRGINILLO	CAMERON	KATE ANDREWS H.S.	COALDALE	10	15	M	87.25
HAMILTON	GLENN	WINSTON CHURCHILL SCHOOL	LETHBRIDGE	11	00	M	87.25
BERGERON	DEBBIE	CREMONA SCHOOL	CREMONA	10	14	F	87.25
KOCH	DAVID	TEMPO SCHOOL	EDMONTON	10	15	M	87.00
SANON	AZHISH	HARRY AINLAY COMP. H.S.	EDMONTON	11	17	M	87.00
SMILH	PATRICK	ARCHBISHOP MACDONALD H.S.	EDMONTON	10	15	M	86.75
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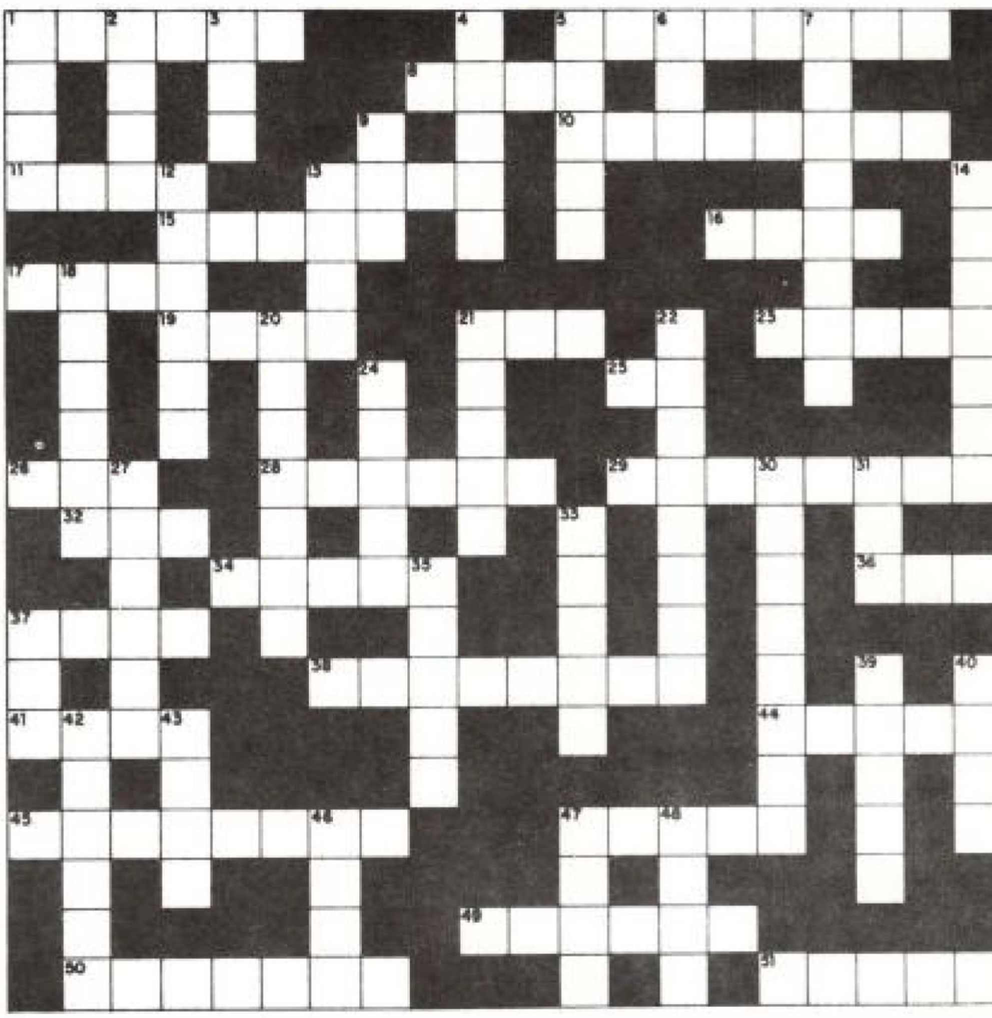
A Crossword Puzzle

by *Roger H. Wolters*
Pillager High School
Pillager, MN 56473

Answers on page 44.

Across

1. ILCERC
5. YGRTOEEM
8. RFUO
10. DTIINAOD
11. NEEV
13. EMNA
15. NOUIN
16. NELI
17. RGMA
19. SBEA
21. AYR
23. AEPNL
25. IP
26. MSU
28. TRCMEI
29. MDATIERE
32. TSE
34. LNAEG
36. NEO
37. XIAS
38. OTNQAEUI
41. EOCN
44. MIPER
45. GALDNIOA
47. TRYOF
49. CATRFO
50. MNALUER
51. MRIPS



Down

1. CEBU
2. TARE
3. EGL
4. NITOP
5. APRHG
6. DOD
7. ETNIRLAG
9. NTE
12. MNUERB
13. OEMD
14. GITRENE
18. SIDRUA
20. ETMSGEN
21. TORIA
22. IDISOINV
24. LTAOT
27. DNAIME
30. LYTMLIUP
31. OTW
33. GTIID
35. LEAQU
37. CRA
39. LRETI
40. RTME
42. IRINGO
43. DEGE
46. EARA
47. CEAF
48. OTOR

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Freddy the Frog and Number Line

by *Robert C. Branch*, Ph.D.
Gonzaga University
Spokane, Washington

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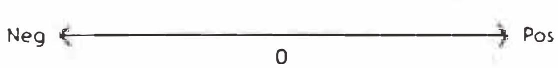
Among the most difficult concepts to teach for understanding from the elementary mathematics curriculum are the operations on negative numbers. A number of mnemonic devices have been developed ("Two like signs yield a positive," "Minus a minus is a plus," "Two unlike signs yield a negative," et cetera) but only help the learner get the right answer mechanically. They do not provide a framework for developing or understanding operations involving negative numbers. In this paper I will describe a model I have developed to help my students accept and understand the use of the basic operations on positive and negative numbers.

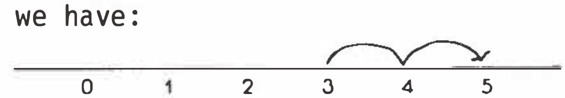
The lessons center around Freddy Frog, who has perfected his jumps in a manner not unlike Jonathan Livingston Seagull's flight. Freddy, also a perfectionist, has developed his jumping skills to the point that every jump, whether forward or backward, is uniform in length. Thus, when Freddy hops on the straight path that goes past his home, he will land on certain predictable spots. Consider this diagram of Fred's home and the path.



"O" marks the location of Freddy's home on the path. The arrows on the path indicate that it does not end, but continues to the right and left toward the nebulous villages of "Pos" and "Neg," respectively. If Freddy starts at home, faces Pos, and takes 3 hops forward, we could represent that action thus:

The numerals 1, 2, and 3 indicate where Freddy was at the end of the first, second, and third jumps, respectively.

If Freddy starts at 3, still facing Pos, and jumps two hops forward, we have:



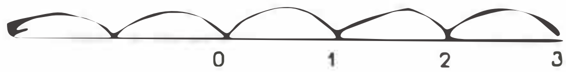
which leaves him the same place he would be if he had merely jumped forward 5 hops. Hence, 3 + 2 = 5.

Now consider the instance in which Freddy starts at 6, still facing Pos, and jumps 2 hops backwards:

Upon completion, he will be only 4 hops from home. Thus, hopping backwards is like subtracting (6 - 2 = 4).

It should be noted that the direction Freddy hops determines whether the operation to be performed is addition or subtraction. If he hops forward, we are adding; if backward, subtracting.

Consider the situation in which Freddy starts (still facing Pos) at 3 and jumps backward 5 hops. This can be diagrammed as follows:

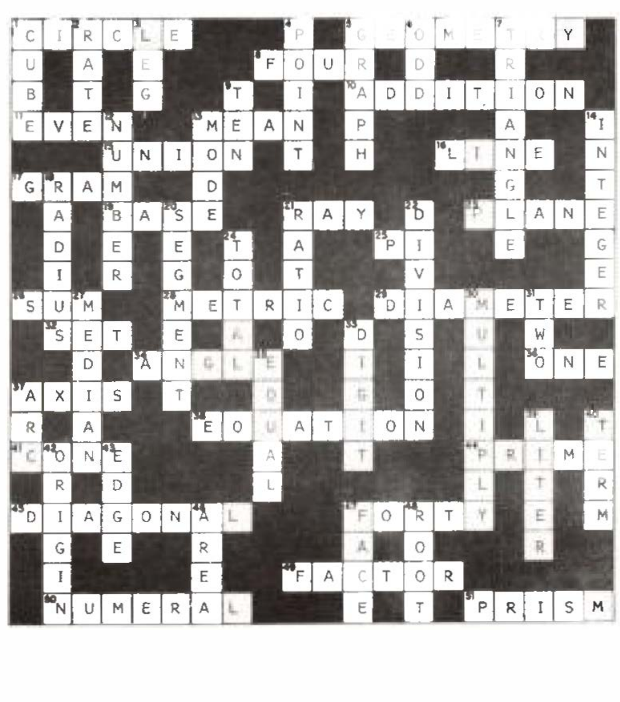


Freddy is now 2 hops from home, but on the opposite side as before. We could therefore call this new location "opposite of 2" and write it -2. The points on the path left of home (0) can now be easily established as opposites of those on the Pos side. It is thus possible to handle addition and subtraction of a positive number from any integral location on the path (number line) by merely having Freddy face Pos, start on the first term, and jump the number of hops indicated by the second term, in the direction dictated by the operation sign. For example, 3 + 6 means, "start at 3 facing Pos and jump forward 6," -3 - 6 means, "Start at -3 facing Pos and jump backward 6," and -3 + 6 means, "start at -3 facing Pos and jump forward 6." Before making an in-depth analysis of all that is entailed in this translation process, let us consider one more facet of the system.

Suppose Freddy starts at home, faces Neg and jumps forward four hops. Where will he finally land? Performing this oneself or with a model helps the pupil realize that Freddy will be at -4. Let us now translate Freddy's jumps into our mathematical symbols. He started at home and jumped forward, so we have 0 + , but what did we add? If we added 4, he would have landed at 4 on the Pos side. But he did not; he landed at -4. Therefore, jumping while facing Neg is represented as operating with a Neg number, or in this case, -4. The instance described represents: 0 + (-4) = -4 because Freddy started at home and jumped forward four jumps while facing Neg.

It should now be noted that the first term tells where Freddy starts, the operation sign (+ or -) tells the direction he will jump (forward or backward), the sign of the second term (+ or -) indicates the direction he is facing (Pos or Neg), and the value of the second term determines how many hops he will take. Those are all we need in order to add or subtract any pair of integers.

Answers to Crossword Puzzle
(on page 42)



Bonus Algebra Activities

reprinted from the Quebec Association of Mathematics Teachers journal.

1. How would you use parentheses to make these equations true?

   $72 \div 2 \times 4 \div 4 + 5 = 1$
   $72 \div 2 \times 4 \div 4 + 5 = 16$
   $72 \div 2 \times 4 \div 4 + 5 = 41$
   $72 \div 2 \times 4 \div 4 + 5 = 10\frac{2}{7}$
   $72 \div 2 \times 4 \div 4 + 5 = 6$

2. What value does the following expression approach as $x \to \infty +$

   $\log_3 (6x - 5) - \log_3 (2x + 1)$

3. If $f(x) = \frac{x+1}{x-1}$, evaluate $f\left(\frac{f(f(f(x))) + 1}{f(f(f(x))) - 1}\right)$

4. Consider the following method for evaluating

$$\sqrt{20 + \sqrt{20 + \sqrt{20 + \sqrt{20 + \cdots}}}}$$

let $x = \sqrt{20 + \sqrt{20 + \sqrt{20 + \sqrt{20 + \cdots}}}}$

then $x = \sqrt{20 + x}$

$x^2 = 20 + x$
$x^2 - x - 20 = 0$
$(x - 5)(x + 4) = 0$
$x = 5$ or -4, but $x \neq -4 \therefore x = 5$.

Now you try a similar method for

a) $1 + \cfrac{2}{1 + \cfrac{2}{1 + \cfrac{2}{1 + \cfrac{2}{\circ\circ\circ}}}}$

b) and to evaluate x given:

$$x^{x^{x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}}} = 2 \text{ and } x > 0$$

ANSWERS: 2.) Expression $\to \log_3 3 = 1$

3.) x

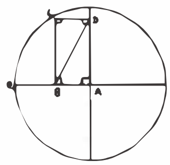
4.) a) 2 b) $x = \sqrt{2}$

Reprinted from Vector, *Newsletter/Journal of the British Columbia Association of Mathematics Teachers.*

I Gottcha!

contributed by *Jay Caturay*
Prince Rupert Secondary School, Prince Rupert, British Columbia

I wonder how many of our readers can give the answers to the following problems within 10 seconds for each question?

1. What is 32% of 25?
2. 3/8 + 2/3 - 1/24 =
3. Thirty divided by a half.
4. 

Given:

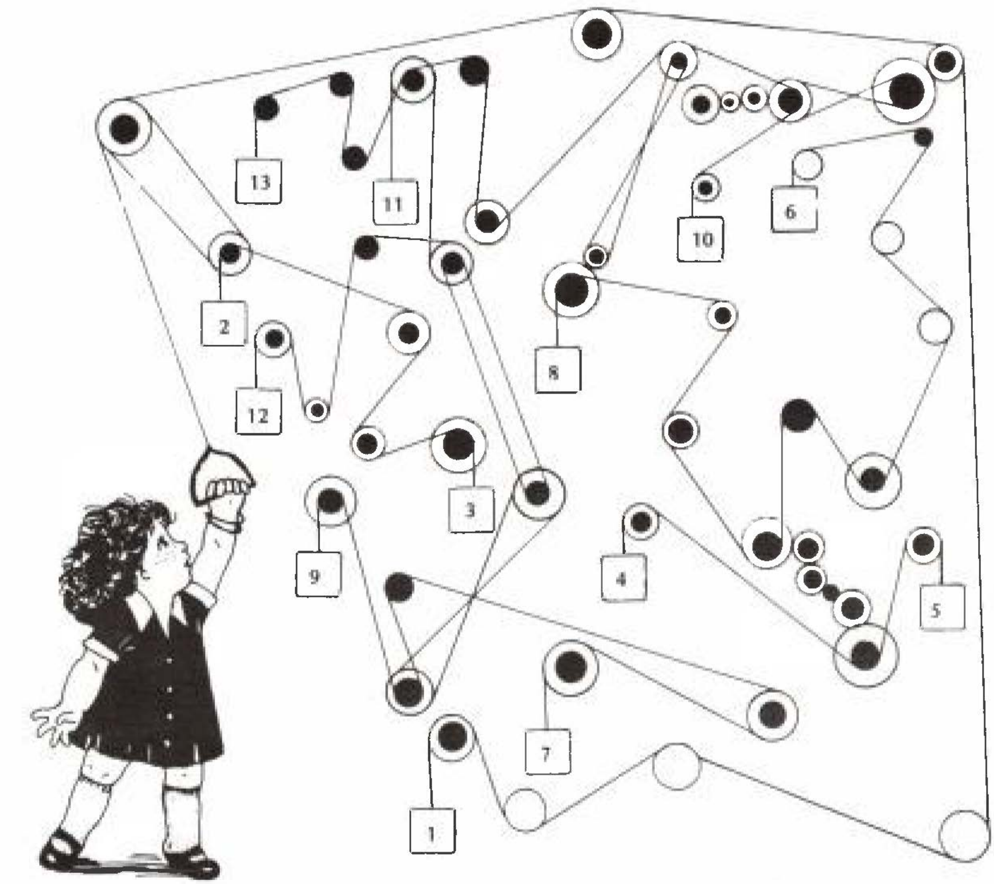
radius $\overline{AE} = 10$ cm

$\overline{BE} = 2$ cm

Find $\overline{BD}$

Answers:

1. 8 $\quad 32/100 \times 25 = 32 \times 25/100 = 32 \times 1/4 = 8$
2. 1 $\quad a/b + c/d = ad + bc/bd$
   $3/8 + 2/3 = 9 + 16/24 = 25/24$
3. 60, not 15
   $\frac{30}{1/2} = \frac{30 \times 2/1}{1/2 \times 2/1} = 60$
4. $\overline{BD} = 10$ cm $\quad$ Draw AC $\quad \overline{AE} = \overline{AC}$, $\overline{AC} = \overline{BD}$
   Hence $\overline{AE} = \overline{BD} = 10$ cm



PULLEY PROBLEM

When Josie pulls down on the lever, the whole system of loads, belts, and pulleys will begin to move. Which loads will go up, and which loads will come down?

Remember: wheels belted to each other revolve in the same direction, unless the belt crosses itself. In that case, the wheels revolve in opposite directions. Wheels in contact with each other on their outer rims will rotate in opposite directions.

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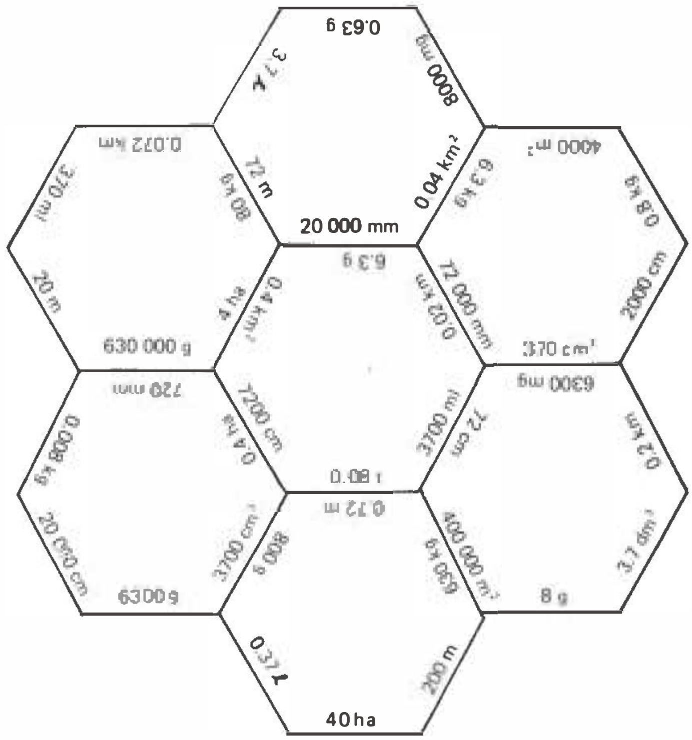
Hexametric

by *Dr. H. Don Allen*, Nova Scotia Teachers' College

Reprinted from Saskatchewan Mathematics Teachers' Society.

Cut out the seven hexagons. Rearrange them in the same pattern...but have touching edges name the same measure, i.e., 1 cm is equivalent to 10 mm.



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101-3rd Avenue North
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RECENT METRIC PUBLICATIONS FOR THE TEACHER

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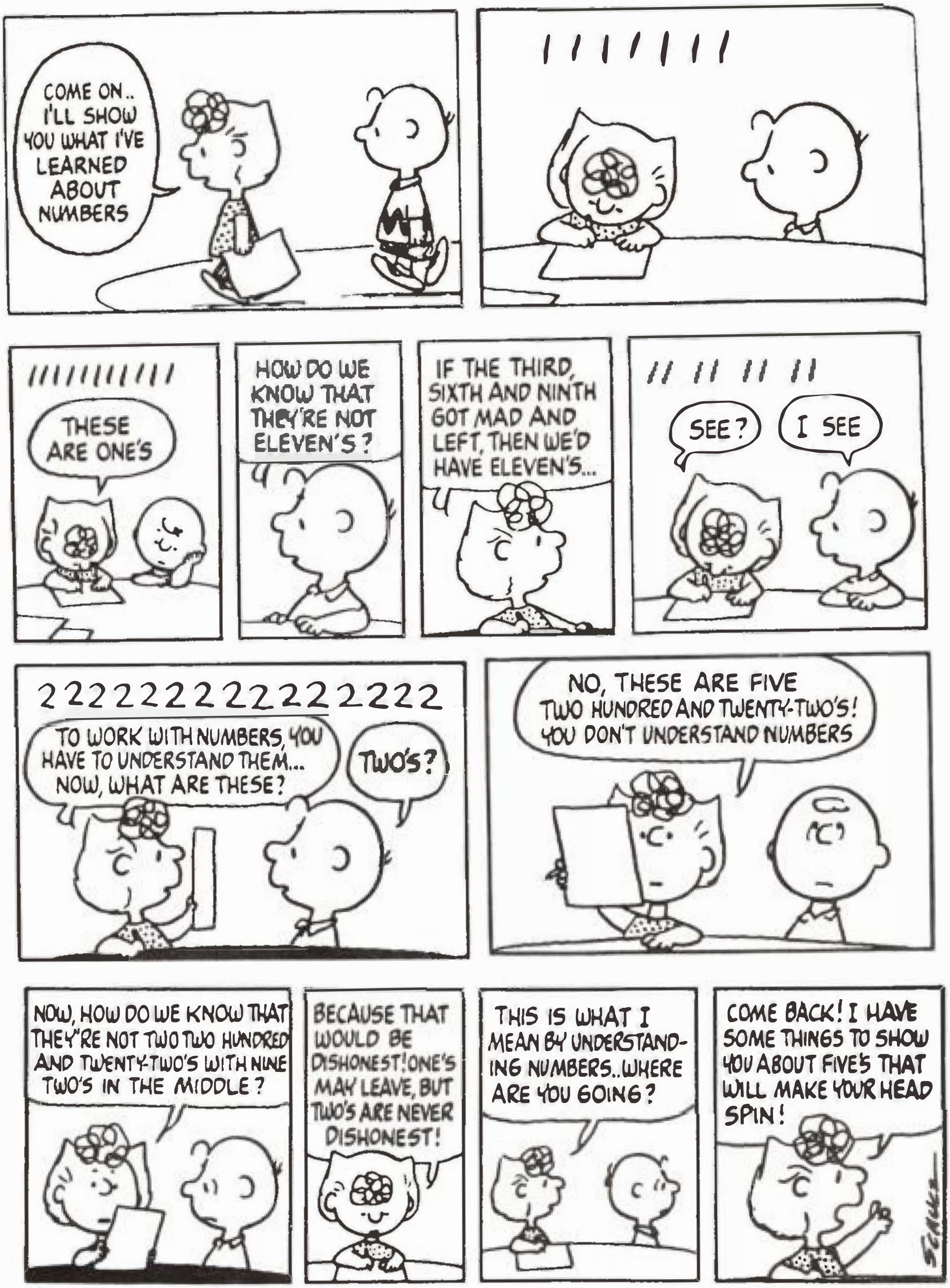

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MATHEMATICS COUNCIL, ATA

Twentieth Annual Conference and Business Meeting

November 7 and 8, 1980
Capri Hotel, Red Deer

FEATURING:

Friday

- Opening Session, 8:30-9:30 p.m.
  Robert Reys
- Social and Commercial Displays

Saturday

- 12 Sessions
- 24 Workshops
- Luncheon Address by *Robert Reys*

TOPICS INCLUDE:

- micro-computers
- activity centers
- math "tubs"
- paper folding
- prism project
- gifted programs
- testing
- NCTM recommendations

REGISTRATION FEE:

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Watch for registration forms in *The ATA News* and the September 1980 school mailing.