Selected References

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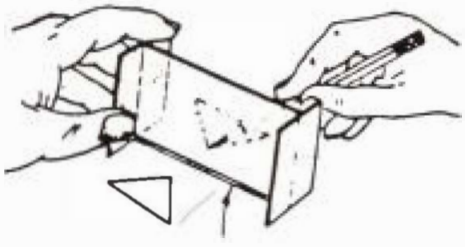
Mira Math for Grades 7-12

Reprinted from The Math Post.

Junior High and High School Teachers: Have you tried these activities with miras? The Mathematics Team has class sets of miras which can be borrowed for one-month intervals.

1. Investigate *images* of 3-D objects and of plane figures. For plane figures, draw the image and the reflection line. Practise with a variety of figures (points, segments, angles, triangles). Conjecture on the position, size, shape and sense of the image. Point to a part of the object and observe the corresponding part of the image. Draw the image when all or part of the object is behind the mira. Define reflection. Grades 7-10.

2. Conjecture on *properties of reflection.* Test each of these in a number of cases. Use the mira to draw the image and a variety of ways to test the property (paper folding,



cut-outs, acetate sheets, measuring instruments). Grades 7-10.

3. Investigate *line-symmetry* in a variety of figures (non-geometric and geometric), establish the meaning of line symmetry, relate plane symmetry in a 3-D object with line symmetry in a 2-D drawing of it.

Practise drawing lines of symmetry and creating figures and designs that have line symmetry. Grades 7-10.

4. Investigate the *line symmetries of various geometric figures:* triangles, quadrilaterals, regular polygons, others. Investigate properties associated with two or more lines of symmetry, and the images in a pair of hinged mirrors. Vary the angle, use the mira to draw diagrams to represent what you see, kaleidoscope, set the mirrors at right angles and look straight into the hinge. Examine what you see. Grades 8-12.

5. *Basic constructions related to line symmetry:* perpendicular lines, line through a point perpendicular to a line, perpendicular bisector, angle bisector, arc bisector, centre of a circle, test for parallel lines, draw parallel lines, mid-line et cetera. Application of these techniques to the study of altitudes, perpendicular bisectors, medians, angle bisectors of triangles. Investigate other construction problems. Draw various geometric figures: isosceles triangle, rectangle, rhombus, regular octagon. Grades 7-10.

6. Relate the mira to *paper folding* activities. Grades 7-12.

7. *Symmetry in the integer line.* Grades 7 and 8.

8. *Symmetry* in the *addition matrix* and the *multiplication matrix.* Grades 7 and 8.

9. *Reflection-congruent figures:* meaning of congruence, corresponding parts, draw the reflection line of two reflection-congruent figures, trace two congruent figures (points, circles, line segments, arcs, angles, triangles) and find the least number of reflections to map the one onto the other, direct congruence and opposite congruence, definition of congruence. Draw two triangles given SSS, SSA, SAS, ASA, SAA, AAA, HS and HA, and test for congruence, sufficient conditions for congruence. Grades 7-10.

10. *Properties of geometric figures:* Many of the properties stated in theorems and deductions of Euclidean geometry may be discovered (conjectured) by constructing the figures with a mira and/or testing with the mira for example, opposite angles of two intersecting lines, isosceles triangle theorem, properties of a rhombus, rectangle, parallelogram, equal chords of a circle. Grades 7-10.

11. *Applications to drafting,* 2-D representations of 3-D objects - drawing lines that meet at a vanishing point off the page. Grade 10 or later.

12. A formal study of *Euclidean geometry* can be based on reflections: reflection is defined, a few conjectures from number 2 are assumed as postulates and others are proved, then theorems and deductions follow. This approach is taken in *Geometry, A Transformation Approach,* Coxford and Usiskin, Laidlaw Publishing (Doubleday in Canada). The mira can be used to discover many properties, demonstrate many of the proofs and theorems, draw diagrams and stimulate ideas for proofs. Grade 10 or later.

13. *Composites of reflections.* The mira can be used to discover that rotation and translation are composites of two reflections in intersecting and parallel lines respectively. Glide reflection is the composite of three reflections. Given two congruent figures, the isometry by which they are related can be determined with the mira. Properties relating the reflections of these isometries can be discovered with the mira. Grade 8 or later.

14. Images under *dilations, stretches,* and *shears* can be drawn using the mira; some of their properties may then be conjectured. Grade 10 or later.

15. *Conics*. The envelope of a parabola, ellipse, and hyperbola can be drawn with the mira. These conics can be plotted point by point to specified dimensions revealing their locus laws. Tangents from a point, at a point, and with a given direction can be drawn accurately. Reflection and other properties of conics can be discovered. Common tangents can be drawn to any pair of conics (including circles). Grade 11 or later.

16. *Graphing on plain paper*. Draw the axes, scale the x-axis, reflect this onto the y-axis. Plot (a,b) by reflecting the y-axis perpendicular to the x-axis through (a,0). Investigate the image of (a,b) under reflection in y=0, x=0, y=x, y=-x, and the origin (composite of two reflections). Use the line symmetry of a function to assist in plotting it. Given functions f and g, using the mira graph f^{-1}, $|f|$, 1/f, f+g, f-g, and f∘g. Grades 8-12.

17. *Vectors*. Demonstrate a (geometric) vector as an equivalence class of directed line segments. Sum two or more vectors geometrically and by components. Solve force, navigation, and velocity problems geometrically. Grades 10-12.

18. *Trigonometry*. Plot sin, cos, and tan using a point-by-point technique. Graph their inverses and reciprocals. Graph the image of sin or cos under a translation. Demonstrate or discover identities related to reflection, for example, $\cos\theta=\cos(-\theta)$, $\sin(\frac{\pi}{2}-\theta)=\sin(\frac{\pi}{2}+\theta)$, $\sin(\frac{\pi}{2}-\theta)=\cos\theta$. For Grades 11 and 12.

19. *Optics*. Investigate i=r, a divergent pencil reflected from a mirror, a parallel beam reflected from a mirror, reflection of parallel rays striking a concave or convex spherical mirror (aberration). Draw diagrams for concave spherical mirror, convex spherical mirror, convex lenses, concave lenses. Investigate reflection of parallel rays from a concave and convex parabolic mirror. Grade 11 or later.

READING MATH

(Junior/Senior High)

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