PERM FIL
File #12 H.M.L.

12

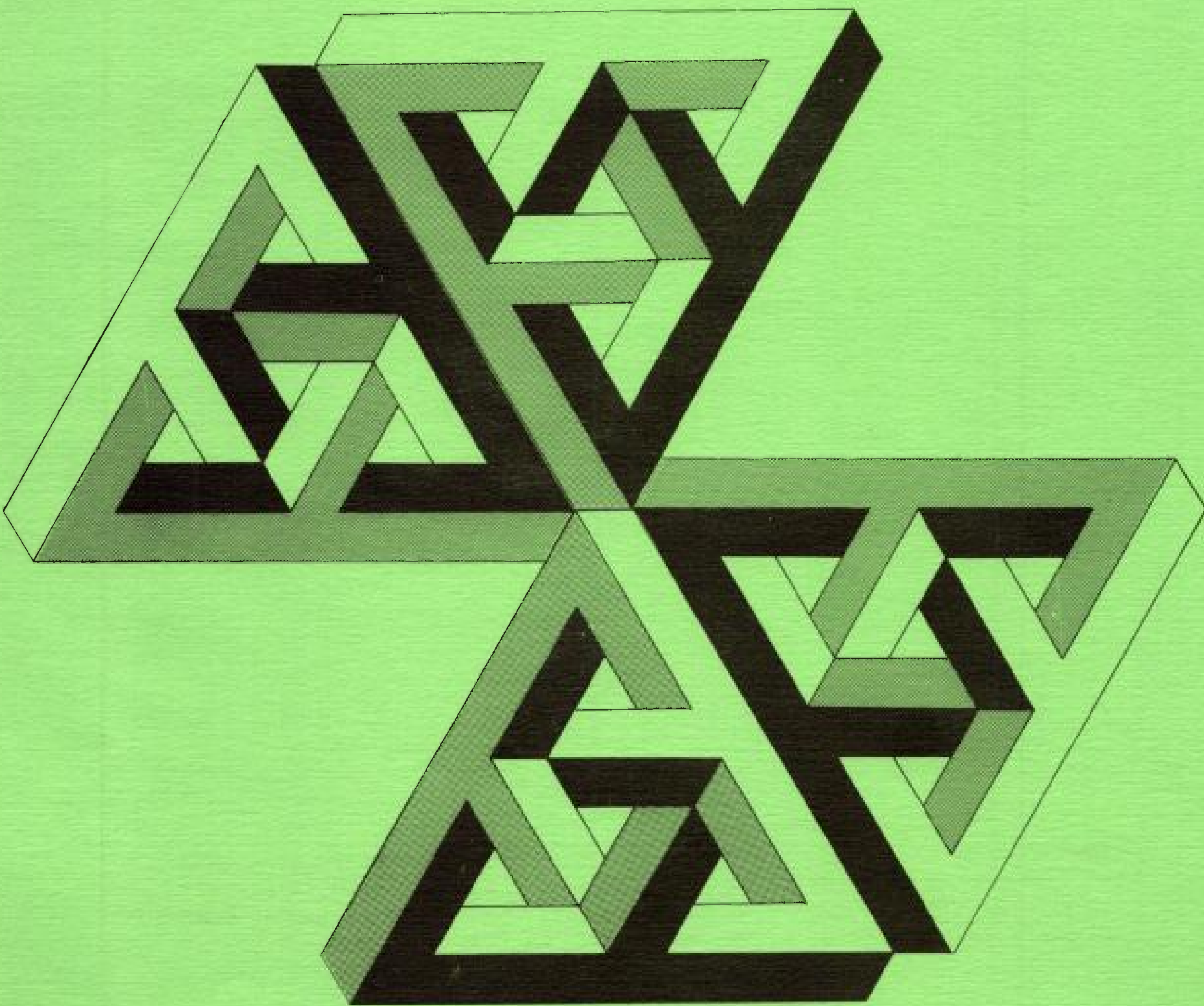
ISSN 0319 - 8367

delta-k

THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

Volume XX, Number 2

February 1981



delta-k

THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

Volume XX, Number 2 — *February 1981*

Contents

EDPRESS

delta-K is published by The Alberta Teachers' Association for the Mathematics Council. Editor of this issue: Ed Carriger, R.R.1, Site 2, Box 4, Bluffton T0C 0M0. Editorial and Production Services: Communications staff, ATA. Opinions of writers are not necessarily those of either the Mathematics Council or The Alberta Teachers' Association. Please address all correspondence regarding this publication to the editor. *delta-K* is indexed in the Canadian Education Index.

Plotting the Polygonal Numbers

by *Bonnie H. Litwiller* and *David R. Duncan*
Professors of Mathematics
University of Northern Iowa
Cedar Falls, Iowa

Many mathematics teachers are familiar with the polygonal numbers. For example, Figure 1 displays the first five triangular numbers because the dots that they number can be arranged to form equilateral triangles.

Figures 2 and 3 similarly picture the first five square and pentagonal numbers.

Polygonal numbers of higher order can also be pictured. Figure 4 displays the first 10 numbers of eight different orders of polygonal numbers.

Figure 1

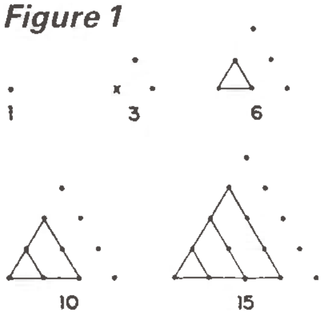


Figure 2

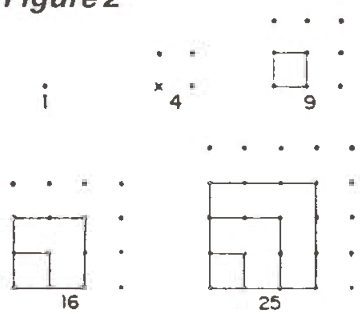


Figure 3

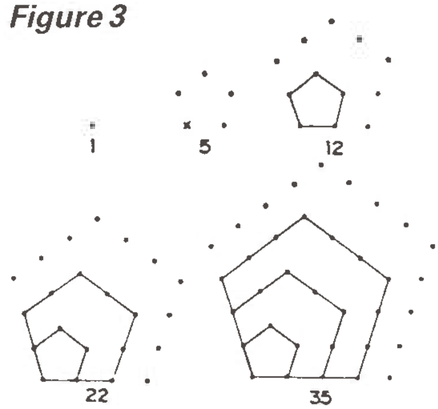


Figure 4

Naturals	1	2	3	4	5	6	7	8	9	10	...
Triangular	1	3	6	10	15	21	28	36	45	55	...
Squares	1	4	9	16	25	36	49	64	81	100	...
Pentagonal	1	5	12	22	35	51	70	92	117	145	...
Hexagonal	1	6	15	28	45	66	91	120	153	190	...
Heptagonal	1	7	18	34	55	81	112	148	189	235	...
Octagonal	1	8	21	40	65	96	133	176	225	280	...
Nonagonal	1	9	24	46	75	111	154	204	261	325	...
Decagonal	1	10	27	52	85	126	175	232	297	370	...

The square numbers are easy to find on the interior of the multiplication table. They lie on the major diagonal and are shown in Diagram 1. Beginning with the first square number, successive square numbers are generated by proceeding to the right one step and then down one step.

DIAGRAM 1: Square Numbers

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57
0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114
0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133
0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152
0	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171
0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190
0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209
0	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228
0	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247
0	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266
0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285
0	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304
0	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323
0	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342
0	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361

The hexagonal numbers are shown in Diagram 2. Beginning with the first hexagonal number, successive hexagonals are generated by proceeding to the right one step and then down two steps.

DIAGRAM 2: Hexagonal Numbers

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57
0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114
0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133
0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152
0	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171
0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190
0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209
0	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228
0	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247
0	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266
0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285
0	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304
0	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323
0	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342
0	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361

Diagrams 3 and 4 depict the octagonal numbers and the decagonal numbers. The octagonals are generated by moving to the right one step and down three, while the decagonals are generated by moving to the right one step and down four.

DIAGRAM 3: Octagonal Numbers

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57
0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	103	114
0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133
0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152
0	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171
0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190
0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209
0	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228
0	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247
0	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266
0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285
0	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304
0	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323
0	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342
0	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361

DIAGRAM 4: Decagonal Numbers

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57
0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114
0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133
0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152
0	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171
0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190
0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209
0	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228
0	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247
0	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266
0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285
0	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304
0	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323
0	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342
0	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361

Diagrams 1 through 4 contain polygonal numbers of even order. Can the dodecagonal (12 sides) numbers be generated by moving one step to the right and down five?

Polygonal numbers of odd order can also be located on the interior of the multiplication table. Diagrams 5 and 6 depict two methods of generating the triangular numbers. The triangular numbers in Diagram 5 are generated by moving to the right two steps, circling the triangular number, and then moving down one step and circling the triangular number. This right-two-circle, down-one-circle process is repeated indefinitely.

DIAGRAM 5: Triangular Numbers

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57
0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114
0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133
0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152
0	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171
0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190
0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209
0	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228
0	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247
0	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266
0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285
0	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304
0	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323
0	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342
0	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361

Diagram 6 shows another method of generating the triangulars by addition. The sums of the pairs of circled numbers are, consecutively, 1, 3, 6, 10, ..., the triangular numbers.

DIAGRAM 6: Triangular Numbers

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57
0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114
0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133
0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152
0	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171
0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190
0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209
0	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228
0	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247
0	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266
0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285
0	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304
0	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323
0	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342
0	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361

Diagram 7 displays the pentagonal numbers. They are also generated by adding the pairs of circled numbers. In contrast to the triangular numbers, the pairs do not overlap.

DIAGRAM 7: Pentagonal Numbers

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57
0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114
0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133
0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152
0	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171
0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190
0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209
0	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228
0	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247
0	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266
0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285
0	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304
0	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323
0	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342
0	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361

Diagrams 8 and 9 show the heptagonal and nonagonal numbers. Again, they are generated by adding the pairs of numbers that are circled.

DIAGRAM 8: Heptagonal Numbers

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57
0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114
0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133
0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152
0	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171
0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190
0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209
0	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228
0	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247
0	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	256
0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285
0	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304
0	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323
0	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342
0	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361

DIAGRAM 9: Nonagonal Numbers

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57
0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114
0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133
0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152
0	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171
0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190
0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209
0	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228
0	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247
0	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266
0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285
0	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304
0	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323
0	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342
0	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361

Observe that increasing odd orders of polygonal numbers result from increasing the number of spaces between the pairs of circled numbers to be added. By continuing the patterns used in Diagrams 6 through 9, generate polygonal numbers of order 11.

The readers and their students are encouraged to conjecture and to investigate further patterns concerning polygonal numbers.

? ? ? PROBLEM CORNER ? ? ?

edited by *William J. Bruce* and *Roy Sinclair*
University of Alberta
Edmonton, Alberta

Problems suggested here are aimed at students of both the junior and senior high schools of Alberta. Solutions are solicited and a selection will be made for publication in the next issue of *delta-K*. Names of participants will be included. All solutions must be received (preferably in typewritten form) within 30 days of publication of the problem in *delta-K*.

Mail solutions to: Dr. Roy Sinclair
Department of Mathematics
University of Alberta
Edmonton, Alberta T6G 2G1

* * * * * * * * * * * * * * * * *

Problem 2:

(submitted by William J. Bruce, University of Alberta, and reprinted from the June 1980 issue of delta-K.*)*

Clearly $1 = 1^2$ is a perfect square, but 11 and 111 are not. Consider all numbers $11111 \cdots 1 = S$, in which all digits are unity, and prove or disprove that, except for $s = 1$, no such number is a perfect square.*

*EXTENSION (Proof not to be submitted for publication.)

The theorem is true for any number that can be written in the form $100m + 10 + 1$ (m a positive integer). Also, true for $100m + k + 1$ when k is *not* divisible by 4.

Problem 3:

(submitted by William J. Bruce, University of Alberta.)

Point P is located in a rectangular region such that its distances from three of the vertices of the rectangle are given by "a ft.," "b ft.," and "c ft." Let "d ft." be the unknown distance to the fourth vertex and find a relation among the four distances, "a," "b," "c," and "d," so that whenever any three are known, the fourth can be computed.

Abstract of
Problem-Solving is a Reading Problem

by *Jeffrey Klein*
Director of Mathematics
Manalapan-Englishtown Regional School District
Englishtown, New Jersey

This abstract was presented by Dr. Klein at the 58th annual meeting of the National Council of Teachers of Mathematics, held in Seattle, Washington, April 16-19, 1980.

Perceiving Symbols

Perceiving is defined as recognizing and pronouncing. The term "symbols" refers to words essential to mathematical reading, as well as other symbols such as + or =. Thus, perceiving symbols involves *recognition* and *pronunciation*, but it does not involve comprehension.

Paper and Pencil Exercise

Directions: This exercise is designed as a race against yourself in spotting key words quickly and accurately. Look at the first word in each row, then circle that word every time it appears in that row.

1.	Average	Average	Averaged	Average	Average
2.	Minus	Mean	Minus	Minute	Minimum
3.	Add	Added	Add	Add	Adds

Attaching Literal Meaning

Once the reader has recognized and pronounced the symbols essential to a particular task, he is ready to attach denotative or literal meaning to these symbols. Comprehension at this level depends on two basic elements: *symbol meaning* (vocabulary) and *symbol order* (phrases, sentences, et cetera).

VOCABULARY

1. \- - C - - - -
2. \- I - - - -
3. \- - R - - - - - - - -
4. C - - - - - - - - - -
5. \- - - - - - L -
6. \- - - E

Hints:

1. the numbers to be multiplied
2. the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
3. do the operation within these first
4. the sum is the same in either order
5. the symbols used for the numbers
6. used as a grouping number

Analyzing Relationships

This requires a student to grasp and identify important relationships (stated and unstated) among several literally stated facts or ideas and state them in the form of inferences, generalizations, conclusions, or equations. *Caution:* It is easy for the teacher to assume that a simple relationship is obvious.

Example: Circle two words on each line which have something in common, so that when you have done so, the remaining three will also have something in common.

1. INCH PINT YARD GALLON MILE
2. LITRE FOOT GRAM ROD QUART
3. OUNCE GALLON CENTILITRE CUP TON
4. MILLIGRAM GRAM MEGAGRAM DECIGRAM MICROGRAM

Solving Word Problems

Dependent on all three levels of the reading process previously described, the solution of word problems is the most sophisticated reading task in mathematics. A major goal of instruction in mathematics is to develop readers who can solve problems successfully and independently.

Example: Jack had 50¢. He bought four postcards at 5¢ each. How many postcards at 10¢ each could he buy with the money he had left.

A. In this problem you are trying to find: (check one)

1. the number of 10¢ cards Jack can buy,
2. the price of the cards Jack bought,
3. the money Jack had at the beginning.

B. As you read the problem, write the correct amount from Column B before each space in Column A.

Column A	*Column B*
1. the amount of money Jack had at the beginning: __________	30¢
2. the amount he paid for four cards at 5¢ each: __________	50¢
3. the amount he had left after he bought four cards at 5¢ each: __________	20¢

You Can't Keep a Secret

by *William J. Bruce*
Department of Mathematics
University of Alberta
Edmonton, Alberta

The following is an updated version of a very simple arithmetic fun game suitable for elementary grade students:

1. Select an integer between 1 and 10.
2. Double.
3. Add 5.
4. Multiply by 50.
5. Add 1730.
6. Subtract the year in which you were born ($1880 < \text{year} \leq 1980$).
7. State the result obtained.

Key:

The first digit in your answer is the number selected. The remaining digits give your age in the year 1980.

Proof:

Let n be an integer such that $2 \leq n \leq 9$, x be an integer such that $-20 < x \leq 80$, so that $1900 + x$ will be the year of birth, and let y be your age in 1980.

Apply the required steps to obtain $50(2n + 5) + 1730 - (1900 + x) = 100n + y$, which simplifies to yield $x + y = 80$, which is independent of the choice of n.

Since $-20 < x \leq 80$, it follows that $0 \leq y < 100$. Hence, the procedure will always work as long as the year of birth is restricted as stated.

Note:

For years 1880 and before, the amount added in Step 5 has to be *reduced*. In this case, the procedure will not work up to 1980, so a corresponding *reduction* in the number 1980 will be necessary.

After 1980, an update is accomplished by *increasing* the number in Step 5.

ELEMENTARY ACTIVITIES

Place Value

TENS AND ONES (Grade 1)

This is very important as an introduction before any written work is done. Children are given containers of counters such as bread tags, shells, bottle caps, paper clips or pennies, and several ten cards or egg cartons. Children fill ten cards and record how many tens and how many ones are left over.

Ten Card:

HUNDRED BLOCK (Grade 1)

Cut a grid of numerals 1-100, printed on card, into small squares. Children reassemble the grid.

Measurement

PUMPKIN ACTIVITIES (Grade 1)

Measure the weight of pumpkin and seeds, the length or circumference of pumpkin and seeds end to end, and count the seeds by tens.

Money

VALUE PICTURES

Use pictures made of shapes. Give each shape a monetary or numerical value: 5¢, 10¢, 25¢.

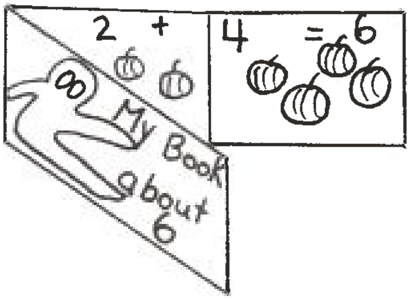
Variation: Give the letters of your name these values: vowels - 25¢, A-M - 5¢, N-Z - 10¢.

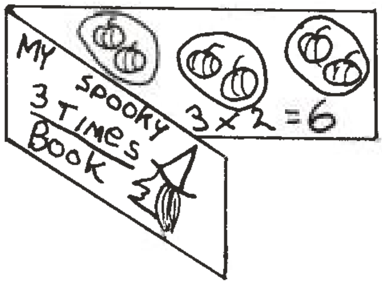
Number Facts

NUMBER FACTS BOOK (Grade 1) and
TIMES FACTS BOOK (Grades 2 and 3)

These can be used either as a discovery activity when introducing a new number or times table, or as a review exercise once all facts are learned.

For example:

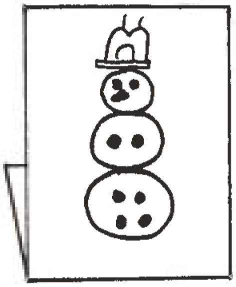


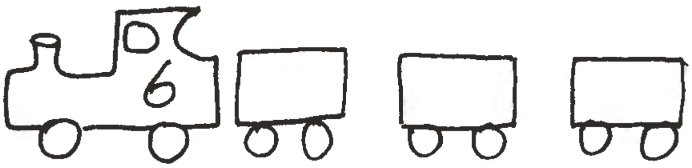


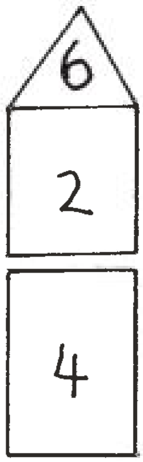
SNOWMEN, TRAINS, ROCKETS (Grade 1)

Children find pieces to form a number fact. If pieces are laminated first, numerals are erasable.

pocket for pieces



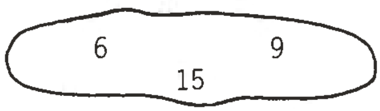




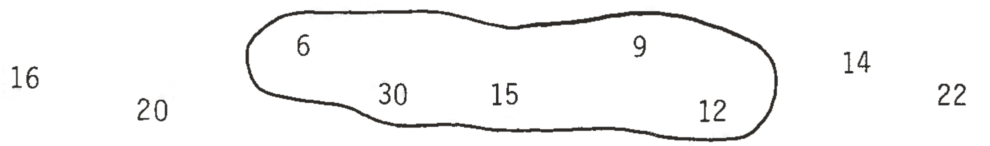
FACTORS (Grades 2 and 3)

This activity must be preceded by extensive work on counting by twos, fives, tens, et cetera, and completing number patterns.

Draw cloud on blackboard.
Print several numerals in it.

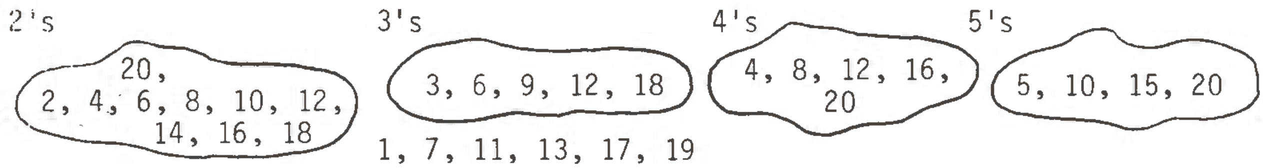


Ask children if they can think of other numerals that belong in the cloud. As the answers are given, print them either inside or outside the cloud.



Children study the numerals inside and outside to discover the secret pattern. The correct answer form should be: ""3 is a factor of all these numerals."

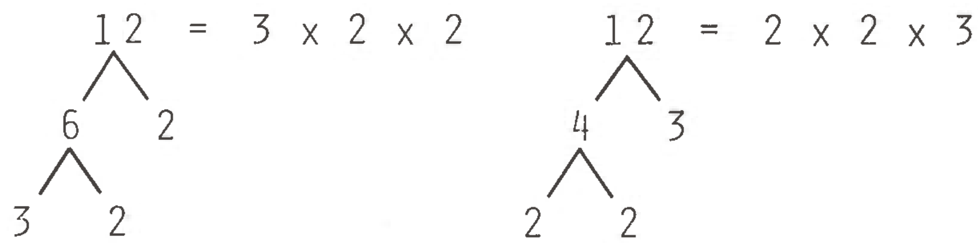
Draw several clouds on blackboard and label with appropriate factors. Pick several children to come to the blackboard, one to be responsible for each cloud and one having no cloud. As numbers are called out, each child puts number in correct cloud. Those not belonging in any cloud are put outside.



As children become familiar with these procedures, they will notice that such numbers as 6 belong in several clouds.

FACTOR TREES

Show the children how to make factor trees. Start with an easy number (like 12) and ask for factors. Then, ask for factors of the factors. Write it like this:



Make sure the children notice that it doesn't matter how you first factor 12; you always end up with the same factors.

BLOCK IT

Topic:
Whole Number Combinations

Grade Level:
5 - 9

Time:
30 - 45 minutes

Number of Players:
2 - 4

Materials Needed:
Game Board
3 Spinners or 3 Dice
10 or 12 colored Markers for each player

Object:
To get four in a row, horizontally, vertically, or diagonally, while trying to block opponents from getting four in a row.

Rules:

1. Each player spins one spinner, highest spin goes first.
2. Each play consists of spinning all three spinners (or rolling all three dice). The player combines his three numbers using any operation to arrive at a number.
3. The player then places his marker on that square to cover that number.
4. Each player takes turns spinning and combining three numbers to cover numbers on the board.
5. Player tries to get four of his markers in a row, while trying to block his opponent. If questioned, players must justify their number.

Variations:

The use of powers may be used in addition to the four basic operations. Perhaps some students will be more challenged by trying to get five markers in a row.

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	44	45	48	50	54	55
60	64	66	72	75	80	90	96
100	108	120	125	144	150	180	216

MARTINETTI

Topic:
Addition

Grade Level:
1 - Adult

Time:
15 minutes

Number of Players:
2 - 4

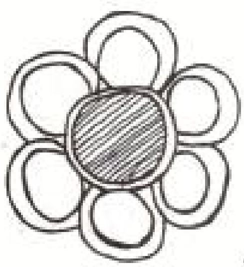
Materials Needed:
Game Board
3 Dice
Game Piece for each player

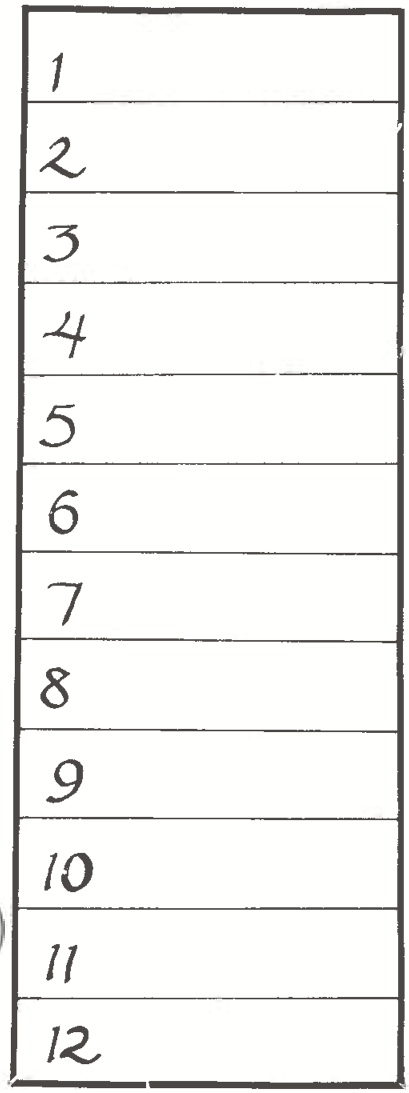
Object:
Each player tries to be the first to move his counter, in accordance with the rolls of the dice, from 1 to 12 and back again.

Rules:

1. High roller goes first. Each player, in turn, rolls the three dice once. Player's throw must contain a "1" before he can put his marker in the box so numbered.
2. After his throw, the dice are passed to the next player, and so on.
3. Once a player has thrown a "1," he must try for a "2." He can make a "2" by throwing either a "2" or two "1s." He continues to move his marker in this way from box to box.
4. Some throws may enable him to move through more than one box on a single throw. For example, a throw of 1, 2, and 3 would not only take him through the first three boxes, but on through the fourth (1 + 3 = 4), the fifth (2 + 3 = 5), and the sixth (1 + 2 + 3 = 6).
5. Players should watch the throws of their opponents. If a player throws a number he needs, but overlooks and does not use that number, the opponent should wait until the dice are passed, explain the move, and then move his own marker one space forward.

Variation:
Use the face value of the dice, but permit any combination of operations - multiplication, division, addition, or subtraction - to make the numbers.





Studying Puddles

Reprinted from The Math Post

1. If it was a light rain and the sun is out, time how long it takes various size puddles to shrink or disappear.

2. What is the rate of shrinkage during a class period?

3. Count the number of puddles in various areas. Make a graph of the results. What conclusions can you draw?

4. If it was a hard rain, measure the width, depth, and length of a variety of puddles. Keep records throughout the day and into the next. Graph the results.

Rocks

Reprinted from The Math Post

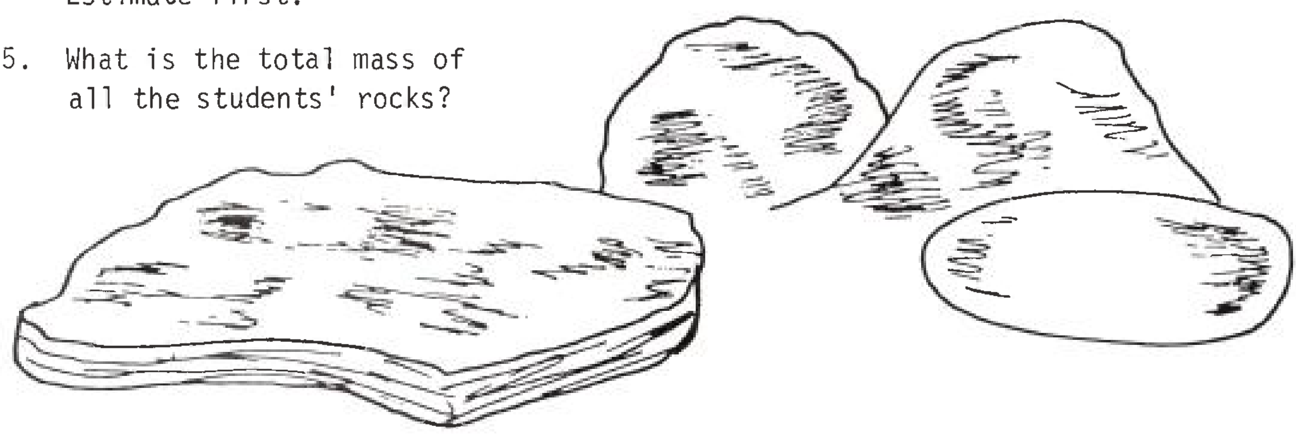
1. Bring a rock to class.

2. Estimate the mass of your rock, then weigh it exactly.

3. Line the rocks up largest to smallest. Is the larger rock always the heavier? Why? Why not?

4. Find something in your classroom desk which has the same mass as your rock. Estimate first.

5. What is the total mass of all the students' rocks?



How Many Blades of Grass are in the Schoolyard?

Reprinted from The Math Post

1. Estimate first, then sample one square metre of area.
2. Find the average of several square metres.
3. Measure the schoolyard and find how many square metres there are in all.
4. Approximately how many blades of grass are in the schoolyard?

KIDS' STUFF MATH

(Grades 1 - 6)

-Frank

Activities, games, and ideas for the elementary classroom for teachers who enjoy working with busy, enthusiastic young mathematicians. The experiences here vary widely and can be used in all kinds of classrooms. They may easily be adapted to different ability levels and teaching methods. Each idea is designed to fit a specific math skill. A complete list of skills for the elementary years is included in the Appendix. This helps you to determine which skills need teaching or reinforcing for individual students.

Contents: Numeration and Number Theory; Sets and Number Concepts; Whole Numbers and Integers; Practice Paper; Fractional Numbers; Problem-Solving; Measurement; Geometry; Probability, Stats, and Graphing. Cost is $12.95; over 300 pages.

Available from:
Western Educational Activities Ltd.
Box 3806
Edmonton, Alberta T5H 2S7

Publisher: Incentive Publications

SECONDARY ACTIVITIES

It's In the Bag

Reprinted from The Math Post

Al, Bob, Chuck, Don, and Ed are running in a sack race. Halfway through the race, they are in these positions:

Al is 20 m behind Bob.

Bob is 50 m ahead of Chuck.

Chuck is 10 m behind Ed.

Don is 30 m ahead of Al.

Ed is 50 m behind Don.

Can you figure out who, at this point, is winning the race? Who is second? Third?

I've Earned My Stripes

Reprinted from The Math Post

(7.5, -3)
(5, -3)
(-5, -1)
(-7, 0)
(-10, 0)
(-12, -1)
(-13, -4)
(-13, -10)
LINE ENDS

(-9, 0)
(-10, -5)
★ (-10, -7)
(-9, -5)
(-7, 0)
LINE ENDS

(-3, -1.5)
★ (-5, -8)
(-5, -10)
(-1, -2)
LINE ENDS

(2, -2.5)
★ (0, -10)
(4, -3)
LINE ENDS

(-12, -10)
(-14, -10)
(-15, -12)
(-15, -14)
(-13, -14)
LINE ENDS

(-10, -10)
(-11, -9)
(-12, -10)
(-13, -13)
(-13, -14)
(-10, -15)
(-9, -14)
(-9, -13)
(-10, -10)
(-9, -10)
(-7, -11)
(-7, -14)
(-9, -14)
LINE ENDS

(-13, -9)
(-16, -9)
(-19, -11)
(-20, -13)
(-20, -15)
(-17, -18)
(-14, -19)
(-10, -20)
(-8, -21)
(-7, -23)
(-5, -23)
(-5, -21)
(-6, -19)
(-11, -17)
(-15, -16)
(-17, -14)
(-17, -13)
(-16, -12)
(-15, -12)
LINE ENDS

(-9, -18)
★ (-10, -19)
(-10, -17.5)
LINE ENDS

(-14, -19)
★ (-13, -17)
(-13, -19)
LINE ENDS

(-15, -16)
★ (-17, -17)
(-16, -15)
LINE ENDS

(-19, -16)
★ (-18, -15)
(-20, -15)
LINE ENDS

(-17, -14)
★ (-19, -12)
(-17, -13)
LINE ENDS

(-17, -10)
★ (-16, -11)
(-16, -9)
LINE ENDS

(9, 7)
(9, 5)
(8, -2)
(2, -15)
(2, -16)
(3, -16)
(3, -17)
(5, -17)
(6, -18)
(7, -17)
(8, -17)
(9, -16)
(13, -16)
(14, -17)
(16, -16)
(18, -16)
(18, -15)
(19, -15)
(19, -13)
(15, -7)
(12, -2)
(11, 5)
(11, 7)
LINE ENDS

(3, -16)
(4, -14)
(6, -12)
(8, -11)
(12, -12)
LINE ENDS

(9, -16)
(10, -14)
(11, -13)
(12, -12)
(14, -11)
(15, -11)
(17, -13)
(18, -15)
LINE ENDS

(15, -13)
(16, -15)
(16, -16)
LINE ENDS

(13, -13)
(13, -16)
LINE ENDS

(11, -15)
(11, -16)
LINE ENDS

(8, -14)
(7, -16)
(7, -17)
LINE ENDS

(6, -14)
(4, -16)
(5, -17)
LINE ENDS

(6, -8)
(8, -11)
LINE ENDS

(15, -9)
(14, -11)
LINE ENDS

(1, 17)
(0, 21)
(0, 24)
(3, 22)
(17, 22)
(20, 24)
(20, 21)
(19, 17)
LINE ENDS

★Shade these areas.

(6, 10)
(3, 10)
(0, 8)
(1, 13)
(0, 12)
(2, 19)
(4, 21)
(7, 22)
(7, 21)
(8, 19)
★ (9, 18)
(11, 18)
(12, 19)
(13, 21)
(13, 22)
(16, 21)
(18, 19)
(20, 12)
(19, 13)
(20, 8)
(17, 10)
(14, 10)
LINE ENDS

(4, 16)
(5, 17)
(4, 15)
(4, 13)
(5, 12)
(8, 12)
(10, 14)
LINE ENDS

(8, 12)
(9, 11)
(9, 13)
LINE ENDS

(8, -1)
★ (10, -2)
(8, -2)
LINE ENDS

(10, 15)
(11, 13)
(12, 12)
(15, 12)
(16, 13)
(16, 15)
(15, 17)
(16, 16)
LINE ENDS

(11, 13)
(11, 11)
(12, 12)
LINE ENDS

(7, 12)
(6, 8)
(7, 7)
(13, 7)
(14, 8)
(13, 12)
LINE ENDS

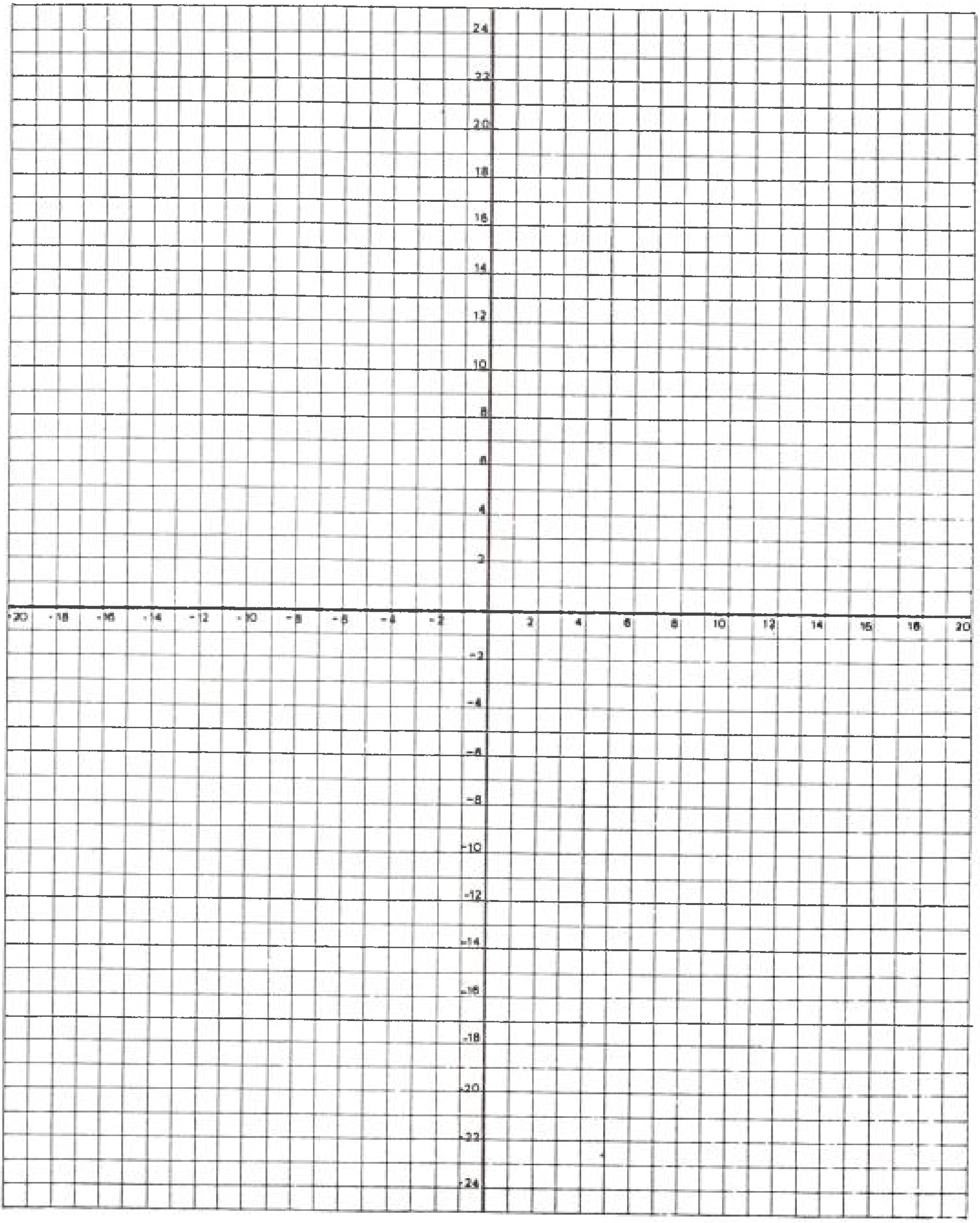
(3, -13)
(-7, -13)
LINE ENDS

(12, -2)
★ (10, -4)
(12.5, -3)
LINE ENDS

(13.5, -5)
★ (12, -7)
(14.5, -6)
LINE ENDS

(7, -4)
★ (9, -6)
(6, -6)
LINE ENDS

Name ____



24
22
20
18
16
14
12
10
8
6
4
2
-20
-18
-16
-14
-12
-10
-8
-6
-4
-2
2
4
6
8
10
12
14
16
18
20
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
-22
-24

Trash

Reprinted from The Math Post

Materials Needed:

- Washers or tiles
- One empty waste basket
- One ream of paper (500 sheets)
- Balance scales and metric masses

NOTE: You may have to use the washers or tiles to make additional metric masses for this investigation.

1. How much trash has been placed in the waste basket in your room since school started this morning? Use your balance scales and other materials to find the mass of the trash. Keep a record of your work.

2. When school is out, how much trash do you think will be in the waste basket?

3. How much trash would all the rooms in your school throw away in one day? In one week? In one year? How accurate do you think your answers are? How could you improve the accuracy of your answers?

4. What is the mass of a ream (500 sheets) of paper? What should be the mass of 1000 sheets of paper?

5. How many sheets of paper are thrown away by your school each year? Hint: Use results from 3 and 4.

EXTENSION:

If all the trash from your school was compacted (pressed together) for one year, how much space would it take up?

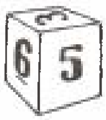
Logicombo

Reprinted from The Math Post

Math Skills: Simultaneous linear equations
Whole number operations
Logic

Number of Players: 2

Materials:



Three hexahedra dice
Game sheet (duplicate one sheet per player); one watch with a second hand, or a one-minute timer; one pencil per player.

Rules:

The object of the game is to reach or exceed a total score of 200 points.

The player whose first name begins with the letter closer to A is Player 1; the other player is Player 2.

Player 1 rolls the dice but does not allow Player 2 to see them. Player 1 places the dice in order from highest to lowest ($A \geq B \geq C$).

Player 2 asks for the following clues and records them on the game sheet in the appropriate columns:
- What is the sum of the larger two numbers? $(A + B)$
- What is the difference between the larger two numbers? $(A - B)$
- What is the sum of the smaller two numbers? $(B + C)$
- What is the product of the smaller two numbers? (BC)

Player 2 then has one minute to guess what the three numbers (A, B, C) are. Player 1 watches the watch or timer.

Scoring is as follows:
- If all three numbers are correctly guessed, score three times the sum of the three numbers.
- If only two numbers are correctly guessed, score two times the sum of the two numbers.
- If only one number is correctly guessed, score that number.
- If no numbers are correctly guessed, subtract the sum of the three numbers from the running total.

On the next turn, Player 2 rolls the dice, and Player 1 tries to guess the three numbers.

The first player to reach or exceed a total score of 200 is the winner.

Variations:

Ask for clues of $A + B$, $A - B$, $A - C$, and AC.

Ask for clues of $B + C$, BC, $A - C$, and AC.

Use 3 octahedra dice.

Name ____________________

LOGICOMBO

If $A \geq B \geq C$				Guess:			Actual:				
$A+B$	$A-B$	$B+C$	BC	A	B	C	A	B	C	Score	Running Total

Correct Guesses	Score
3	3 times sum of 3 numbers
2	2 times sum of 2 numbers
1	that number
0	subtract sum of 3 numbers

Mathematics Council Executive 1980–81

123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789

PRESIDENT
Lyle Pagnucco — Res 271-0259
14135 Parkside Dr. SE — Bus 276-5521
Calgary T2J 3X3

ACTING PAST PRESIDENT
Robert Holt — Res 986-1134
14 Brenda Vista — Bus 455-2008
Box 58, Site 9, R.R.2
Leduc T9E 2X2

VICE-PRESIDENT
Judy Steiert — Res 282-6593
203, 1608 - 20 Ave. NW — Bus 283-8434
Calgary T2M 1G8

SECRETARY
Dr. Arthur Jorgensen — Res 723-5370
Box 2069 — Bus 723-5515
Edson TOE OPO

TREASURER
Brian Chapman — Res 782-3551
Box 1525 — Bus 782-3812
Lacombe TOC 1SO

delta-K EDITOR
Dr. George Cathcart — Res 435-1945
Dept. of Elementary Education — Bus 432-4153
University of Alberta
Edmonton T6G 2G5

FACULTY OF EDUCATION REP. and MONOGRAPH EDITOR
John Percevault — Res 328-1259
Faculty of Education — Bus 329-2111 ext. 257
U. of Lethbridge
Lethbridge T1K 3M4

DEPT. OF EDUCATION REP.
Bruce Stonell — Res 346-7814
Box 5002 — Bus 343-5262
3rd Floor, West
Provincial Building
4920 - 51 Street
Red Deer T4N 5Y5

MATHEMATICS REPRESENTATIVE
Dr. Geoffrey J. Butler — Res 435-1433
Dept. of Mathematics — Bus 432-3988
University of Alberta
Edmonton T6G 2G1

ATA STAFF ADVISOR
C.E. Connors — Bus 453-2411 ext. 249
Barnett House
11010 - 142 Street
Edmonton T5N 2R1

PEC LIAISON REPRESENTATIVE
Norval A. Horner — Res 456-2744
15235 - 117 Street — Bus 973-3301
Edmonton T5M 3V7

DIRECTORS

Ron Cammaert — Res 223-4948
Box 1771 — Bus 223-2902
Taber TOK 2GO

Gary Hill — Res 327-5601
310 Laval Blvd. — Bus 329-1243
Lethbridge T1K 3W5

Donald Hinde — Res 782-6849
Box 741 — Bus 782-3812
Lacombe TOC 1SO

Klaus Puhlmann — Res 795-2568
Box 1570 — Bus 723-4471
Edson TOE OPO

Mrs. Hilary Ward — Res 346-2739
4529 - 48 Street — Bus 346-4397
Red Deer T4N 1S2

SOUTH WEST REGIONAL

Joe Krywolt — Res 328-8439
President — Bus 327-3945
1229 - 32A Street S
Lethbridge T1K 3A3

Mrs. M. Jo Maas — Res 553-4848
Secretary — Bus 737-3963
Box 484
Fort Macleod TOL OZO

PRINTED AT
BARNETT HOUSE