Why Study Mathematics?

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Recently a group of students wrote the provincial education minister and asked, "Why do we have to learn division of polynomials? We will never have to use it."

I presume the students were learning how to do the following:

$$\begin{array}{r|l} & x^{360} + x^{359} + x^{358} + \cdots + x + 1 \\ \cline{2-2} x - 1 & x^{361} - 1 \end{array}$$

We are all faced with these frustrating questions from students. No one asks the gym coach where the push-ups that are being taught will be used ten years from now. The gym coach's answer would probably be that the exercise is helpful in developing good muscle tone. A similar answer about the effects on the mind of mathematical exercises would not be accepted.

Any student who wants to do more in life than shovel snow will have to learn mathematics. I will explain some of the reasons why, and then give a specific answer to the question about division of polynomials asked by the students.

You can always ask a student, "What do you want to be when you grow up?" One typical answer might be "cattle rancher, just like Dad." At the University of Alberta, there is a department in the Faculty of Agriculture and Forestry called animal science. Many of the students in this department are interested in ranching. Very few of them are scholarly or academically oriented in a traditional sense.

To get a B.Sc. degree from the Department of Animal Science, you must take a calculus course. To learn this subject, you must have studied subjects such as division of polynomials. The degree programs in food science, plant science, rural economy, and soil science all require calculus.

Perhaps your student suddenly loses interest in ranching and wants to work in a drug store as a pharmacist. After all, these people translate doctors' prescriptions, mix drugs, and count money, and that doesn't appear to require advanced training. At the University of Alberta there is a Faculty of Pharmacy. Many of the students in this faculty are preparing to become pharmacists. To get a degree in this faculty, you are required to take calculus!

Now your question might be, "Why do these programs require calculus?" These requirements are not set by the Mathematics Department, but by the teachers in these other areas. Why do they require a subject that a few years ago was reserved for engineers and physicists?

There are several reasons for the requirement of sophisticated mathematics. Much of the mathematics they are required to learn is, in fact, useful in understanding the work they will do. I will give an example of this when I discuss division of polynomials.

Another reason for requiring the study of advanced mathematics is that it helps develop a logical, disciplined approach to problem solving. Such "mental tone" is useful in any subject, although it is difficult to explain how any one item of mathematics contributes to it.

Probably the most important reason for requiring advanced mathematics is overkill. Certainly the students will have to know and use simple algebra. The easiest way to give them confidence in that subject is to push them on to a more advanced one. Learning to do mathematics is similar to learning to play a musical instrument. What seems hard today becomes easy tomorrow, with practice. Meanwhile, what you try to learn tomorrow seems impossible, until the next day. For a student with average ability taking mathematics, you can assume that real understanding lags about one year behind the subject matter.

For many subjects, such as nursing, dentistry, geography, or education, a knowledge of statistics is required. Statistics is easier to understand and use if it is preceded by a calculus course.

Here are some surprising facts of life about mathematics at the university level. Calculus 202 is designed for science students who have not had Math 31 in high school. (We have other calculus classes for mathematics majors, business majors, and engineers.) We regard any student who has not had Math 31 as being poorly prepared for university mathematics. Out of 1,000 students who start in September, 500 will drop out or fail by the end of the first term, and only about 300 will achieve a passing grade by the end of the second term in June. That means that 70 per cent of the students entering the course either have to repeat it or have changed their career plans to accommodate their lack of success in mathematics. A large number of these students avoided mathematics in high school, and never dreamed that the subjects or careers they decided upon would end up requiring so much mathematics.

Suppose your student wants to be a real estate agent. You could suggest a commerce degree at a university, but it too requires a first-year course in calculus.

A student can, of course, become a real estate agent without a commerce degree. He or she doesn't really have to know advanced mathematics or, for that matter, how to divide polynomials. But if we want to be effective teachers, we have to have faith that life is better if one understands as much as one can about why things work the way they do.

Let's return to the problem of using polynomial division. Suppose you want to get a mortgage for $60,000 over 30 years, with an interest rate that is equivalent to 1.5 per cent compounded monthly. (Legally, all such mortgages are advertised on the basis of interest compounded every six months. For simplicity, assume that the equivalent monthly rate is given.)

What should the monthly payments be? There are books that are published in which you can look up the amount required. There are also calculators that can produce the monthly payment at the push of a button. You don't have to know how the numbers got into the book, just as you don't have to know what causes rain, what the moon is, or why we have lungs. On the other hand, knowing how the tables were assembled removes some of the sorcery from the process.

If someone misses two payments, is late with a third, has his interest changed at the end of one year, and must pay an interest penalty for paying off the whole mortgage upon sale of a house, then no book is going to provide details on what amounts should be paid. Now it is necessary to know how the information got into the mortgage books so that it can be modified to suit a special situation.

Set up a line indicating that 30 x 12 = 360 payments worth R each will be made at the end of each month. In return for these payments, the mortgage company will provide \$60,000 now. Let i = monthly interest rate = .015.

How much is the payment due at the end of the first month worth now? If A = how much it is worth now, then at the end of one month it is worth A again plus the interest for one month on A, or Ai. Thus, $R = A + A_i = A\ (1 + i)$. This means that the first payment of R dollars is worth $\frac{R}{1+i}$ dollars today. A similar argument for the payment after two months shows that it is worth $\frac{R}{(1+i)^2}$ today, and the last payment after 360 months is worth $\frac{R}{(1+i)^{360}}$ today. All these added together should equal \$60,000. Thus,

$$60{,}000 = \frac{R}{1+i} + \frac{R}{(1+i)^2} + \frac{R}{(1+i)^3} + \cdots + \frac{R}{(1+i)^{360}}$$

or

$$60{,}000 = R\left(\frac{1}{1+i} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \cdots + \frac{1}{(i+1)^{360}}\right).$$

Examine the polynomial division result in the second paragraph and let $x = \frac{1}{1+i}$. Then $\frac{1}{1+i} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \cdots + \frac{1}{(i+1)^{360}} = \left(\frac{1}{1+i} - 1\right)\left(\frac{1}{(1+i)^{361}} - 1\right) - 1$, which simplifies to $\dfrac{1 - \dfrac{1}{(1+i)^{360}}}{i}$. Thus, the monthly rent R is found by solving the equation: $60{,}000 = R\left(\dfrac{1 - \dfrac{1}{(1+i)^{360}}}{i}\right)$

for R, given that $i = .015$.

The number $\left(\dfrac{1 - \dfrac{1}{(1+i)^{n}}}{i}\right)$ is usually symbolized with $a_{\overline{n}|i}$, which in this case is $a_{\overline{360}|.015}$. These numbers often come up in formulas for finance, and there are tables published which calculate them for various numbers of months and interest rates.

We have shown how division of polynomials can be used to understand how a formula for mortgage payments is arrived at. Now your potential real estate agent will insist that he or she expects to be very successful and could afford to hire experts to handle the understanding of such matters.

I recently served as a consultant for a well-established builder who had obtained an 18-month loan for $12 million. This wealthy and experienced man relied on "expert" advice, and apparently had little idea of how his experts went about what they were doing. At one point, just before final documents were signed, a lender changed an interest rate from an amount compounded semi-annually to one compounded monthly. When the builder asked for advice on the matter, he was told it was only a minor change. If it were interest on $50, the amount involved would have been minor. In relation to the total loan amount, perhaps it still was not significant. But the difference amounted to over $35,000, and certainly the lender knew that!

I was called in because what had started out as a simple mortgage loan of $12 million at 15 per cent turned out, once the various provisions were accounted for, to be a monster with an interest rate of 38 per cent (not including the fee I charged for my services).

This is the kind of trap that someone can fall into who doesn't spend the time to learn something about what experts do. People who will eventually have relatively unsophisticated jobs are asked to prepare themselves by taking advanced courses in mathematics so that they have some idea as to how the experts arrive at their conclusions. One consequence is that they are not so intimidated by expert knowledge.

At one time, calculus was regarded as a subject for the last years in university. It is now taught in high school. Some of the courses I took in graduate school not so long ago are now required of first-year students. This trend will continue.

Learning mathematics is hard work for many students, and it is very tempting for them to try to avoid the work by claiming it is of no use to them. Each instructor must radiate with confidence the feeling that every bit of mathematics the student is asked to learn in the early years has some positive effect on the student's preparation for life.