Pythagoras Revisited

by *Vernoy Johnson*

New Trier High School East
Winnetka, Illinois

Reprinted from The Illinois Mathematics Teacher, *Vol. 31, No. 3, May 1980.*

In his collection of 371 proofs of the Pythagorean Theorem, Loomis offers what he calls four kinds of demonstrations, those based on linear relations (algebraic proofs), on comparison of areas (geometric proof), on vector operations (quarternionic proofs), and on mass and velocity relationships (dynamic proofs). The point of this brief paper is to add a "proof" to Loomis' collection in a fifth category that I will call "people packing" (a humanistic proof).

The reader will quickly observe that it is not a proof at all, but rather a device for reinforcing in the student's mind the original meaning of Euclid's 47th proposition. For, although in *using* the theorem the relationship is most frequently translated into an algebraic formulation, the *meaning* of the theorem is a comparison of the areas of three squares.

The following "proof" works beautifully for a class of 20 to 30 students. You may have to round up a few more people if you make the figure larger than suggested. Mark the floor with masking tape to form a right triangle with sides of approximately three feet, four feet, and five feet, and mark the corresponding squares on each of the three sides. Invite people to stand, packed tightly, inside each of the two smaller squares. At this time you might remind the students what the theorem says about areas, and point out that the number of people standing in the squares is an indirect and, of course, very crude measure of the areas of those two squares. Then move the students to the largest square, the one on the hypotenuse. VIOLA! They all just fit! Of course you could get in one extra person, or leave one out and the "tightness" wouldn't be noticeable. But the image is there, in a dramatic and indelibly memorable way, one that can be easily referred to if uncertainties arise about the meaning of the theorem.

In spite of the rather ponderous and almost pontificial nature of Loomis' work, I think he would approve of the spirit, if not the rigor, of this 372nd "proof" of his beloved theorem. I believe it would be seen by him as an appropriate response to his "hope that this simple exposition of this historically renowned and mathematically fundamental proposition ... may interest many minds... Read and take your choice; or better, find a new, different proof...."

REFERENCE

Loomis, Elisha S. *The Pythagorean Proposition, Its Demonstrations Analyzed and Classified*, 2nd edition. Ann Arbor, Michigan: Edwards Bros., Inc., Litho printers, 1940.