Operation of Union and Intersection on Sets

by Marlow Ediger

Pupils in the elementary school should have learning activities in mathematics which are interesting, meaningful, and purposeful. "Learn by discovery" is a key concept in having pupils develop conclusions and generalizations in elementary school mathematics.

Pupils in the first grade can discover meanings pertaining to the operations of union and intersection on sets, which are disjoint as well as not disjoint, providing that the learning activities provided for them are interesting, meaningful, and purposeful. The operation of union of disjoint sets will be discussed first.

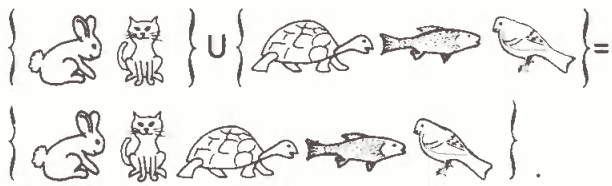
1. Use actual objects or pupils in the classroom. Pupils who have developed understandings pertaining to rational counting can also develop important understandings pertaining to the union of sets. Two boys can stand in front of the classroom representing one set, with three other boys in the second set. The question can be asked, "How many boys do we have if the two sets are joined?" The order of the sets could be changed when these pupils representing the two sets are standing in front of the classroom. Pupils could inductively develop the understanding that the operation of union on sets is commutative (pupils would develop the generalization in their own terminology which is meaningful to them). Real objects such as books, rulers, pencils, crayons, and toys can also be used to help pupils understand meanings pertaining to the operation of union on sets as well as the commutative property of union.

2. Use pictures. Pupils enjoy looking for pictures in discarded magazines in school as well as in the home. They can look for pictures of boats, cars, trucks, buses, and people. Set one could be made up of two cars such as a Chevrolet and a Ford, while the second set has three members - Plymouth, Rambler, and Dodge. The question that can be answered by pupils is, "If we join the cars in both sets to make a new set, how many cars do we have in this new set?" The commutative property of union should also be emphasized by changing the order of the two sets.

3. Use the flannel board. There should be felt cut-outs of various animals, people, cars, trucks, and geometric designs for pupils to utilize while responding to questions involving the joining of two sets. Using cut-outs which name a variety of animals, people, and so forth, provide for variety in learning activities for pupils, thus helping to maintain pupil interest. (The same would be true for varying the actual objects and pupils in developing sets as well as in the use of pictures.) When the flannel board is used, as well as in previous times, pupils should be able to describe a set accurately. For instance, the teacher could put the following on the flannel board and have pupils tell about the set:

The discussion would end with an accurate description of the set. Another set could be placed on the flannel board, such as:

This set would also be described accurately. The question that can now be raised is, "If the two sets are joined, how many members do we have in the new set?" The new set that results can be visualized by pupils, such as:



Pupils should understand that what is located within the braces makes up the members of a given set. The commutative property of union can also be visualized by pupils when changing the order of the two sets. Pupils at this stage of learning need also to understand a related understanding to the operation of union and that is the operation of addition on numbers such as 2 + 3 = 5 in the previously discussed example. The number of members of the first set was two, and the number of members of the second set was three, therefore, 2 + 3 = 5.

4. Review previous learnings and utilize abstract symbols more frequently. In this stage of achievement pupils can deal effectively with more abstract symbols than previously. Sets on a flannel board can be labelled, such as:

Pupils can work problems using numerals only, such as:

$3 + 2 = \square$, $3 + \square = 5$,

$2 + 3 = \square$, $\begin{matrix} 3 \\ +2 \end{matrix}$ $\begin{matrix} 2 \\ +3 \end{matrix}$,

and other addition facts that pupils have developed understandings to previously when persons, objects, pictures, and the flannel board were used. The teacher needs to remember that learning activities should be varied to develop and maintain pupil interest as well as provide for individual differences among pupils.

Pupils, toward the end of the first grade, can also discover the operation of intersection on sets. The operation of intersection on sets should be presented shortly after pupils develop understandings pertaining to the union of sets which are not disjoint.

1. Use dramatizations. These dramatizations should be realistic and lifelike. Don, Bill, and John are on a committee to feed pets in the classroom for one week. Ann, Judy, and Bill are on a different committee to take care of the plants in the classroom during the same week. Pupils could be asked, "Who are the members of the committee to take care of the pets in the classroom this week?" The names of the members of the committee to take care of the pets in the classroom this week should be written on the chalkboard:

{Don, Bill, John}.

"Which pupils are members of the committee to water plants this week?" These names should also be written on the chalkboard:

{Ann, Judy, Bill}.

Pupils could now see the two sets placed side by side, such as:

{Don, Bill, John}, {Ann, Judy, Bill}.

The next question that can be raised is, "How many members make up the two sets if they are joined?" If pupils

respond with "six," the teacher should have the members of both committees come to the front of the room in order that all pupils can understand that there are five members making up the "union" of the two sets. On the chalkboard, the teacher can finish writing

{Don, Bill, John} ∪ {Ann, Judy, Bill} =

{Don, Bill, John, Ann, Judy}

during the final dramatization. Several dramatizations should be viewed by pupils so that they clearly understand the meaning of the operation of union sets which are not disjoint. Pupils should also understand the commutative property of union through dramatizations at this point.

The teacher should now have pupils develop inductively an understanding of the operation of intersection on sets. Pupils in the classroom can be asked, "Who is on the committee to feed our pets in the classroom this week?" Pupils will respond with the following names: "Don, Bill, and John." On the chalkboard, the teacher can write

{Don, Bill, and John}.

The next question asked of pupils could be the following: "Who is on the committee to take care of our plants in the classroom this week?" Pupils should respond with the correct names, "Ann, Judy, and Bill." The teacher, on the same line on the chalkboard, writes

{Ann, Judy, Bill}.

The teacher can now ask, "Which member is on both committees?" After pupils have responded correctly, the teacher can finish writing

{Don, Bill, John} ∩ {Ann, Judy, Bill} =

{Bill}.

Pupils should develop an accurate, meaningful understanding of the abstract symbol for the operation of intersection. Several dramatizations should be used in order that pupils understand the meaning of a member being common to two sets.

2. Use the flannel board. Cut-outs of animals, people, cars, trucks, and geometrical designs can be used. The teacher can place a felt cut-out of a duck, pig, and rabbit in one set, such as:

Beside it, a second set could be placed made up of a duck and a cat, such as:

Pupils, under teacher guidance, could describe accurately each set. The teacher could now ask, "How many different kinds of farm animals would there be if we joined the two sets to make a new set?" If some pupils respond with "five," in a discussion pupils can develop the understanding that duck is a member of both sets; there are four members in the new set which can be visualized by pupils as:

Since "duck" is common to both sets,

3. Use abstract symbols. The letters of the alphabet, the days of the week, and/or the months of the year can be written on the chalkboard. For instance, pupils are asked to name the first three days of the week for

one set; as pupils mention the names, the teacher can write the set as

{Sunday, Monday, Tuesday}.

The teacher can then ask pupils to name the last five days of the week; the teacher or another pupil can write on the chalkboard the second set consisting of the last five days of the week:

{Tuesday, Wednesday, Thursday, Friday, Saturday}.

The teacher then asks pupils, "If the two sets are joined, what members make up the new set consisting of the days of the week?" The teacher writes the names as they are mentioned by pupils. If pupils respond with the following as being the union of the two sets mentioned previously, "Sunday, Monday, Tuesday, Tuesday, Wednesday, Thursday, Friday, Saturday," the teacher can have pupils look at a calendar in order to name the days of the week. Most pupils can, of course, at this point recite the days of the week. Pupils can inductively develop the understanding that

{Sunday, Monday, Tuesday} $\cup$ {Tuesday, Wednesday, Thursday, Friday, Saturday} = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}.

At this point, pupils can also be asked, "Which member is common to both sets?" The teacher, after receiving the correct response from pupils, can write on the chalkboard:

{Sunday, Monday, Tuesday} $\cap$ {Tuesday, Wednesday, Thursday, Friday, Saturday} = {Tuesday}.

Pupils should notice the symbol "$\cap$", and how it differs from the symbol "$\cup$" used in joining two sets. A discussion should follow in which the symbols "$\cup$" and "$\cap$" become differentiated and understood by pupils so that meaningful learning may take place.

Further learning activities for pupils in understanding what, in adult terms, would be the "union and intersection of sets which are not disjoint" could be the following.

Have pupils name the first two months of the year for the first set. Next, have pupils name the first three months of the year as the second set. Write the specific sets on the chalkboard at the time they are given by pupils. Disagreements among pupils as to the correct sets wanted can make for excellent discussions in the classroom; in these discussions pupils reveal correct as well as incorrect understandings. If pupils want to mention the names of months more than once in the union of the two previously mentioned sets, such as incorrectly stating that

{January, February} $\cup$ {January, February, March} = {January, February, January, February, March},

the teacher can ask the question, "What set is made up of the first three months of the year?" After a discussion, pupils will generalize that the set consisting of the first three months of the year is

{January, February, March} and not {January, February, January, February, March}.

Pupils could then be asked which member or members are common to both sets. The teacher can write pupil responses on the chalkboard as they are given, using the appropriate symbols for sets named and the correct symbol for intersection of the two sets.

The order of sets can also be changed so pupils can inductively understand that in union and intersection of sets the order is not important.