activities

Beyond the Usual Constructions

by Melfried Olson

University of Wyoming
Laramie, Wyoming 82071

Teacher's Guide

Grade level: 7–12.

Introduction: Basic constructions concerning triangles and the nine-point circle were introduced in this column by Allen in 1972. There are other interesting relationships concerning triangles and circles that can be explored by using constructions. The activities below provide additional construction experiences.

Materials: Compass, protractor, straightedge, and copies of the next three sheets for each student.

Objectives: After a review of similarity of triangles, congruence of circles, work with half planes, and the constructions of centroid, orthocenter, circumcenter, incenter, and the nine-point center and nine-point circle of a triangle, the following set of exercises can be used as motivational material for further discussion of special relationships derived from a triangle and previously mentioned basic constructions.

Directions: Make copies of the following pages—enough to allow for student mistakes. You may need to review the basic constructions if you haven't worked with them recently. These include bisecting angles, constructing a perpendicular from a point to a line, and constructing a perpendicular bisector of a segment. Appropriate discussion should follow each activity, and the interested student could pursue similar material as a class project.

Solutions

1B. Angle bisectors of $\triangle JKL$ are altitudes of $\triangle ABC$.

Note: If a nonacute $\triangle ABC$ is used, the altitudes will intersect outside the triangle so the relationship in 1B will not hold.

2C. The segments are concurrent. The point at which they are concurrent is called the *Fermat point* of the $\triangle ABC$ (denoted F).

D. $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$ all have the same length.

E. The measures of the six angles are congruent.

3C. $\overline{PQ} \parallel \overline{JK}$, $\overline{PR} \parallel \overline{JL}$, and $\overline{QR} \parallel \overline{KL}$, and, as such, $\triangle PQR$ and $\triangle JKL$ are similar.

4D. The points M, N, and O are collinear. This line is called the ***Simson line of P*** for the triangle.

5. Note: This activity should be done as a parallel construction to Euler's nine-point circle (see Allen).

5D. The nine-point center of $\triangle ABC$ is Z. The nine-point circle of $\triangle ABC$ is the circumcircle of $\triangle JKL$.

REFERENCES

Allen, Charles E. "Activities: Mission—Construction." *Mathematics Teacher* 15(November 1972): 631–34.

Eves, Howard. *Fundamentals of Geometry*. Boston: Allyn & Bacon, 1969.

BEYOND THE USUAL CONSTRUCTIONS

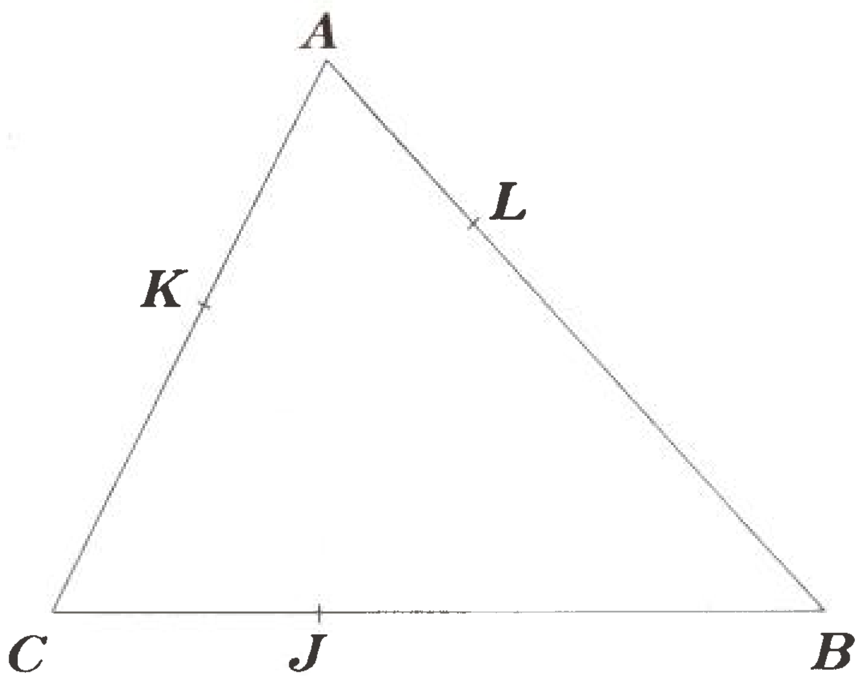
ACTIVITY 1

A. Points J, K, and L are the feet of the altitudes from A, B, and C, respectively. Draw $\triangle JKL$.

$\triangle JKL$ is called the orthic triangle of $\triangle ABC$.

B. Construct the bisectors of the angles of $\triangle JKL$, labeling the point of intersection of the angle bisectors H. What do you notice?

With respect to $\triangle ABC$, H is called the orthocenter, but with respect to $\triangle JKL$, H is called the incenter.



ACTIVITY 2

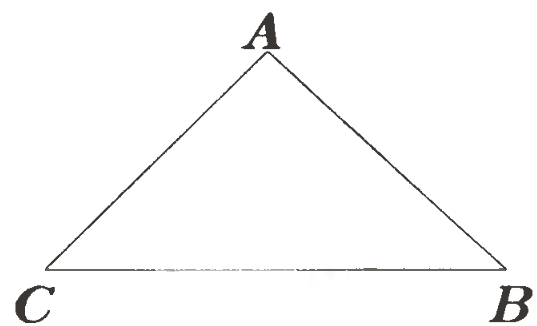
A. Construct an equilateral $\triangle ABC'$ with sides of length AB so that C' is not in the same half plane as C determined by $\overleftrightarrow{AB}$.

B. Repeat (A) for the other sides of $\triangle ABC$ so as to have three equilateral triangles, $\triangle ABC'$, $\triangle AB'C$, and $\triangle A'BC$.

C. Draw $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$. What do you observe?

D. Measure $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$. What do you observe?

E. Label $\overline{AA'} \cap \overline{BB'} \cap \overline{CC'}$ as F. Measure the six angles formed around F. What do you observe?



SHEET 1

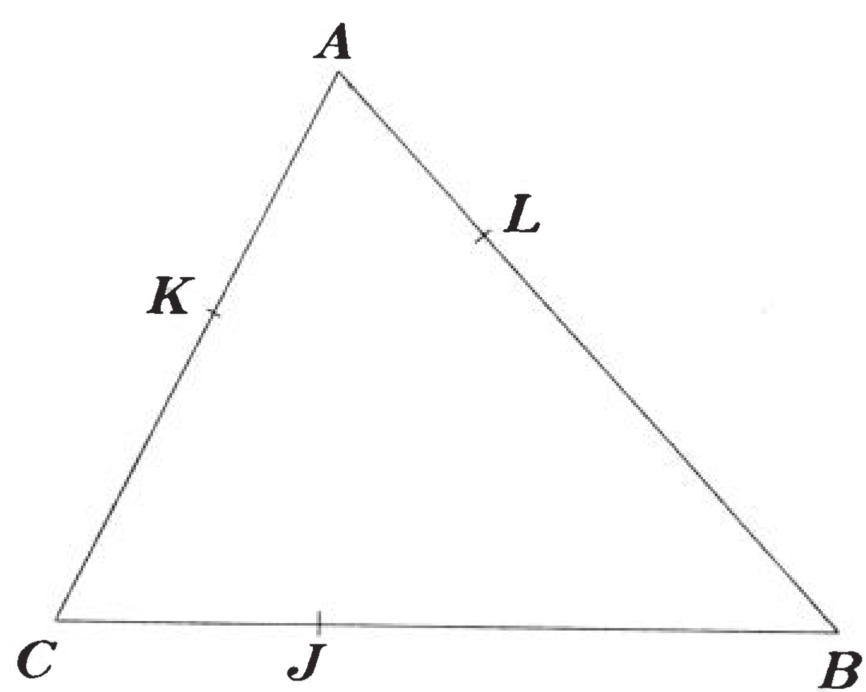
BEYOND THE USUAL CONSTRUCTIONS

ACTIVITY 3

A. Points J, K, and L are the feet of the altitudes from A, B, and C, respectively. Draw $\triangle JKL$.

B. Draw the perpendicular bisectors of $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$. Label the point of intersection of the perpendicular bisectors D. With respect to $\triangle ABC$, D is called the circumcenter. Using D as the center and $\overline{AD}$ as the radius, draw the circle that also contains B and C. This is called the circumcircle of $\triangle ABC$.

C. Let $\overrightarrow{AJ}$, $\overrightarrow{BK}$, and $\overrightarrow{CL}$ intersect the circumcircle in points P, Q, and R, respectively. Draw $\triangle PQR$. How are $\triangle PQR$ and $\triangle JKL$ related?



SHEET 2

BEYOND THE USUAL CONSTRUCTIONS

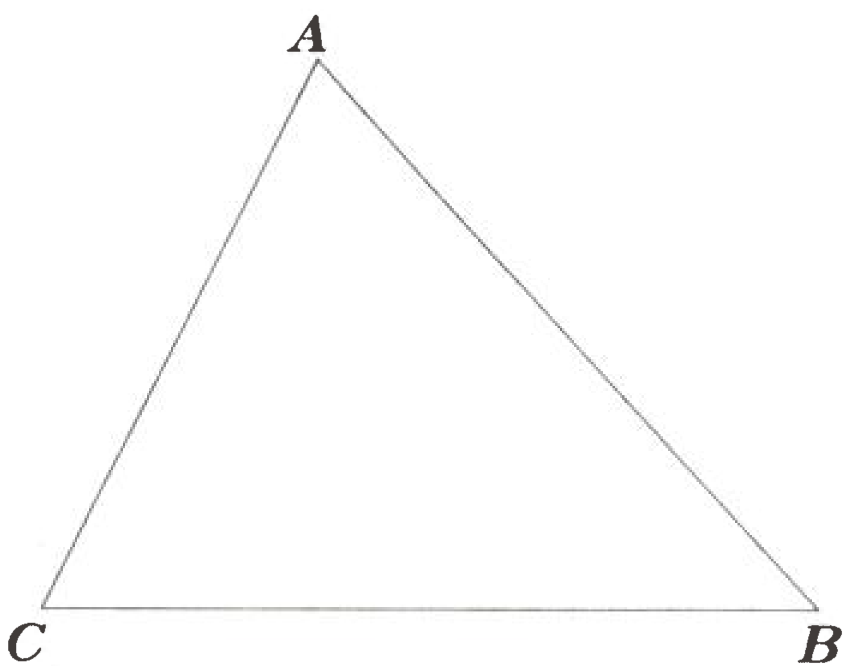
ACTIVITY 4

A. Find the circumcenter and circumcircle of $\triangle ABC$.

B. Choose a point P (different from A, B, or C to start with) on the circumcircle.

C. Construct $\overline{PM} \perp \overleftrightarrow{BC}$ at M, $\overline{PN} \perp \overleftrightarrow{AC}$ at N, and $\overline{PO} \perp \overleftrightarrow{AB}$ at O.

D. What do you observe about M, N, and O?



ACTIVITY 5

A. Points J, K, and L are feet of the altitudes from A, B, and C, respectively. Draw $\triangle JKL$ and find the orthocenter H of $\triangle ABC$.

B. Find the circumcenter of $\triangle JKL$, call it Z, and draw the circumcircle of $\triangle JKL$. Find the circumcenter D of $\triangle ABC$.

C. If we consider $\triangle ABC$ and find the midpoints of the sides, the feet of the altitudes, and the midpoints of $\overline{HA}$, $\overline{HB}$, and $\overline{HC}$, we note that these nine points lie on a circle (called the nine-point circle of $\triangle ABC$) with center N the midpoint of $\overline{HD}$.

D. Compare Z to the nine-point center of $\triangle ABC$, and compare the circumcircle of $\triangle JKL$ to the nine-point circle of $\triangle ABC$. What do you find?