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THE
ALBERTA
TEACHERS'
ASSOCIATION
MATHEMATICS COUNCIL

Volume XX, Number 3

April 1981

NCTM

59th
ANNUAL
MEETING

22 - 25 APRIL

ST. LOUIS 1981

An Agenda for Action

Recommendations for School Mathematics of the 1980s

The National Council of Teachers of Mathematics recommends that—

1. problem solving be the focus of school mathematics in the 1980s;
2. basic skills in mathematics be defined to encompass more than computational facility;
3. mathematics programs take full advantage of the power of calculators and computers at all grade levels;
4. stringent standards of both effectiveness and efficiency be applied to the teaching of mathematics;
5. the success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing;
6. more mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population;
7. mathematics teachers demand of themselves and their colleagues a high level of professionalism;
8. public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society.

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ALBERTA
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MATHEMATICS COUNCIL

Volume XX, Number 3 **April 1981**

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EDPRESS

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From the Desk of the New Editor

Ed Carriger's first issue of *delta-K* was Volume 13, Number 1, December 1973. His last was Volume 20, Number 2, February 1981. During these seven years, Ed gave unselfishly of his time and effort to produce a journal that is well recognized across North America. I pledge to maintain the high quality of *delta-K*. Of course, I will need your assistance. Please feel free to send me anything from a one-paragraph announcement to a one-page idea, position statement, or whatever, to a 10- (or more) page article.

Our past treasurer, Don Hinde, also retired this year after serving as treasurer for seven years. Don always had information at his fingertips and kept the books meticulously. We are pleased that Don is continuing on the executive as a director.

Dick Kopan, our president, found it necessary to resign in mid-year because he accepted a teaching position in British Columbia.

We will miss these officers, but wish them success in their continuing professional endeavors.

What's inside? In addition to some important announcements and reports and some very useful ideas, this issue contains two major articles. Dr. Jim Timourian, the Mathematics Department representative on MCATA, answers the question, "Why Study Mathematics?" His article will be of special interest to secondary teachers. The second article, by Ediger, discusses some concrete ways in which the operations of union and intersection of sets can be made meaningful to elementary school children. While many elementary teachers may not like the symbolism used, the ideas are certainly basic and the approach concrete.

- *George Cathcart*

Reports and Announcements

Two successful mathematics conferences were held recently:

Regina (NCTM)
October 23-25, 1980

Many mathematics teachers from western Canada and some from eastern Canada and the United States met in Regina in late October for an NCTM name-of-site meeting. NCTM President Max Sobel and several current members of the board of directors were joined by two former presidents, Glenadine Gibb (Texas) and John Egsgard (Ontario), and by several past board members.

Chuck Allen from California was the keynote speaker. The program featured 77 sessions, which enabled participants to pick sessions which were of interest to them.

Red Deer (MCATA)
November 7 and 8, 1980

Over 270 mathematics teachers from all parts of Alberta gathered at the Capri Motor Lodge for the 20th Annual Conference of MCATA. Dr. Robert E. Reys, from the University of Missouri, was the feature speaker. He brought us up-to-date on some of the results of the second mathematics assessment in the United States. He also spoke on some issues involving the use of calculators in schools.

Thirty-four other sessions and workshops provided a wide variety of topics and issues for teachers to think about, as well as ideas to use.

NEXT MEETING

NCTM

59th
ANNUAL
MEETING

22-25 APRIL

ST. LOUIS — 1981

PLAN TO ATTEND

Report on the First Annual Conference of the Alberta Association for Computers in Education

- by *Ron Cammaert*

The conference held October 24 and 25 at the University of Alberta was designed to provide an introduction to microcomputers and their application in education within Alberta. A major thrust of the conference was to determine the direction which the development of computers in education should take and the role each agency (government, university, colleges, and the school system) should play in this development.

Dr. Berthofer, Assistant Deputy Minister of Advanced Education and Manpower, pointed out that there seemed to be a diminishing return on money put into education and that there was some question as to the ability or political desire to increase allocations of money to education. He mentioned the rapid change occurring in society due mainly to the development of electronic communication (telephone, radio, T.V., and computers). Through this change, the individual has access to more and more information and on an individual basis rather than in a broadcast mode. He contends that this will cause a demand for a shift from teacher-centred to student-centred education and that use of computers will help facilitate this change.

The government, Dr. Berthofer felt, should provide incentive and regulation to provide this change in technological base. The government should coordinate efforts in the field, should establish guidelines for hardware, and help move to standardization of software. He felt the issue was not hardware, but the software and the delivery system for these.

Dr. Gene Romaniuk, Division of Educational Research Services, University of Alberta, gave a short history of computers and then explained the various uses for computers in education. The shortage of software for microcomputers is not surprising in light of the fact that Apple, Pet, and TSR-80's were only introduced in 1977. Some of the applications he mentioned included the following:

- Administration -
  scheduling, report cards, attendance, payroll, inventory, purchasing, security, information service, and library. (It is not likely that a micro would be able to cover all of these areas.)
- Subject of study -
  literacy, data processing.
- Computer-assisted instruction (CAI) -
  drill and practice, tutorial, socratic, gaming, simulation, diagnosis and prescription.
- Computer-managed instruction (CMI)
- Testing -
  item bank, test generation, test scoring and analysis, printing and graphics.
- Guidance

Rationale for CAI:

1. individualization - speed, curriculum, method, depth, remedial, diagnosis of errors, immediate feedback
2. available outside working hours
3. possible cost saving

Why has there been a slow growth of CAI?

1. cost
2. incentive
3. lack of skilled personnel
4. program exchange
5. protection for publishing
6. technological problems

Margaret Penney, Co-ordinator of Instructional Development, Grant MacEwan College, said that when using a computer for CAI, the teacher should ask the following questions:

1. What is the best way to teach a given skill or concept to a given student?
2. How can the computer aid in this process?

Ms. Penney added that teachers should apply a systematic problem-solving method to their teaching, including assessing, planning, implementing, and evaluation. Teachers should use media, including computers, in the implementation and evaluation segments to allow them to spend more time with those students needing teacher attention and have more time for planning.

Establishment of a Society

An executive was selected to establish a name, the objectives of the society and committees. The executive is as follows:

President:	*Russ Sawchuk* Grant MacEwan College
Vice-President:	*Peter Wright* Edmonton Public School System
Secretary:	*Dale Bent* University of Alberta
Treasurer:	*Steve Hunka* University of Alberta
Editor:	*Ed Carriger*
Directors:	*Jim Thiessen* Alberta Education
	Doug Crawford Advanced Education and Manpower
	Ron Cammaert (MCATA representative) Taber School Division
	Hank Boer Lethbridge Public School System
	Al Stamp
	Nelly MacEwan Edmonton Separate School Board
	Teresa Gatien

Short Courses in Calgary and Edmonton

Calgary Invades Edmonton

The Stampeders invaded Commonwealth Stadium and beat the Eskimos twice in 1980. The Flames could do the same to the Oilers. What the Boomers will be able to do to the Drillers is an open question. Since there is no professional baseball team in Calgary, the new Edmonton franchise should be safe.

The University of Calgary also invaded Edmonton in 1980. The invasion was a co-operative venture between the University of Calgary and the Edmonton Public School Board. The venue was Satoo Elementary School; the venture was an in-service course entitled Innovations in Mathematics in Primary Grades: An Activity Approach to Teaching Numeration, Addition, and Subtraction. The delivery mode was the neighborhood teacher centre.

Mr. Rick Johnson of Satoo and Mr. Sid Rachlin of the University of Calgary were the leaders. Dr. George Cathcart of the University of Alberta assisted with the course.

The 13 registered students were given experiences with a variety of concrete aids for teaching numeration, addition, and subtraction in the primary grades and then were encouraged to take the aids to their own classroom, try them, and report their impressions. Successful participants received credit for a quarter course at the University of Calgary.

For further information about this or other in-service courses at the University of Calgary, contact Mr. Jack Loughton, In-service Co-ordinator, Faculty of Education, University of Calgary.

Edmonton Fights Back

The mathematics staff in Elementary Education at the University of Alberta have designed a special continuing education offering for summer 1981. Six one-credit courses will be offered which have been especially designed for practising elementary school teachers. The six topics that have tentatively been scheduled include problem solving, diagnosis, calculators, microcomputers, enrichment, and geometry.

For the time being, these topics will be offered as Ed. CI 501, Current Developments in Elementary School Mathematics, which is a three-credit course. Therefore, registrants must select three of the six topics (all six for six credits). The course will be offered during the first three weeks of summer session by the elementary mathematics education staff. Three credits could be taken in either a two-week or three-week block.

For further information, contact:

Dr. Joan Worth
Department of Elementary Education
University of Alberta
Edmonton, Alberta T6G 2G5

NOTICE:

The Canadian Inventory of Historic Building is about to begin a study on early schools in Canada. As a base for this work, we would like to locate any buildings constructed as schools in Canada before 1930. If there is such a building in your area and you would like to see it included in the study, please write to:

School Study, Canadian Inventory of Historic Building
Parks Canada
Ottawa, Ontario K1A 1G2

Monograph on Problem Solving

Readers will be interested to learn of the publication of a monograph entitled "Studying Problem Solving Behavior in Early Childhood," by Doyal Nelson, a mathematics education professor in the Department of Elementary Education, Faculty of Education, University of Alberta.

The monograph provides valuable insights and speculations of relevance for classroom practice about the problem-solving strategies of young children, and suggests a promising methodology for research in problem solving.

Cost of this publication is $9. per single copy, $8. for five or more copies. It is available from:

Office of the Dean
845 Education South
Faculty of Education
University of Alberta
Edmonton, Alberta
T6G 2G5

PRIMARILY TIME

(for primary classes)

-Soderstrom

This book contains 24 duplicating masters to aid in the teaching of time. These can be used with or without teacher direction, depending on the child's age.

Hours, half-hours, quarter-hours, and five-minute intervals are presented in a variety of ways. Exercises include matching, writing the times, drawing hands, and coloring corresponding clocks. Each page includes clocks which review the concepts previously taught. Activities have been designed to be fun and to increase in difficulty throughout the book. The pages have been illustrated with large, basically uncluttered illustrations to allow for easy coloring when the exercises have been completed. Cost is $8.95.

Available from:
Western Educational Activities Ltd.
Box 3806
Edmonton, Alberta T5H 2S7

Publisher: Good Apple, Inc.

Plus + + +

The following material is reprinted from Issue No. 6 of Plus + + +, *a short magazine informing mathematics educators across Canada about important events, research, curriculum development, and items of national interest.*

Provincial Mathematics Programs in Canada (1978-79)

A draft document, entitled *Provincial Mathematics Programs* in Canada as of 1978-79, has been prepared by LaJune Naud, Consultant in Mathematics for the Nova Scotia Department of Education, for the Curriculum Committee of the Council of Ministers of Education of Canada. The data was obtained in part from the official provincial guidelines and in part through direct communication with each department/ministry. In Grades 1-6, it is noted generally that there is less emphasis on abstraction, more exploration of space and shape, more recourse to basic scales, and a tendency to postpone work on fractions. In Grades 7-9, more emphasis is detected on problem solving, application, interdisciplinary aspects, and basic numeric skills. Treatment of algebraic manipulation, factoring, graphing, and statistics varies from province to province. In the upper school, trigonometry is spread over several years. Transformations are increasingly taught and there is a move toward the emphases of Grades 7-9. Lists of topics common to all curricula are given. Appendices include summaries of answers to questionnaires by provincial officials, lists of guides and texts, and summary of programs by provinces.

The Canadian Mathematics Olympiad

The twelfth Canadian Mathematics Olympiad was held on April 30, 1980. Participating were 205 candidates, 178 of whom were nominated under provincial quotes and 27 by their schools. There were five questions, each worth 20 marks, and one candidate received at least 80. The top three students were:

- John J. Chew
  (University of Toronto Schools, Toronto)
  $1,000 prize
- David W. Ash
  (Fort William C.I., Thunder Bay)
  $750 prize
- Stanislav N. Valnicek
  (Evan Hardy C.I., Saskatoon)
  $500 prize

Eleven of the top 16 students received their certificates and cash awards in person from the Honorable Pauline McGibbon, Lieutenant-Governor of Ontario, at a seminar hosted by the University of Toronto in June. This was the second seminar which has been held for the winners; a previous one was held at the University of Waterloo in 1979.

For the past three years, the administration of the contest has been held under the direction of Professor J.H. Burry of the Department of Mathematics, Memorial University, St. John's, Newfoundland. Now, the contest moves to the University of Alberta.

The Olympiad is open to students nominated either by the provincial Olympiad co-ordinator or by their school principal. The thirteenth Olympiad will be written on Wednesday, May 6, 1981; nominations should be made by Friday, April 3, 1981. For further information, write:

Dr. J.G. Butler, Olympiad Committee
Department of Mathematics
University of Alberta
Edmonton, Alberta T6G 2G1

Mathematical Education Day in Vancouver

In conjunction with its annual winter meeting, the Canadian Mathematical Society organized a day of special interest to school teachers and community college instructors on the theme "Modern Developments in the Uses of Mathematics" for Friday, December 12, 1980. The eight speakers were from the University of British Columbia and Simon Fraser University. Further information on this meeting can be had from Professor George Bluman, Department of Mathematics, University of British Columbia, 2075 Wesbrook Mall, Vancouver, British Columbia V6T 1W5.

Provincial Contests and Exams

The June 1980 issue of *delta-K*, journal of the Mathematics Council of The Alberta Teachers' Association, edited by Ed Carriger (R.R.1, Site 2, Box 4, Bluffton, Alberta TOC OMO), carries a report on the C.M.S. 1980 Alberta High School Prize Examination. This contest, won by Robert Morewood of Medicine Hat, consists of two papers: (1) 20 multiple-choice questions, (2) 6 problems (*delta-K* publishes solutions). One of the problems asked the contestants to show that a solid figure, all of whose cross-sections are circles, must be a __________ (the reader will undoubtedly identify the missing word).

In the same issue is a report on a diagnostic test of 25 multiple-choice questions given to students in introductory calculus courses at the University of Alberta. In the most poorly answered question, the respondent had to identify a parabola through (-1,0), (0,-1), (1,0) (diagrammed) as one of: $y=(x-1)^2$, $y=x^2-1$, $y^2=1-x^2$, $y=1-x^2$, none of the foregoing. On the other hand, 89 per cent of 2071 students identified the real values of k for which kx^2+kx+1 had no real roots.

The question paper for the 1977 Concours mathematique de ℓ'A.M.Q. appears in *La Gazette des Sciences Mathêmatiques du Québec*, rédacteur en Chef Marc Bourdeau, Ecole Polytechnique, C.P. 6079, Succ. A, Montréal H3C 3A7. There are seven problems, each labelled with a title.

Sample:
"La boule de billard" Considérons une table de billard de 360 cm par 180 cm dont les six poches A,B,C,D,E et F sont disposées comme dans la figure. (A,B,C,D,E,F in clockwise order; B midway between A and C, E midway between D and F.) Une balle est frappé au point O, centre du carré ABEF, et rebondit pour la première fois au point P, à 48 cm du trou B (et 132 cm du trou A). Dans quelle poche ira-t-elle tomber (on négligera la friction, le rayon de la boule et de l'entrée des poches).

Canadian Mathematics Education Study Group/Groupe canadien d'etude en didactique des mathematiques

1981 Meeting: June 5 to 9 at the University of Alberta; open to mathematicians and mathematics educators. Various groups will study mathematics education research, aspects of teacher training, mathematics and language, the history and pedagogy of mathematics, etc. Guest speaker: Dr. Jeremy Kilpatrick, University of Georgia. Information can be obtained from J. Hillel, Department of Mathematics, Concordia University, 7141 Sherbrooke Street West, Montreal, Quebec H4B 1R6.

NATIONAL COUNCIL OF
Teachers of Mathematics

NCTM

NOTICES

New Resource Focusses on Math Avoidance

Guidelines for schools to assess the extent of mathematics avoidance by girls and to promote the study of mathematics by girls (and boys) are provided in the new NCTM information resource, "Mathematics Education of Girls and Young Women." An instrument for assessing enrolment in mathematics is included as well as a listing of resource organizations.

Many students are exposed to powerful influences that discourage them from continuing their study of mathematics beyond that required by school policy. Individuals and organizations must make commitments to assure truly equal opportunities for girls and young women to achieve in mathematics. Teachers must help students recognize their accomplishments and potential in mathematics. Information must be given to students explaining how the study of mathematics will affect their future. Too many doors to both employment and continuing education opportunities are closed to those without a sufficient mathematics background.

Single free copies of this resource and NCTM's position statement "The Mathematics Education of Girls and Young Women" are available from the Headquarters Office in Reston.

How to Study Mathematics

One NCTM publication that takes the mystery out of learning mathematics is *How to Study Mathematics*, by James Margenau and Michael Sentlowitz. This clever, enjoyable book gets right to the bottom of the student's math difficulties by exposing the most common study problems and offering easy, effective cures. Available from the NCTM for $1.70, this 32-page book continues to be one of the NCTM's best sellers.

Crisis in Mathematics Classrooms

"The shortage of qualified mathematics teachers in United States classrooms is one of the most pressing problems facing education today," according to Dr. Max A. Sobel, president of the National Council of Teachers of Mathematics. Sobel claims that "if we are not able to supply our students with qualified teachers of mathematics, we will not be able to prepare them for participation in the technological age of the 1980s. Already there are reports that the Soviet Union is far ahead of the United States in providing all of their secondary students with advanced programs in mathematics. We face a serious crisis in this technological decade if steps are not taken to insure an adequate supply of mathematics teachers for our schools."

In a survey just completed by the NCTM in co-operation with the Association of State Supervisors of Mathematics and the National Council of Supervisors of Mathematics, the majority (61 per cent) of mathematics supervisors reported that "certified teachers of mathematics are *very* difficult to find." In some large cities there was a ratio as low as one applicant for each ten mathematics teaching vacancies. The supervisors felt that over the next two years the situation would worsen, with 73 per cent predicting it would be *very* difficult to fill mathematics teaching vacancies with qualified people.

The survey also found that almost 25 per cent of the reported teaching positions in mathematics for the 1980-81 school year were filled by teachers not permanently certified in mathematics. Faced with classes of students and no certified mathematics teachers, school systems have found that the most popular strategy for dealing with the shortage of qualified applicants is to assign teachers from other fields of preparation to teach mathematics. Competition with industries' salaries and the difficult teaching conditions are the two most frequently cited causes for the shortage.

According to NCTM's *An Agenda for Action: Recommendations for School Mathematics of the 1980s,* "public support for mathematics instruction must be raised to a level commensurate with the importance of mathematical understanding to individuals and society."

Free Calculator Information

The NCTM, in co-operation with the Calculator Information Center, has just announced the availability of *Uses of Calculators in Secondary Mathematics,* by Betty J. Krist, West Seneca Central Schools/State University of New York at Buffalo. In addition to suggested activities, this four-page information bulletin provides pragmatic suggestions and comments with regard to using calculators as an aid to student learning of secondary mathematics. Also, NCTM's eight-page listing of "Calculator Information Resources" has just been updated and is available without charge from the NCTM Headquarters Office.

Toronto: 60th Annual Meeting

The Meetings Committee and the Toronto Program Committee have developed three opportunities to involve you in the 60th Annual Meeting in Toronto, April 14-17, 1982.

Theme Papers

The Toronto Meeting will have an added dimension in that, instead of one theme, it will have three: *motivation, problem solving,* and the *role of technology.* Would you like to speak on one of these topics? Speakers will be selected on the basis of a review of submitted papers. Send five copies, typed and double spaced. Include a title, brief program description (25 words or less), the proposer's name and school as they should appear in the program, and a preferred mailing address. The reviewing teams will make their selections known by July 1, 1981.

Short Subjects

Any member who has a novel teaching strategy, a special treatment of some content area, or any combination of these is invited to submit a proposal for a 30-minute presentation for the annual "Short Subjects" feature. Proposals should be submitted in triplicate, typed and double spaced, and should contain (1) the title; (2) the proposer's name and school as they should appear on the program; (3) NCTM membership number; (4) preferred mailing address and telephone number;

(5) interest level (early childhood, intermediate grades, junior high school, senior high school, two-year college, teacher education, research, or general interest); and (6) a brief overview of the idea (not more than 200 words). The Program Committee will review the proposals, select those to be presented, and notify participants on or before July 1, 1981.

Research Proposals

Program space has also been reserved for those interested in research and its implications for classroom teachers, supervisors, and curriculum developers. If you wish to be considered for a place on the program, write for full details. Final submissions must be postmarked no later than July 3, 1981 to be considered.

* * * * * * * * * *

Papers or requests for information should be sent to Jesse Rudnick, Program Chairman, Ritter Hall 334, Temple University, Philadelphia, PA 19122.

COME OUT FOR MATH

(Grades 1 - 6)

-Rue, Hardesty, Fannin

Seventy-eight games for teaching basic math skills. Exciting math games with practical math applications can motivate children to learn and retain what they have learned. The games included in this book have all been used successfully in classrooms and remedial math labs. They are suitable for enrichment, practice, and as teaching tools. They may be incorporated into individualized math programs or used as free-time activities in more traditional classroom settings. Some of these games are self-checking; most are self-directed. Cost is $9.50; over 170 pages.

Available from:
Western Educational Activities Ltd.
Box 3806
Edmonton, Alberta T5H 2S7

Publisher: Incentive Publications

Why Study Mathematics?

by *J.G. Timourian*

Department of Mathematics
University of Alberta

This article is adapted from a lecture given by the author at the MCATA convention in Red Deer, November 1980.

Recently a group of students wrote the provincial education minister and asked, "Why do we have to learn division of polynomials? We will never have to use it."

I presume the students were learning how to do the following:

$$\begin{array}{r@{}l} & x^{360} + x^{359} + x^{358} + \cdots + x + 1 \\ \cline{2-2} x - 1 \,\big) & x^{361} - 1 \end{array}$$

We are all faced with these frustrating questions from students. No one asks the gym coach where the push-ups that are being taught will be used ten years from now. The gym coach's answer would probably be that the exercise is helpful in developing good muscle tone. A similar answer about the effects on the mind of mathematical exercises would not be accepted.

Any student who wants to do more in life than shovel snow will have to learn mathematics. I will explain some of the reasons why, and then give a specific answer to the question about division of polynomials asked by the students.

You can always ask a student, "What do you want to be when you grow up?" One typical answer might be "cattle rancher, just like Dad." At the University of Alberta, there is a department in the Faculty of Agriculture and Forestry called animal science. Many of the students in this department are interested in ranching. Very few of them are scholarly or academically oriented in a traditional sense.

To get a B.Sc. degree from the Department of Animal Science, you must take a calculus course. To learn this subject, you must have studied subjects such as division of polynomials. The degree programs in food science, plant science, rural economy, and soil science all require calculus.

Perhaps your student suddenly loses interest in ranching and wants to work in a drug store as a pharmacist. After all, these people translate doctors' prescriptions, mix drugs, and count money, and that doesn't appear to require advanced training. At the University of Alberta there is a Faculty of Pharmacy. Many of the students in this faculty are preparing to become pharmacists. To get a degree in this faculty, you are required to take calculus!

Now your question might be, "Why do these programs require calculus?" These requirements are not set by the Mathematics Department, but by the teachers in these other areas. Why do they require a subject that a few years ago was reserved for engineers and physicists?

There are several reasons for the requirement of sophisticated mathematics. Much of the mathematics they are required to learn is, in fact, useful in understanding the work they will do. I will give an example of this when I discuss division of polynomials.

Another reason for requiring the study of advanced mathematics is that it helps develop a logical, disciplined approach to problem solving. Such "mental tone" is useful in any subject, although it is difficult to explain how any one item of mathematics contributes to it.

Probably the most important reason for requiring advanced mathematics is overkill. Certainly the students will have to know and use simple algebra. The easiest way to give them confidence in that subject is to push them on to a more advanced one. Learning to do mathematics is similar to learning to play a musical instrument. What seems hard today becomes easy tomorrow, with practice. Meanwhile, what you try to learn tomorrow seems impossible, until the next day. For a student with average ability taking mathematics, you can assume that real understanding lags about one year behind the subject matter.

For many subjects, such as nursing, dentistry, geography, or education, a knowledge of statistics is required. Statistics is easier to understand and use if it is preceded by a calculus course.

Here are some surprising facts of life about mathematics at the university level. Calculus 202 is designed for science students who have not had Math 31 in high school. (We have other calculus classes for mathematics majors, business majors, and engineers.) We regard any student who has not had Math 31 as being poorly prepared for university mathematics. Out of 1,000 students who start in September, 500 will drop out or fail by the end of the first term, and only about 300 will achieve a passing grade by the end of the second term in June. That means that 70 per cent of the students entering the course either have to repeat it or have changed their career plans to accommodate their lack of success in mathematics. A large number of these students avoided mathematics in high school, and never dreamed that the subjects or careers they decided upon would end up requiring so much mathematics.

Suppose your student wants to be a real estate agent. You could suggest a commerce degree at a university, but it too requires a first-year course in calculus.

A student can, of course, become a real estate agent without a commerce degree. He or she doesn't really have to know advanced mathematics or, for that matter, how to divide polynomials. But if we want to be effective teachers, we have to have faith that life is better if one understands as much as one can about why things work the way they do.

Let's return to the problem of using polynomial division. Suppose you want to get a mortgage for $60,000 over 30 years, with an interest rate that is equivalent to 1.5 per cent compounded monthly. (Legally, all such mortgages are advertised on the basis of interest compounded every six months. For simplicity, assume that the equivalent monthly rate is given.)

What should the monthly payments be? There are books that are published in which you can look up the amount required. There are also calculators that can produce the monthly payment at the push of a button. You don't have to know how the numbers got into the book, just as you don't have to know what causes rain, what the moon is, or why we have lungs. On the other hand, knowing how the tables were assembled removes some of the sorcery from the process.

If someone misses two payments, is late with a third, has his interest changed at the end of one year, and must pay an interest penalty for paying off the whole mortgage upon sale of a house, then no book is going to provide details on what amounts should be paid. Now it is necessary to know how the information got into the mortgage books so that it can be modified to suit a special situation.

Set up a line indicating that 30 x 12 = 360 payments worth R each will be made at the end of each month. In return for these payments, the mortgage company will provide \$60,000 now. Let i = monthly interest rate = .015.

How much is the payment due at the end of the first month worth now? If A = how much it is worth now, then at the end of one month it is worth A again plus the interest for one month on A, or Ai. Thus, $R = A + A_i = A\,(1 + i)$. This means that the first payment of R dollars is worth $\frac{R}{1 + i}$ dollars today. A similar argument for the payment after two months shows that it is worth $\frac{R}{(1 + i)^2}$ today, and the last payment after 360 months is worth $\frac{R}{(1 + i)^{360}}$ today. All these added together should equal \$60,000. Thus,

$$60{,}000 = \frac{R}{1+i} + \frac{R}{(1+i)^2} + \frac{R}{(1+i)^3} + \cdots + \frac{R}{(1+i)^{360}}$$

or

$$60{,}000 = R\left(\frac{1}{1+i} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \cdots + \frac{1}{(i+1)^{360}}\right).$$

Examine the polynomial division result in the second paragraph and let $x = \frac{1}{1+i}$. Then

$$\frac{1}{1+i} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \cdots + \frac{1}{(i+1)^{360}} = \left(\frac{1}{1+i} - 1\right)\left(\frac{1}{(1+i)^{361}} - 1\right) - 1,$$

which simplifies to $\dfrac{1 - \dfrac{1}{(1+i)^{360}}}{i}$. Thus, the monthly rent R is found by solving the equation: $$60{,}000 = R\left(\frac{1 - \frac{1}{(1+i)^{360}}}{i}\right)$$

for R, given that i = .015.

The number $\left(\frac{1 - \frac{1}{(1+i)^{n}}}{i}\right)$ is usually symbolized with $a_{\overline{n}|i}$, which in this case is $a_{\overline{360}|.015}$. These numbers often come up in formulas for finance, and there are tables published which calculate them for various numbers of months and interest rates.

We have shown how division of polynomials can be used to understand how a formula for mortgage payments is arrived at. Now your potential real estate agent will insist that he or she expects to be very successful and could afford to hire experts to handle the understanding of such matters.

I recently served as a consultant for a well-established builder who had obtained an 18-month loan for $12 million. This wealthy and experienced man relied on "expert" advice, and apparently had little idea of how his experts went about what they were doing. At one point, just before final documents were signed, a lender changed an interest rate from an amount compounded semi-annually to one compounded monthly. When the builder asked for advice on the matter, he was told it was only a minor change. If it were interest on $50, the amount involved would have been minor. In relation to the total loan amount, perhaps it still was not significant. But the difference amounted to over $35,000, and certainly the lender knew that!

I was called in because what had started out as a simple mortgage loan of $12 million at 15 per cent turned out, once the various provisions were accounted for, to be a monster with an interest rate of 38 per cent (not including the fee I charged for my services).

This is the kind of trap that someone can fall into who doesn't spend the time to learn something about what experts do. People who will eventually have relatively unsophisticated jobs are asked to prepare themselves by taking advanced courses in mathematics so that they have some idea as to how the experts arrive at their conclusions. One consequence is that they are not so intimidated by expert knowledge.

At one time, calculus was regarded as a subject for the last years in university. It is now taught in high school. Some of the courses I took in graduate school not so long ago are now required of first-year students. This trend will continue.

Learning mathematics is hard work for many students, and it is very tempting for them to try to avoid the work by claiming it is of no use to them. Each instructor must radiate with confidence the feeling that every bit of mathematics the student is asked to learn in the early years has some positive effect on the student's preparation for life.

Pythagoras Revisited

by *Vernoy Johnson*

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Reprinted from The Illinois Mathematics Teacher, *Vol. 31, No. 3, May 1980.*

In his collection of 371 proofs of the Pythagorean Theorem, Loomis offers what he calls four kinds of demonstrations, those based on linear relations (algebraic proofs), on comparison of areas (geometric proof), on vector operations (quarternionic proofs), and on mass and velocity relationships (dynamic proofs). The point of this brief paper is to add a "proof" to Loomis' collection in a fifth category that I will call "people packing" (a humanistic proof).

The reader will quickly observe that it is not a proof at all, but rather a device for reinforcing in the student's mind the original meaning of Euclid's 47th proposition. For, although in *using* the theorem the relationship is most frequently translated into an algebraic formulation, the *meaning* of the theorem is a comparison of the areas of three squares.

The following "proof" works beautifully for a class of 20 to 30 students. You may have to round up a few more people if you make the figure larger than suggested. Mark the floor with masking tape to form a right triangle with sides of approximately three feet, four feet, and five feet, and mark the corresponding squares on each of the three sides. Invite people to stand, packed tightly, inside each of the two smaller squares. At this time you might remind the students what the theorem says about areas, and point out that the number of people standing in the squares is an indirect and, of course, very crude measure of the areas of those two squares. Then move the students to the largest square, the one on the hypotenuse. VIOLA! They all just fit! Of course you could get in one extra person, or leave one out and the "tightness" wouldn't be noticeable. But the image is there, in a dramatic and indelibly memorable way, one that can be easily referred to if uncertainties arise about the meaning of the theorem.

In spite of the rather ponderous and almost pontificial nature of Loomis' work, I think he would approve of the spirit, if not the rigor, of this 372nd "proof" of his beloved theorem. I believe it would be seen by him as an appropriate response to his "hope that this simple exposition of this historically renowned and mathematically fundamental proposition ... may interest many minds... Read and take your choice; or better, find a new, different proof...."

REFERENCE

Loomis, Elisha S. *The Pythagorean Proposition, Its Demonstrations Analyzed and Classified*, 2nd edition. Ann Arbor, Michigan: Edwards Bros., Inc., Litho printers, 1940.

Operation of Union and Intersection on Sets

by *Marlow Ediger*

Pupils in the elementary school should have learning activities in mathematics which are interesting, meaningful, and purposeful. "Learn by discovery" is a key concept in having pupils develop conclusions and generalizations in elementary school mathematics.

Pupils in the first grade can discover meanings pertaining to the operations of union and intersection on sets, which are disjoint as well as not disjoint, providing that the learning activities provided for them are interesting, meaningful, and purposeful. The operation of union of disjoint sets will be discussed first.

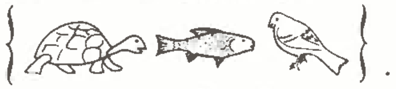
1. Use actual objects or pupils in the classroom. Pupils who have developed understandings pertaining to rational counting can also develop important understandings pertaining to the union of sets. Two boys can stand in front of the classroom representing one set, with three other boys in the second set. The question can be asked, "How many boys do we have if the two sets are joined?" The order of the sets could be changed when these pupils representing the two sets are standing in front of the classroom. Pupils could inductively develop the understanding that the operation of union on sets is commutative (pupils would develop the generalization in their own terminology which is meaningful to them). Real objects such as books, rulers, pencils, crayons, and toys can also be used to help pupils understand meanings pertaining to the operation of union on sets as well as the commutative property of union.

2. Use pictures. Pupils enjoy looking for pictures in discarded magazines in school as well as in the home. They can look for pictures of boats, cars, trucks, buses, and people. Set one could be made up of two cars such as a Chevrolet and a Ford, while the second set has three members - Plymouth, Rambler, and Dodge. The question that can be answered by pupils is, "If we join the cars in both sets to make a new set, how many cars do we have in this new set?" The commutative property of union should also be emphasized by changing the order of the two sets.

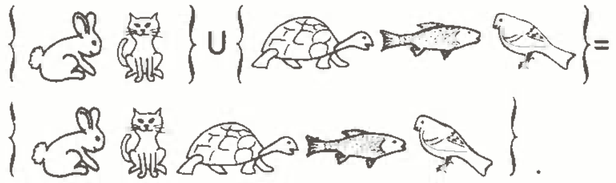
3. Use the flannel board. There should be felt cut-outs of various animals, people, cars, trucks, and geometric designs for pupils to utilize while responding to questions involving the joining of two sets. Using cut-outs which name a variety of animals, people, and so forth, provide for variety in learning activities for pupils, thus helping to maintain pupil interest. (The same would be true for varying the actual objects and pupils in developing sets as well as in the use of pictures.) When the flannel board is used, as well as in previous times, pupils should be able to describe a set accurately. For instance, the teacher could put the following on the flannel board and have pupils tell about the set:


.

The discussion would end with an accurate description of the set. Another set could be placed on the flannel board, such as:

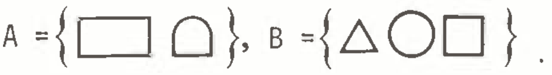


This set would also be described accurately. The question that can now be raised is, "If the two sets are joined, how many members do we have in the new set?" The new set that results can be visualized by pupils, such as:



Pupils should understand that what is located within the braces makes up the members of a given set. The commutative property of union can also be visualized by pupils when changing the order of the two sets. Pupils at this stage of learning need also to understand a related understanding to the operation of union and that is the operation of addition on numbers such as 2 + 3 = 5 in the previously discussed example. The number of members of the first set was two, and the number of members of the second set was three, therefore, 2 + 3 = 5.

4. Review previous learnings and utilize abstract symbols more frequently. In this stage of achievement pupils can deal effectively with more abstract symbols than previously. Sets on a flannel board can be labelled, such as:



Pupils can work problems using numerals only, such as:

$3 + 2 = \square$, $3 + \square = 5$, $2 + 3 = \square$, $\begin{array}{r}3\\ +2\\ \hline\end{array}$ $\begin{array}{r}2\\ +3\\ \hline\end{array}$,

and other addition facts that pupils have developed understandings to previously when persons, objects, pictures, and the flannel board were used. The teacher needs to remember that learning activities should be varied to develop and maintain pupil interest as well as provide for individual differences among pupils.

Pupils, toward the end of the first grade, can also discover the operation of intersection on sets. The operation of intersection on sets should be presented shortly after pupils develop understandings pertaining to the union of sets which are not disjoint.

1. Use dramatizations. These dramatizations should be realistic and lifelike. Don, Bill, and John are on a committee to feed pets in the classroom for one week. Ann, Judy, and Bill are on a different committee to take care of the plants in the classroom during the same week. Pupils could be asked, "Who are the members of the committee to take care of the pets in the classroom this week?" The names of the members of the committee to take care of the pets in the classroom this week should be written on the chalkboard:

{Don, Bill, John}.

"Which pupils are members of the committee to water plants this week?" These names should also be written on the chalkboard:

{Ann, Judy, Bill}.

Pupils could now see the two sets placed side by side, such as:

{Don, Bill, John}, {Ann, Judy, Bill}.

The next question that can be raised is, "How many members make up the two sets if they are joined?" If pupils respond with "six," the teacher should have the members of both committees come to the front of the room in order that all pupils can understand that there are five members making up the "union" of the two sets. On the chalkboard, the teacher can finish writing

{Don, Bill, John} ∪ {Ann, Judy, Bill} =

{Don, Bill, John, Ann, Judy}

during the final dramatization. Several dramatizations should be viewed by pupils so that they clearly understand the meaning of the operation of union sets which are not disjoint. Pupils should also understand the commutative property of union through dramatizations at this point.

The teacher should now have pupils develop inductively an understanding of the operation of intersection on sets. Pupils in the classroom can be asked, "Who is on the committee to feed our pets in the classroom this week?" Pupils will respond with the following names: "Don, Bill, and John." On the chalkboard, the teacher can write

{Don, Bill, and John}.

The next question asked of pupils could be the following: "Who is on the committee to take care of our plants in the classroom this week?" Pupils should respond with the correct names, "Ann, Judy, and Bill." The teacher, on the same line on the chalkboard, writes

{Ann, Judy, Bill}.

The teacher can now ask, "Which member is on both committees?" After pupils have responded correctly, the teacher can finish writing

{Don, Bill, John} ∩ {Ann, Judy, Bill} =

{Bill}.

Pupils should develop an accurate, meaningful understanding of the abstract symbol for the operation of intersection. Several dramatizations should be used in order that pupils understand the meaning of a member being common to two sets.

2. Use the flannel board. Cut-outs of animals, people, cars, trucks, and geometrical designs can be used. The teacher can place a felt cut-out of a duck, pig, and rabbit in one set, such as:

 .

Beside it, a second set could be placed made up of a duck and a cat, such as:

 .

Pupils, under teacher guidance, could describe accurately each set. The teacher could now ask, "How many different kinds of farm animals would there be if we joined the two sets to make a new set?" If some pupils respond with "five," in a discussion pupils can develop the understanding that duck is a member of both sets; there are four members in the new set which can be visualized by pupils as:

 .

Since "duck" is common to both sets,

3. Use abstract symbols. The letters of the alphabet, the days of the week, and/or the months of the year can be written on the chalkboard. For instance, pupils are asked to name the first three days of the week for

one set; as pupils mention the names, the teacher can write the set as

{Sunday, Monday, Tuesday}.

The teacher can then ask pupils to name the last five days of the week; the teacher or another pupil can write on the chalkboard the second set consisting of the last five days of the week:

{Tuesday, Wednesday, Thursday, Friday, Saturday}.

The teacher then asks pupils, "If the two sets are joined, what members make up the new set consisting of the days of the week?" The teacher writes the names as they are mentioned by pupils. If pupils respond with the following as being the union of the two sets mentioned previously, "Sunday, Monday, Tuesday, Tuesday, Wednesday, Thursday, Friday, Saturday," the teacher can have pupils look at a calendar in order to name the days of the week. Most pupils can, of course, at this point recite the days of the week. Pupils can inductively develop the understanding that

{Sunday, Monday, Tuesday} ∪ {Tuesday, Wednesday, Thursday, Friday, Saturday} = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}.

At this point, pupils can also be asked, "Which member is common to both sets?" The teacher, after receiving the correct response from pupils, can write on the chalkboard:

{Sunday, Monday, Tuesday} ∩ {Tuesday, Wednesday, Thursday, Friday, Saturday} = {Tuesday}.

Pupils should notice the symbol "∩", and how it differs from the symbol "∪" used in joining two sets. A discussion should follow in which the symbols "∪" and "∩" become differentiated and understood by pupils so that meaningful learning may take place.

Further learning activities for pupils in understanding what, in adult terms, would be the "union and intersection of sets which are not disjoint" could be the following.

Have pupils name the first two months of the year for the first set. Next, have pupils name the first three months of the year as the second set. Write the specific sets on the chalkboard at the time they are given by pupils. Disagreements among pupils as to the correct sets wanted can make for excellent discussions in the classroom; in these discussions pupils reveal correct as well as incorrect understandings. If pupils want to mention the names of months more than once in the union of the two previously mentioned sets, such as incorrectly stating that

{January, February} ∪ {January, February, March} = {January, February, January, February, March},

the teacher can ask the question, "What set is made up of the first three months of the year?" After a discussion, pupils will generalize that the set consisting of the first three months of the year is

{January, February, March} and not {January, February, January, February, March}.

Pupils could then be asked which member or members are common to both sets. The teacher can write pupil responses on the chalkboard as they are given, using the appropriate symbols for sets named and the correct symbol for intersection of the two sets.

The order of sets can also be changed so pupils can inductively understand that in union and intersection of sets the order is not important.

??? Problem Corner ???

edited by *William J. Bruce* and *Roy Sinclair*

University of Alberta
Edmonton, Alberta

Problems suggested here are aimed at students of both the junior and senior high schools of Alberta. Solutions are solicited and a selection will be made for publication in the next issue of *delta-K*. Names of participants will be included. All solutions must be received (preferably in typewritten form) within 30 days of publication of the problem in *delta-K*.

Mail solutions to: Dr. Roy Sinclair or Dr. Bill Bruce
Department of Mathematics
University of Alberta
Edmonton, Alberta T6G 2G1

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Solutions to Problems 2 and 3 will be published in the next issue of *delta-K*.

Problem 4:

(submitted by Dr. A. Meir, University of Alberta)

Let p be a prime number and a be a positive integer. Show that $(a-p)^3 + a^3 = (a+p)^3$ cannot be true.

Rec. Corner

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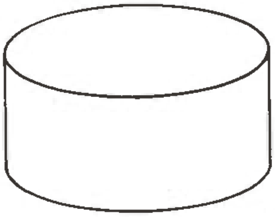
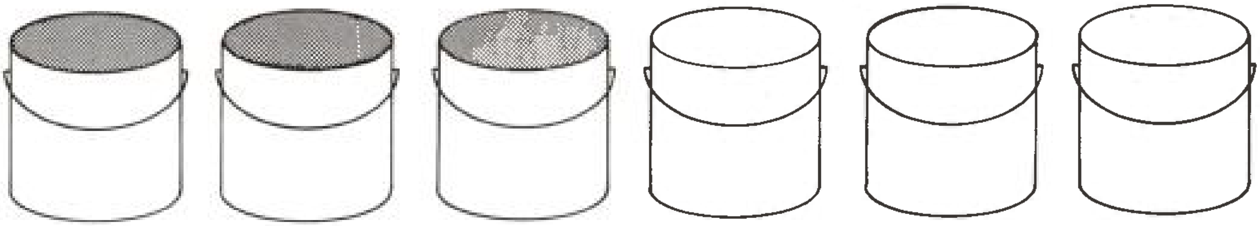
1. Cut the cake into 8 pieces using only 3 cuts.



2.



Move just one pail and end up with an alternating pattern of full and empty pails.

3. Study the following examples:

$$\begin{array}{r} 47 \\ \times 43 \\ \hline 2021 \end{array} \qquad \begin{array}{r} 52 \\ \times 58 \\ \hline 3016 \end{array} \qquad \begin{array}{r} 36 \\ \times 34 \\ \hline 1224 \end{array}$$

These products can be written in one line in seconds.

What characteristics are common to all three examples?

What is the short-cut method?

Try your method on these:

$$\begin{array}{r} 85 \\ \times 85 \\ \hline \end{array} \qquad \begin{array}{r} 71 \\ \times 79 \\ \hline \end{array}$$

Why does it work?

IDEAS

Gleaned from the 20th Annual MCATA Meeting

Ideas in this article were adapted from a session by Jim Barnes, presented at the 20th Annual MCATA Meeting held in Red Deer on November 7 and 8, 1980.

These are Special!

Grade Level: 2-12

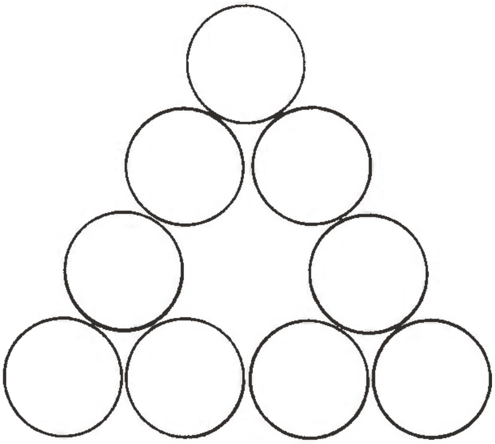
	These are special.	These are *not* special.	Which of these are special?
Decide on any mathematical concept. Put examples of the concept in the left column, non-examples in the centre column, and some examples and some non-examples in the right column. Students are to decide which of those in the right column are examples.			
EXAMPLE: Prime numbers	2,11,13,47	1,4,15,39	5,9,21,53

* *

Number Puzzle

Grade Level: 4-8

1. Use each of the digits 1-9 once so that the sum of each side is 17.
2. Rearrange so that the sum of each side is 20.
3. Rearrange again to make the largest sum. What is the largest sum?



Other IDEAS

27	3	64	4
1	1	343	7
	6	8	2

What number goes in the empty cell to maintain the pattern?

Q R Q

R T

Q

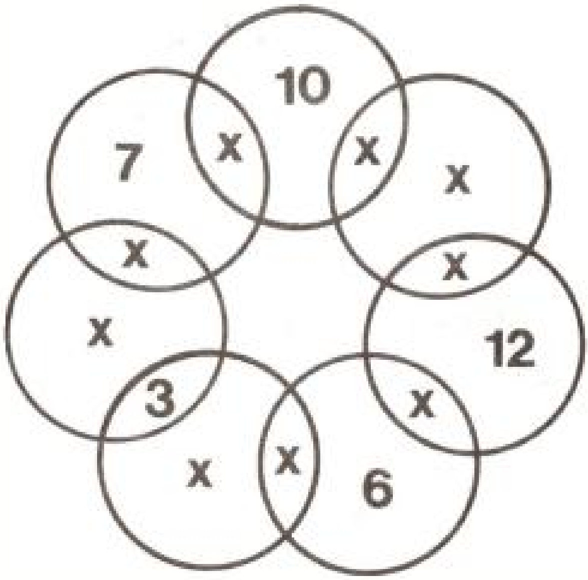
In this figure, each letter stands for a different digit. Row one represents a three-digit number. Row two represents a two-digit number that is the sum of the digits in row one. Row three is a single-digit number that is the sum of the digits in row two. Find the digits. All numbers are in a base ten notation.

Make 21

	5	4	
8			2
1			7
	9	0	

Put the *same* number in each corner to make the sum of each row and each column 21.

Make 21



Make the sum of the three numbers in each circle 21. Use only the numbers 1-14 and each number only *once*.

activities

Beyond the Usual Constructions

by *Melfried Olson*

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Teacher's Guide

Grade level: 7–12.

Introduction: Basic constructions concerning triangles and the nine-point circle were introduced in this column by Allen in 1972. There are other interesting relationships concerning triangles and circles that can be explored by using constructions. The activities below provide additional construction experiences.

Materials: Compass, protractor, straightedge, and copies of the next three sheets for each student.

Objectives: After a review of similarity of triangles, congruence of circles, work with half planes, and the constructions of centroid, orthocenter, circumcenter, incenter, and the nine-point center and nine-point circle of a triangle, the following set of exercises can be used as motivational material for further discussion of special relationships derived from a triangle and previously mentioned basic constructions.

Directions: Make copies of the following pages—enough to allow for student mistakes. You may need to review the basic constructions if you haven't worked with them recently. These include bisecting angles, constructing a perpendicular from a point to a line, and constructing a perpendicular bisector of a segment. Appropriate discussion should follow each activity, and the interested student could pursue similar material as a class project.

Solutions

1B. Angle bisectors of $\triangle JKL$ are altitudes of $\triangle ABC$.

Note: If a nonacute $\triangle ABC$ is used, the altitudes will intersect outside the triangle so the relationship in 1B will not hold.

2C. The segments are concurrent. The point at which they are concurrent is called the *Fermat point* of the $\triangle ABC$ (denoted F).

D. $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$ all have the same length.

E. The measures of the six angles are congruent.

3C. $\overline{PQ} \| \overline{JK}$, $\overline{PR} \| \overline{JL}$, and $\overline{QR} \| \overline{KL}$, and, as such, $\triangle PQR$ and $\triangle JKL$ are similar.

4D. The points M, N, and O are collinear. This line is called the ***Simson line of P*** for the triangle.

5. Note: This activity should be done as a parallel construction to Euler's nine-point circle (see Allen).

5D. The nine-point center of $\triangle ABC$ is Z. The nine-point circle of $\triangle ABC$ is the circumcircle of $\triangle JKL$.

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Allen, Charles E. "Activities: Mission—Construction." *Mathematics Teacher* 15(November 1972): 631–34.

Eves, Howard. *Fundamentals of Geometry*. Boston: Allyn & Bacon, 1969.

BEYOND THE USUAL CONSTRUCTIONS

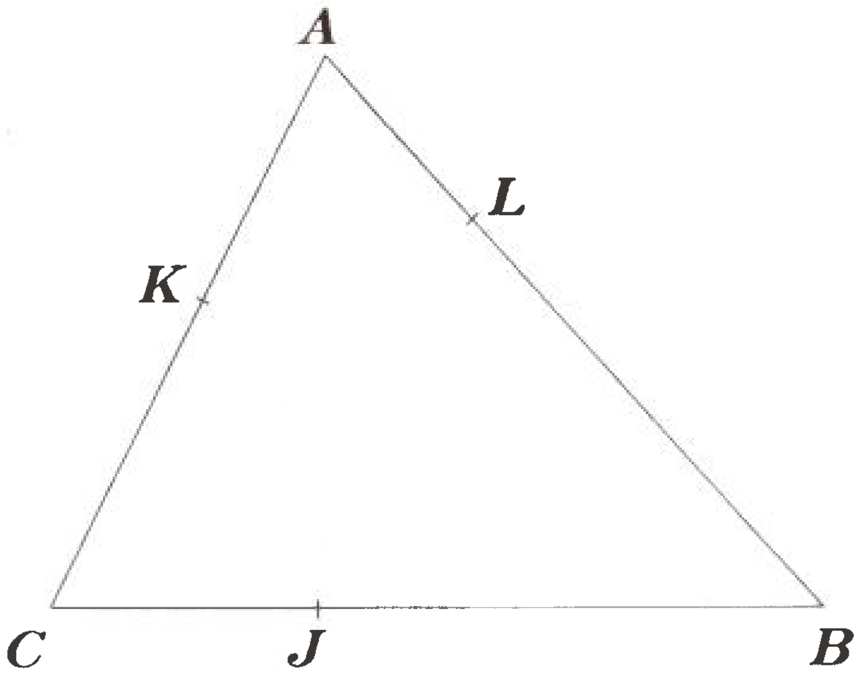
ACTIVITY 1

A. Points J, K, and L are the feet of the altitudes from A, B, and C, respectively. Draw $\triangle JKL$.

$\triangle JKL$ is called the orthic triangle of $\triangle ABC$.

B. Construct the bisectors of the angles of $\triangle JKL$, labeling the point of intersection of the angle bisectors H. What do you notice?

With respect to $\triangle ABC$, H is called the orthocenter, but with respect to $\triangle JKL$, H is called the incenter.



ACTIVITY 2

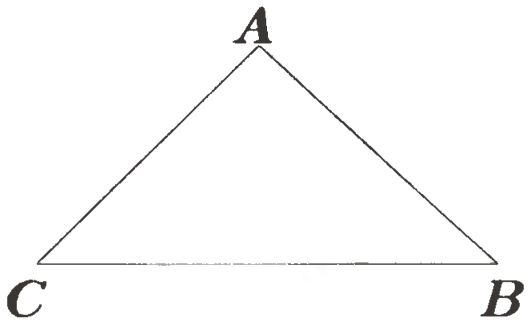
A. Construct an equilateral $\triangle ABC'$ with sides of length AB so that C' is not in the same half plane as C determined by $\overleftrightarrow{AB}$.

B. Repeat (A) for the other sides of $\triangle ABC$ so as to have three equilateral triangles, $\triangle ABC'$, $\triangle AB'C$, and $\triangle A'BC$.

C. Draw $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$. What do you observe?

D. Measure $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$. What do you observe?

E. Label $\overline{AA'} \cap \overline{BB'} \cap \overline{CC'}$ as F. Measure the six angles formed around F. What do you observe?



SHEET 1

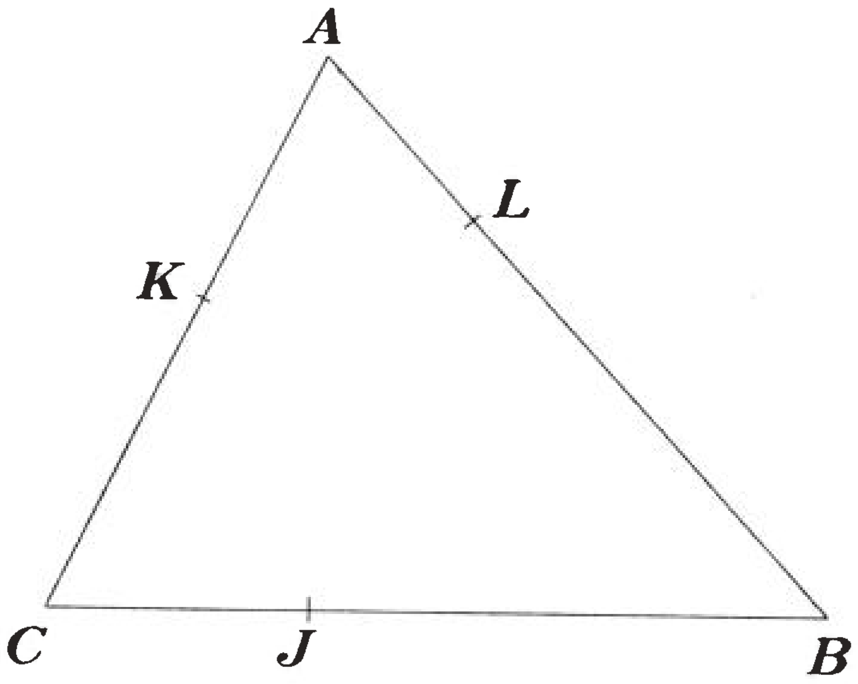
BEYOND THE USUAL CONSTRUCTIONS

ACTIVITY 3

A. Points J, K, and L are the feet of the altitudes from A, B, and C, respectively. Draw $\triangle JKL$.

B. Draw the perpendicular bisectors of $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$. Label the point of intersection of the perpendicular bisectors D. With respect to $\triangle ABC$, D is called the circumcenter. Using D as the center and $\overline{AD}$ as the radius, draw the circle that also contains B and C. This is called the circumcircle of $\triangle ABC$.

C. Let $\overrightarrow{AJ}$, $\overrightarrow{BK}$, and $\overrightarrow{CL}$ intersect the circumcircle in points P, Q, and R, respectively. Draw $\triangle PQR$. How are $\triangle PQR$ and $\triangle JKL$ related?



SHEET 2

BEYOND THE USUAL CONSTRUCTIONS

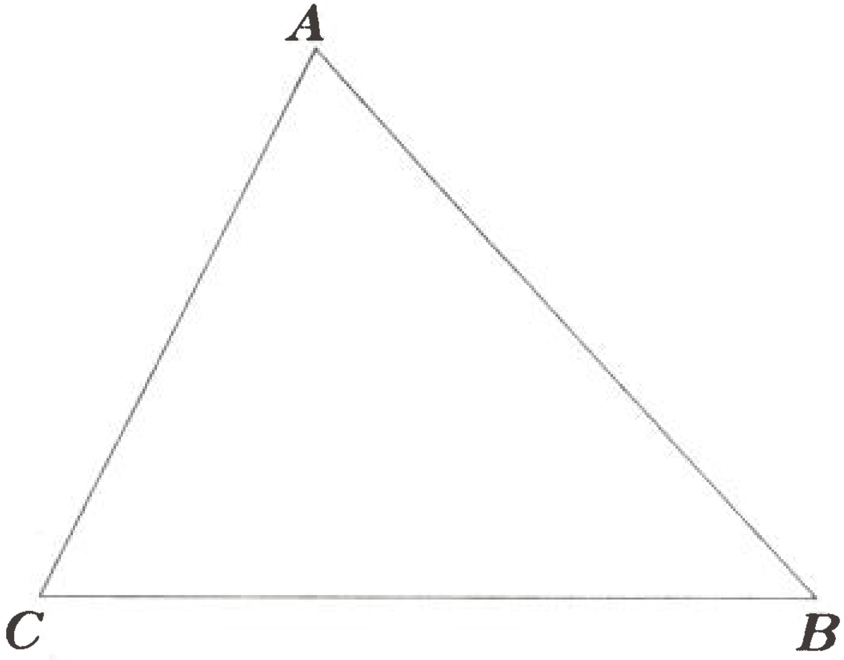
ACTIVITY 4

A. Find the circumcenter and circumcircle of $\triangle ABC$.

B. Choose a point P (different from A, B, or C to start with) on the circumcircle.

C. Construct $\overline{PM} \perp \overleftrightarrow{BC}$ at M, $\overline{PN} \perp \overleftrightarrow{AC}$ at N, and $\overline{PO} \perp \overleftrightarrow{AB}$ at O.

D. What do you observe about M, N, and O?



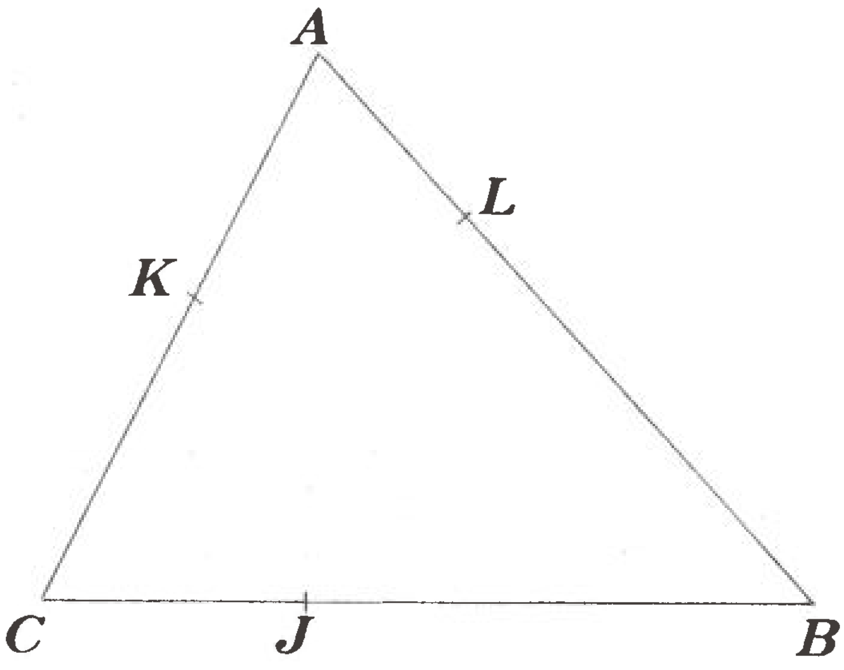
ACTIVITY 5

A. Points J, K, and L are feet of the altitudes from A, B, and C, respectively. Draw $\triangle JKL$ and find the orthocenter H of $\triangle ABC$.

B. Find the circumcenter of $\triangle JKL$, call it Z, and draw the circumcircle of $\triangle JKL$. Find the circumcenter D of $\triangle ABC$.

C. If we consider $\triangle ABC$ and find the midpoints of the sides, the feet of the altitudes, and the midpoints of $\overline{HA}$, $\overline{HB}$, and $\overline{HC}$, we note that these nine points lie on a circle (called the nine-point circle of $\triangle ABC$) with center N the midpoint of $\overline{HD}$.

D. Compare Z to the nine-point center of $\triangle ABC$, and compare the circumcircle of $\triangle JKL$ to the nine-point circle of $\triangle ABC$. What do you find?



Ideas

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The *Ideas* this month asks students to make estimates and to be alert to the reasonableness of results. Estimating answers and getting approximate results are important skills for consumers when they are doing quick calculations. Alertness to the reasonableness of a result is a valuable skill to accompany calculator usage.

IDEAS For Teachers Levels: 1-4

GUESS AND TEST

Objective:

Experience in estimating quantities and gathering data.

Materials needed:

- A clock or watch that measures seconds, or timers for measuring 15, 30, and 60 seconds.
- One copy of the worksheet per student.

Review:

How to measure 15, 30, and 60 seconds.

Directions for teachers:

1. Ask each student to guess how many times he or she could do the activities in the 15-second category. Have them write their estimates in the "Guess" boxes.
2. Then have students work with partners to time one another in doing the activities.
3. Next, students should compare their "Guess" and "Test" columns.
4. Have them follow the same procedure for the 30-second and 60-second questions.
5. When they have finished guessing and testing, ask them to circle their best guesses.

Extension:

1. Have students make up Guess-and-Test activities of their own.
2. Make a class "Record Book" for these and other activities.

IDEAS For Teachers Levels: 3-4

GUESSTIMATES

Objective:

Practice in estimating quantities, gathering data, and inspecting data to find the most reasonable answer.

Materials needed:

- A clock or watch that measures seconds.
- Copies of the worksheet.

Review:

How to measure seconds and the number of seconds in a minute.

Directions for teachers:

1. Ask each student to guess how long it would take her or him to do each of the activities in the box at the top of the page. Have them write their estimates in the "Guess" column.
2. Have the students work with partners to time the activities.
3. For the second exercise, students should answer yes or no based on their past experiences. Have them discuss and defend each answer in this section.

Extension:

Have students make up some exercises like those in the second section to try on each other. They should try them out on themselves first.

IDEAS For Teachers Levels: 5-6

LEAD-FREE MATH

Objective:

Practice in rounding off numbers and estimating the results of addition, subtraction, multiplication, and division with whole numbers.

Materials needed:

- Six markers (chips, cubes, pieces of paper, beans, paper clips, or anything else that will fit in the squares on the worksheet) per student.
- Calculators.

Review:

How to round off numbers and make estimates.

Directions for teachers:

1. Without writing anything down, students should estimate the answer to each example in the squares and put a marker in the square that would give an answer closest to the answer given.
2. For the last two problems, students should put a marker on the number that will give the indicated answers.
3. When they are finished, the students should check their answers with a calculator.

Answers:

184 - 129; 636 ÷ 6; 195 x 3;
1289 - 817; 34; 1000.

IDEAS For Teachers Levels: 7-8

GETTING THE LEAD OUT

Objective:

Practice in rounding off numbers and estimating the results of addition, subtraction, multiplication, and division problems with decimals.

Materials needed:

- Seven markers (chips, cubes, pieces of paper, beans, paper clips, or anything else that will fit in the squares on the worksheet) per student.
- Calculators.

Review:

How to round off numbers and estimate answers.

Directions for teachers:

1. Without writing anything down, students should estimate the answer to each of the examples in the squares and put a marker in the square that would give an answer closest to the number in the answer column.
2. For the last three problems, students should put a marker on the number that will give the indicated approximate answer.
3. Students should check their answers with a calculator when they are finished.

Answers:

9 ÷ 2; 50.3 - 30.28; $0.6\overline{)18.6}$;
$0.1\overline{)22}$; 300; 48.8; 0.84.

IDEAS

Name ____________________

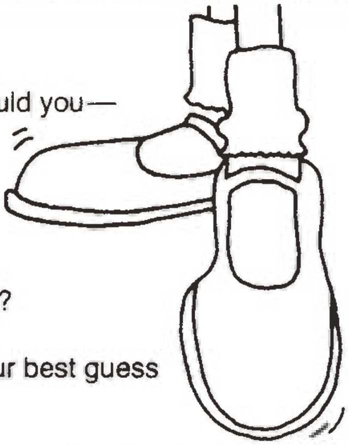
Guess and Test

In 15 seconds:

How many times could you—

- tap your foot?
- make a fist?
- blink your eyes?
- snap your fingers?

Go back and circle your best guess



Guess	Test

In 30 seconds:

How far could you count by fives?
How many foods could you name?
How many times could you tie your shoe?
How many times could you tap your foot?

Guess	Test

Go back and circle your best guess.

In 60 seconds:

How far could you count by ones?
How many people in your class can you name?
How many numbers could your write?
How many times could you tap your foot?

Guess	Test

Go back and circle your best guess.

Are you a good guesser? ☐ Yes ☐ No

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IDEAS

Name ____________________

Guesstimates

How long would it take you to—

	Guess	Test
hop 10 times?		
snap your fingers 20 times?		
count backwards from 20?		
count by 5's to 100?		
write the numbers you say when you count by 2's to 50?		
tie your shoe 10 times? (or someone else's)		
write 40 X's on your paper?		
write your name, address, and telephone number?		

Go back and circle your best guess.
Are you a good guesser? ☐ yes ☐ no

Would you believe—

	Yes	No
Gary hopped 10 times in 5 seconds?		
Ursula counted by 5's to 100 in 10 seconds?		
Emily wrote by 2's up to 40 in 75 seconds?		
Sheila counted backwards from 20 in 5 seconds?		
Steve snapped his fingers 40 times in 3 seconds?		

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IDEAS

Name ____________________

Lead-Free Math

(No pencils allowed)

55 =

6 + 97	15 × 25
100 ÷ 25	184 − 129

106 =

636 ÷ 6	$\begin{array}{r} 79 \\ +\ 87 \\ \hline \end{array}$
$\begin{array}{r} 136 \\ -\ 68 \\ \hline \end{array}$	$\begin{array}{r} 26 \\ \times\ 6 \\ \hline \end{array}$

585 =

$\begin{array}{r} 834 \\ -\ 429 \\ \hline \end{array}$	$\begin{array}{r} 195 \\ \times\ 3 \\ \hline \end{array}$
$19\overline{)1235}$	$\begin{array}{r} 439 \\ +\ 316 \\ \hline \end{array}$

472 =

$\begin{array}{r} 75 \\ 233 \\ +\ 115 \\ \hline \end{array}$	$\begin{array}{r} 1289 \\ -\ 817 \\ \hline \end{array}$
$\begin{array}{r} 174 \\ \times\ 3 \\ \hline \end{array}$	1860 ÷ 5

12 ×

12
418
34

is approximately 400

10 000
100
1000

÷ 23 is approximately 40

Use a calculator to check your answers.

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IDEAS

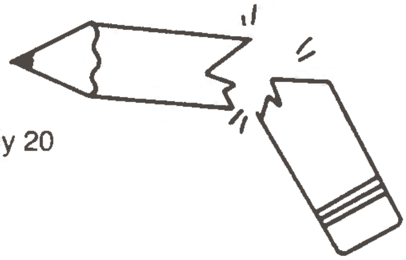
Name ____________________

Getting The Lead Out

(That is, don't use your pencil to figure these out)

The answer is—	Which one is the problem?			
4.5	4.00 + 0.05	8.6 − 2.1	0.9 × 0.5	9 ÷ 2
20.02	400.04 ÷ 2	50.3 − 30.28	1.802 + 2	1001 × 0.2
21	45.6 − 1.46	$6\overline{)18.6}$	17.5 + 1.35	15.5 × 0.2
220	$0.1\overline{)22}$	180 + 0.40	2.2 × 10	320 − 10.0

	0.3	
0.06 ×	3	is approximately 20
	300	



488	
4.88	− 0.1 is approximately 500
48.8	

84	
8.4	÷ 0.03 is approximately 30
0.84	

Check your answers with a calculator

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