# Pursuing Per Cent Problems

*by Bonnie H. Litwiller and David R. Duncan*
*Mathematics Department*
*University of Northern Iowa*
*Cedar Falls, Iowa*

---

The following four problems illustrate different applications of the concept of per cent. In each case, a per cent is to be computed. Although the problems may appear similar at first inspection, they are, in fact, quite different.

Let a and b be fixed constants with $a < b$.

1. a is what per cent of b?
2. b is what per cent of a?
3. b is what per cent greater than a?
4. a is what per cent less than b?

To illustrate these numerically, let $a = 4$ and $b = 5$.

1. 4 is 80% of 5. $(80 = \frac{4}{5} \cdot 100)$
2. 5 is 125% of 4. $(125 = \frac{5}{4} \cdot 100)$
3. 5 is 25% greater than 4. $(25 = \frac{(5-4)}{4} \cdot 100)$
4. 4 is 20% less than 5. $(20 = \frac{(5-4)}{5} \cdot 100)$

In general for $a < b$, the answers to the four problems are:

1. If a is $p_1$% of b, then $p_1 = \frac{a}{b} \cdot 100$.
2. If b is $p_2$% of a, then $p_2 = \frac{b}{a} \cdot 100$.
3. If b is $p_3$% greater than a, then $p_3 = [\frac{(b-a)}{a} \cdot 100]$.
4. If a is $p_4$% less than b, then $p_4 = [\frac{(b-a)}{b} \cdot 100]$.

Let us now find relationships among $p_1$, $p_2$, $p_3$, and $p_4$.

A. $p_3 = \frac{(b-a)}{a} \cdot 100$

$$= \frac{b(100)}{a} - \frac{a(100)}{a}$$

$$= \frac{b(100)}{a} - 100$$

Therefore, $p_3 = p_2 - 100$

For example, if b is 119% of a, then b is 19% greater than a.

B. $p_4 = (\frac{b-a}{b}) \cdot 100$

$$= \frac{b(100)}{b} - \frac{a(100)}{b}$$

$$= 100 - \frac{a(100)}{b}$$

Therefore, $p_4 = 100 - p_1$

For example, if a is 83% of b, then a is 17% less than b.

C. $p_1 \cdot p_2 = (\frac{a}{b} \cdot 100) \cdot (\frac{b}{a} \cdot 100)$

$$= 10\ 000$$

Therefore, $p_1 = \frac{10\ 000}{p_2}$ and $p_2 = \frac{10\ 000}{p_1}$

For example, if a is 50% of b, then b is 200% of a $(200 = \frac{10\ 000}{50})$; if b is 250% of a, then a is 40% of b $(\frac{10\ 000}{250})$.

D. $p_3 = p_2 - 100$

$$= \frac{10\ 000}{p_1} - 100$$

$$= \frac{10\ 000}{100-p_4} - 100$$

$$= \frac{10\ 000 - (100-p_4)100}{100-p_4}$$

$$= \frac{10\ 000 - 10\ 000 + p_4(100)}{100 - p_4}$$

Therefore, $p_3 = \dfrac{100\ p_4}{100-p_4}$

E. Solving for $p_4$:

$p_3(100-p_4) = 100\ p_4$

$100\ p_3 - p_3p_4 = 100\ p_4$

$100\ p_3 = p_3p_4 + 100\ p_4$

$100\ p_3 = (p_3 + 100)p_4$

Therefore, $p_4 = \dfrac{100\ p_3}{p_3+100}$

Examples of D and E follow:

If a is 13% less than b, then b is $\dfrac{100(13)}{100-13} = \dfrac{1300}{87}$ or 14.9% greater than a.

If b is 31% greater than a, then a is $\dfrac{100(31)}{100+31} = \dfrac{3100}{131}$ = 23.7% less than a.

One final real world example is given below. The salary of the superintendent of schools is \$38 000 while that of a mathematics teacher with 20 years' experience is \$18 000. Using the language of per cent, the relationship may be described as follows:

1. The salary of the mathematics teacher is 47.4% of the salary of the superintendent. ($p_1$ = 47.4)

2. The salary of the superintendent is 211.1% of the salary of the mathematics teacher. ($p_2$ = 211.1)

3. The salary of the superintendent is 111.1% greater than the salary of the mathematics teacher. ($p_3$ = 111.1)

4. The salary of the mathematics teacher is 52.6% less than the salary of the superintendent. ($p_4$ = 52.6)

The relations of 3 and 4 may be restated: If the superintendent and mathematics teacher were to exchange salaries, the mathematics teacher would receive a salary increase of 111.1% while the superintendent would receive a salary decrease of 52.6%.

We now verify that relations A through E hold.

A. $p_3 = p_2 - 100$, so 111.1 = 211.1 - 100

B. $p_4 = 100 - p_1$, so $52.6 = 100 - 47.4$

C. $p_1 = \frac{10\ 000}{p_2}$, so $47.4 = \frac{10\ 000}{211.1}$

$p_2 = \frac{10\ 000}{p_1}$, so $211.1 = \frac{10\ 000}{47.4}$

D. $p_3 = \frac{100p_4}{100-p_4}$, so $111.1 = \frac{100(52.6)}{100-52.6}$

E. $p_4 = \frac{100p_3}{p_3+100}$, so $52.6 = \frac{100(111.1)}{111.1+100}$

The reader is to verify these relationships with other sets of data.

---

## SELF-TEACHING CARDS (Vertical Form)

Publisher: Kenworthy Services

These three-purpose cards enable students to learn number combinations, test themselves, and check their answers. Math problems are displayed in vertical format (Traditional Math). Each card is placed on a sheet of paper, test side up. Students write answers through cut-out section. To check answers, the cards are turned over, displaying complete problem, correct answer, and students' answers. Printed and die-cut on durable card stock. Each pack contains 12 identical test cards.

Contents:
- Addition
- Subtraction
- Multiplication
- Division

Price:
$3.90/each

Available from:
Western Educational Activities Ltd.
10929 - 101 Street
Edmonton, Alberta T5H 2S7

(See Catalog - p.175)