An Advisory Exam in Mathematics

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The Mathematics Department at the University of Alberta gives an examination to the students in its introductory calculus courses early in each fall term. This exam serves to locate those students whose background needs upgrading. A remedial program is set up for the benefit of these students. An increasing number of disciplines now require an introductory calculus course of their students, and results in these courses indicate that an inadequate proficiency with the fundamentals is a real stumbling block for the students.

Our remedial program is of a voluntary nature and of about a five-week duration. This year, the student response was very positive. In the coming year, we hope to supplement the program with computer-based (Plato) material.

We would like the teachers of mathematics in Alberta to be kept informed of the nature of these exams and of our remedial efforts. We welcome any suggestions from them on this procedure.

The 1980 examination is given below, and is followed by its statistics.

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1. How many of the following numbers are irrational? $\{1.1, (-32)^{\frac{1}{5}}, 1.111...,$

    $\frac{\sqrt{3}}{\sqrt{12}}, \pi\}$

    (a) one   (b) two   (c) three   (d) four   (e) five

2. $\frac{\sqrt{2} - 1}{\sqrt{2} + 1} =$ (a) $3 + 2\sqrt{2}$ (b) $(\sqrt{2} - 1)^2$ (c) $\frac{1}{\sqrt{3}}$ (d) $1 - 2\sqrt{2}$ (e) none of these

3. $2(3+x) \geq 8x + 3(x+2)$ is equivalent to

    (a) $x \leq \frac{1}{9}$ (b) $x \geq 0$ (c) $x \leq \frac{2}{11}$ (d) $x \leq 0$ (e) none of these

4. The solution set of $|2x - 1| = 7$ is

    (a) $\{4\}$ (b) $\{4,-4\}$ (c) $\{3,-3\}$ (d) $\{-4,3\}$ (e) none of these

5. If $\frac{1}{x} + \frac{1}{y} = \frac{1}{R}$, then $x =$

   (a) $\frac{Ry}{y-R}$ (b) $\frac{y}{R-y}$ (c) $\frac{R+y}{Ry}$ (d) $R - y$ (e) none of these

6. $\dfrac{x^3 + 2x^2 - 1}{x + 1} =$

   (a) $x^2 + 2x - 1$ (b) $x^2 + x + 1$ (c) $x^2 - x + 1$ (d) $x^2 - x - 1$ (e) none of these

7. $\dfrac{1 + \dfrac{1}{y} - \dfrac{20}{y^2}}{1 + \dfrac{9}{y} + \dfrac{20}{y^2}} =$ (a) $\dfrac{1}{9}$ (b) $\dfrac{y - 5}{y + 5}$ (c) $\dfrac{y - 4}{y + 4}$ (d) $\dfrac{y + 4}{y - 4}$ (e) none of these

8. $2x^2 + 3x + c = 0$ has equal real roots if $c =$

   (a) $\dfrac{3}{8}$ (b) $\dfrac{9}{2}$ (c) $-\dfrac{3}{4}$ (d) $\dfrac{9}{8}$ (e) none of these

9. If $g(x) = \dfrac{1}{2x}$, then $g(g(2)) =$ (a) 4   (b) 2   (c) $\dfrac{1}{2}$ (d) 1   (e) none of these

10. If $f(x) = 3x^2$, then $\dfrac{f(1+h) - f(1)}{h} =$

    (a) $9(h+2)$ (b) $3h$ (c) $3(2+h)$ (d) $3(1 + \dfrac{2}{h})$ (e) none of these

11. $\dfrac{(-3y)^2 y^{-\frac{2}{3}}}{(2y)^{-1} 3y^{\frac{1}{3}}} =$ (a) $6y^2$ (b) $\dfrac{3}{2y^2}$ (c) $4^{\frac{1}{3}}(6y)^2$ (d) $6(3^{\frac{2}{3}})y^2$ (e) none of these

12. $(x^{-2}+x^{-4})^{-\frac{1}{2}} =$ (a) $x + x^2$ (b) $\dfrac{x^2}{x + 1}$ (c) $\dfrac{x^2}{\sqrt{x^2+1}}$ (d) $x\sqrt{x^2+1}$ (e) none of these

13. If $s = \dfrac{1}{2}gt^2$, then $\log t =$

    (a) $\dfrac{1}{2}(\log s + \log 2 - \log g)$ (b) $\log\left(\dfrac{s}{g}\right)$ (c) $\log s - \log 2 - \log g$ (d) $\dfrac{1}{2}\log(2s-g)$

    (e) none of these

14. If $\log(x+3) = \log x + \log 3$, then $x =$

    (a) 0   (b) any positive number   (c) $\dfrac{3}{2}$ (d) $\dfrac{2}{3}$ (e) none of these

15. If $9 = 4^x$, then $x =$

    (a) $\log\dfrac{9}{4}$ (b) $\dfrac{\log 4}{\log 9}$ (c) $\log_9 4$ (d) $\log_4 9$ (e) none of these

16. Which of the following represents the graph of $y = 2^x$?
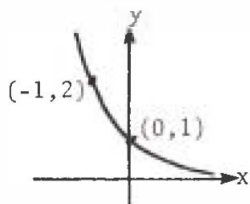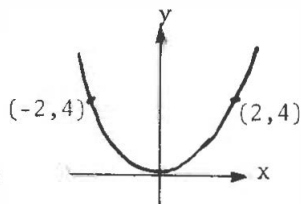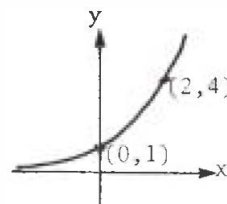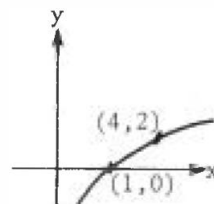

(a)   (b)   (c)   (d)   (e) none of these
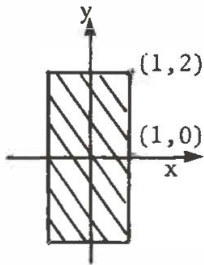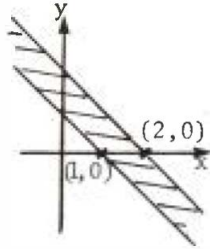
17. The lines $2x - y = 1$ and $2y + bx = 3$ are perpendicular if $b =$

   (a) 4   (b) 1   (c) -1   (d) -4   (e) none of these
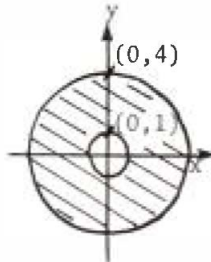
18. Which of the following regions is represented by the inequalities $1 \leq x^2 + y^2 \leq 4$?
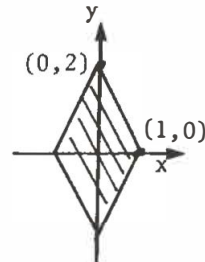

   (a)   (b)   (c)   (d)   (e) none of these

19. The vertex of the parabola $y = x^2 - 2x$ is the point

   (a) (2,0)   (b) (1,-1)   (c) (0,0)   (d) (1,1)   (e) none of these

20. The points of intersection of the line $3y - x = 25$ and the circle $x^2 + y^2 = 25$ are

   (a) (4,3),(-5,0)   (b) (-4,-3),(5,0)   (c) (-3,4),(5,0)   (d) (4,-3),(-5,0)
   (e) none of these

21. The radian measure of the angle 150° is

   (a) $\dfrac{-7\pi}{6}$ (b) $\dfrac{2\pi}{3}$ (c) $\dfrac{-4\pi}{3}$ (d) $\dfrac{5\pi}{6}$ (e) none of these
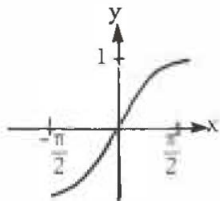
22. $\cos \dfrac{5\pi}{4} =$

   (a) $\dfrac{5}{\sqrt{2}}$ (b) $\dfrac{1}{\sqrt{2}}$ (c) $\dfrac{-\sqrt{3}}{2}$ (d) $\dfrac{1}{2}$ (e) none of these
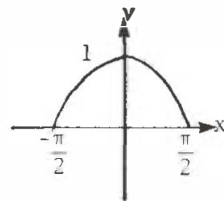
23. $\tan \theta =$

   (a) $\dfrac{\cos \theta}{\sin \theta}$ (b) $\sec \theta - 1$ (c) $\dfrac{1}{\cot \theta}$ (d) $\sec \theta + 1$ (e) none of these
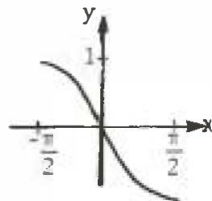
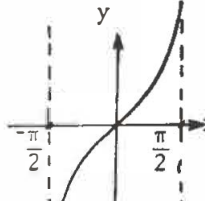24. Which of the following represents the graph of $y = \cos x$ for $|x| \leq \dfrac{\pi}{2}$?

   (a)   (b)   (c)   (d)   (e) none of these

25. A kite string makes an angle of 32° with the (level) ground and 45 feet of string is
    out. The height of the kite above the ground level is

   (a) 45 sin 32°   (b) $\dfrac{\sin 32°}{45}$ (c) 45(1-cos 32°)   (d) $\dfrac{\tan 32°}{45}$ (e) none of these

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Advisory Exam Statistics

Table I gives the percentage of students answering each of the 25 *questions* correctly.

Table II gives the relative frequency (R.F.) and cumulative frequency (C.F.) for each of the possible *scores* (that is, 0 to 25).

TABLE I	
Question	*Percentage*
1	18
2	27
3	54
4	38
5	35
6	64
7	46
8	25
9	47
10	41
11	36
12	22
13	24
14	09
15	34
16	62
17	35
18	09
19	35
20	44
21	67
22	33
23	37
24	33
25	66

TABLE II		
Score	*R.F.*	*C.F.*
0	0.6	0.6
1	1.0	1.6
2	2.3	3.9
3	4.2	8.1
4	4.7	12.8
5	8.7	21.4
6	8.8	30.3
7	8.3	38.5
8	8.5	47.0
9	10.0	57.0
10	7.4	64.4
11	5.6	70.0
12	5.9	75.9
13	5.6	81.4
14	4.0	85.5
15	3.7	89.1
16	2.0	91.1
17	2.4	93.5
18	1.7	95.2
19	1.2	96.4
20	1.2	97.6
21	1.1	98.7
22	0.4	99.1
23	0.3	99.4
24	0.4	99.8
25	0.2	100.0

2 117 students wrote this exam.   The mean score was 9.4.