? ? ? Problem Corner ? ? ?

edited by William J. Bruce and Roy Sinclair
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Problems suggested here are aimed at students of both the junior and senior high schools of Alberta. Solutions are solicited and a selection will be made for publication in the next issue of *delta-K*. Names of participants will be included. All solutions must be received (preferably in typewritten form) within 30 days of publication of the problem in *delta-K*.

Mail solutions to: Dr. Roy Sinclair or Dr. Bill Bruce
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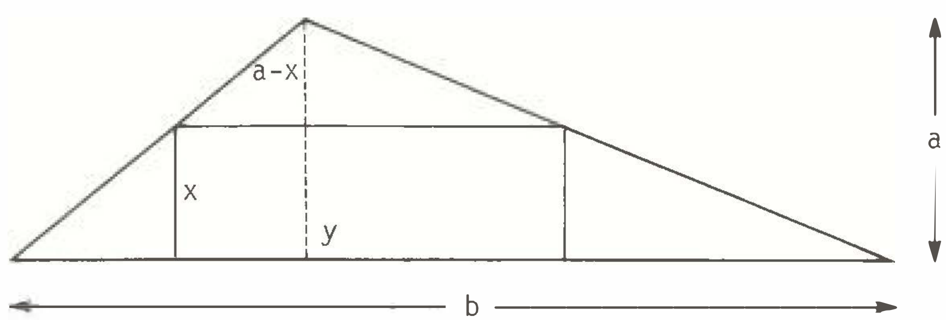
Problem 5:

(submitted by William J. Bruce, University of Alberta)

A rectangle is inscribed in a scalene triangle, as shown, with one side of the rectangle on a side of the triangle and one of its vertices on each of the other sides. Without using calculus, prove that the maximum area of the rectangle is the same no matter on what side of the triangle the one side is located.

NOTE: It has been shown that when the rectangle is drawn with only three of its vertices on the triangle, a rectangle of greater area does not exist.



Problem 4:

(submitted by Dr. A. Meir, University of Alberta)

Let p be a prime number and $\underline{a}$ be a positive integer. Show that $(a-p)^3 + a^3 = (a+p)^3$ cannot be true.

Solution of Problem 4

(suggested by Dr. A. Meir)

Suppose that $(a-p)^3 + a^3 = (a+p)^3$ is true. Then,

$$2a^3 - 3a^2p + 3ap^2 - p^3 = a^3 + 3a^2p + 3ap^2 + p^3$$

or

$$a^3 - 6a^2p = 2p^3 \;. \qquad (1)$$

This means that a^3 must be divisible by p, and since p is prime, $\underline{a}$ must be divisible by p. Say $a = t \cdot p$. Then from (1),

$$t^3p^3 - 6t^2p^3 = 2p^3$$

and, dividing by p^3, we get

$$t^3 - 6t^2 = 2$$

or

$$t^2(t-6) = 2$$

So, $t > 6$, otherwise $t - 6$ would be negative or zero. But then, $t^2 > 36$ and $t-6 \geq 1$, so $t^2(t-6) > 36$ and cannot be 2.