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THE ALBERTA TEACHERS' ASSOCIATION
MATHEMATICS COUNCIL

*Volume XX, Number 4* *June 1981*

# Mathematics Council, ATA
# 21st ANNUAL CONFERENCE

## October 16 and 17, 1981

## University of Lethbridge/ Lethbridge Lodge

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THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

**Volume XX, Number 4** **June 1981**

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EDPRESS

*delta-K* is published by The Alberta Teachers' Association for the Mathematics Council. Editor: Dr. George Cathcart, Department of Elementary Education, University of Alberta, Edmonton T6G 2G5. Editorial and Production Services: Communications staff, ATA. Opinions of writers are not necessarily those of either the Mathematics Council or The Alberta Teachers' Association. Please address all correspondence regarding this publication to the editor. *delta-K* is indexed in the Canadian Education Index.

# Donald H. Hinde

*It is with regret that we announce the death earlier this year of Don Hinde.*

*Don was a long-standing and active member of the Mathematics Council, a highly respected mathematics teacher, and a friend to all of us.*

*Don served on the executive of MCATA as treasurer from 1973 to 1980, and more recently as a director. His efficiency, dedication, and hard work were an inspiration to all who worked with him.*

*Don Hinde will be sadly missed by his wife and family, by his colleagues, by MCATA, and by all of us who knew him.*

*MCATA has set up a Don Hinde Memorial Scholarship, the details of which are still to be worked out.*

# From the Editor's Desk . . .

At the time of writing, your editor has just returned from attending the 59th Annual Meeting of the National Council of Teachers of Mathematics in St. Louis. These meetings are always professionally rewarding and I would encourage you to plan to attend the 60th Annual Meeting, which will be held in Toronto, April 14-17, 1982. This is the first NCTM Annual Meeting to be held outside of the United States.

THE PROBLEM CORNER IS A PROBLEM. Bill Bruce, co-editor of the Problem Corner, tells me that he has not received any solutions to the problems in this corner. We promised to publish students' solutions, but we can't if we don't receive any. Please encourage your junior and senior high students to submit their solutions. If you are not giving the problems to your students, please take a minute to tell us why so that we can mend our ways. Problem 5 and a suggested solution to Problem 4 appear in this issue.

CALCULATORS. Alberta Education is preparing a document on calculators. This is scheduled for distribution to all schools before September. With this in mind, *delta-K* is publishing two articles on calculators in this issue.

Other articles in this issue deal with age problems for beginning algebra students (Bruce), per cent problems (Litwiller and Duncan), and an advisory examination (Trollope).

ACTIVITIES. Ideas have been reprinted from the *Arithmetic Teacher* which provide addition and subtraction practice with whole numbers, fractions, and decimals. These ideas are graded for use with Grades 1 to 8.

- *George Cathcart*

NATIONAL COUNCIL OF
Teachers of Mathematics

# NOTICES

## Mark Your Calendars!

The most mathematically-talented high school students in CANADA will be in the United States for the 23rd Math Olympics from July 8 to July 20, 1981.

They will be participating in a two-day problem-solving competition along with students from 25 other countries. The schedule of activities for the 200 students includes 10 days of sightseeing and recreation in New York City and Washington, D.C.

The participating countries are Australia, Austria, Belgium, Brazil, Bulgaria, Colombia, Cuba, Federal Republic of Germany, Finland, France, Greece, Hungary, Israel, Luxemburg, Mexico, Netherlands, Poland, Romania, Russia, Sweden, Tunisia, United Kingdom, United States, Venezuela, and Yugoslavia.

## Eliminate Sexism from Mathematics Education

— An NCTM Agenda for Action Item for the 1980s —

"Multiplying Options and Subtracting Bias," a videotape and workshop intervention program designed to eliminate sexism from mathematics education, is now available from NCTM. Each of the four 30-minute, full-color videotapes, narrated by Marlo Thomas, is directed to a specific junior/senior high school audience: students, parents, teachers, and guidance counsellors. Each tape uses a variety of formats - candid interviews, dramatic vignettes, and expert testimony - to address the problem of mathematics avoidance and some possible solutions.

A facilitator's guide, which provides special workshop materials to develop an awareness of sexual bias and math anxiety, is an integral part of the program. The guide contains an overview of the workshops and detailed instructions on how the facilitator can prepare for conducting the workshops and four separate step-by-step sets of workshop instructions for each of the target audiences. Overhead transparencies and handout masters are included for use during the workshops.

Elizabeth Fennema, School of Education, University of Wisconsin-Madison, directed the project, which was funded by a grant from the Women's Education Equity Act of the U.S. Education Department. Nearly 3 000 people participated in this project, including 2 500 high school students, 250 teachers and counsellors, 100 "actresses" and "actors," and many academic and technical advisors and staff persons.

Tapes can be ordered from NCTM individually for $125 or as a set of four for $375; either way, a facilitator's guide is included. Individual members receive a 20 per cent discount. Specify three-quarter-inch videocassette, one-half-inch open reel, VHS, or Beta 1.

## An Exceptional Book

*The Mathematical Education of Exceptional Children and Youth*, the second in the NCTM's new Professional Reference Series, is a vivid example of the concern of professional mathematics educators for the exceptional child. Parents of exceptional children may be particularly gratified to find the interdisciplinary approach in this publication that encourages readers to focus on the whole child.

The professional mathematics educators listed on the following publication information sheet prepared their manuscripts during the International Year of the Child (1979) in support of the Council's principal goal, the improvement of classroom instruction in mathematics at all levels.

*The Mathematical Education of Exceptional Children and Youth* sells for $28.

### The Mathematical Education of Exceptional Children and Youth
**An Interdisciplinary Approach**

*Edited by*
Vincent J. Glennon, University of Connecticut

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ISBN 0-87353-171-X

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## CRITICAL THINKING - BOOK 1

by Anita Harnadek
Publisher: Midwest Publications

The purpose of *Critical Thinking - Book 1* is to sharpen thinking skills using class discussion. The author has tried to take advantage of the fact that most students love class discussion and arguments. It is non-graded and at the present time is being used all the way from gifted upper elementary to lower college. The material is aimed primarily at secondary students.

The content is oriented toward the reasoning processes, not subject areas. Since the primary teaching approach is class discussion, the material may be used for supplementary activities in almost any course. However, the book also makes a very good "mini" or semester course taught by one or several disciplines or departments. There are 200 examples and 1 000 problems and questions from which to choose for lively class discussions.

Contents:

Introduction to Critical Thinking, Introduction to Logic, Some Basic Concepts for Critical Thinking, Common Errors in Reasoning, Propaganda Techniques, Advertising and Schemes, Examining Arguments and Value Judgments, Learning to be Open-Minded.

Price:

$12.50/text (178 pages)
$ 5.50/manual

Available from:

Western Educational Activities Ltd.
10929 - 101 Street
Edmonton, Alberta T5H 2S7

(See Catalog - p.76)

# Forthcoming Mathematics Conferences

## MCATA 21st Annual Conference

October 16-17, 1981
University of Lethbridge/
Lethbridge Lodge
Lethbridge, Alberta

*Three Major Themes:*

- Problem Solving
- Gifted Students
- Computers and Technology

*Keynote Speaker:*

Earl Ockenga, Price Laboratory School, Cedar Falls, Iowa

*Accommodation:*

Please make your own arrangements for accommodation. Possible hotels and rates (subject to change):

Lethbridge Lodge
$39 single/$43 double

Travelodge
$32 single/$38 double

Lethbridge Inn
$28 single/$32 double

El Rancho
$24 single/$32 double

*Conference Registration:*

| | Before Sept. 25 | After Sept. 25 |
|---|---|---|
| MCATA Members | $30 | $35 |
| Non-Members* | 40 | 45 |
| Student Members | 10 | 10 |
| Student Non-Members* | 16 | 16 |

*includes membership

Registration forms will be sent to members and schools at the beginning of September.

For further information, contact:

Gary Hill
310 Laval Blvd.
Lethbridge T1K 3W5

## NCTM Honolulu Meeting

August 10-12, 1981
Honolulu, Hawaii

## NCTM Minneapolis/St. Paul Meeting

November 12-14, 1981
St. Paul, Minnesota

For further information and a program booklet, contact:

National Council of Teachers of Mathematics
1906 Association Drive
Reston, Virginia 22091

## British Columbia Association of Mathematics Teachers — 9th Summer Workshop

August 31 - September 2, 1981
Cariboo College
Kamloops, British Columbia

*Sessions on:*

- Mathematics Education
- Statistics
- Computers
- Mathematical Topics

For information, write:

BCAMT Conference Organizers
c/o Cariboo College
P.O. Box 3010
Kamloops, B.C. V2C 5N3

# Plus + + +

*The following material is reprinted from Issue No. 7 of* Plus + + +, *a short magazine informing mathematics educators across Canada about important events, research, curriculum development, and items of national interest.*

## Groupe canadien d'étude en didactique des mathématiques

La groupe se donne pour buts d'encourager la recherche, de faciliter l'échange d'idées et d'informations, de susciter une réflexion critique sur l'enseignement et sur la théorie et la pratique en didactique des mathématiques. La reunion de 1981 aura lieu à l'Université d'Alberta, du vendredi 5 au mardi 9 juin, 1981. La conférence est ouverte aux didacticiens des mathématiques et aux mathématicien intéressés par l'enseignement et par la formation des maîtres en mathématiques. La conference est bilingue.

Le dernier avis sera envoyé au mois d'avril. Pour en recevoir un exemplaire, veuillez écrire au Dr. Joel Hillel, HB 206, Campus Loyola, Université Concordia, 7141 rue Sherbrooke ouest, Montréal, Québec H4B 1R6.

## B.C. Association 9th Summer Workshop

The 9th summer workshop of the B.C. Association of Mathematics Teachers will be held at Cariboo College in Kamloops from August 31 until September 2, 1981. Technical sessions will focus on mathematics education, computers, mathematics, and statistics. The organizers, Derek Chambers and John Siggers, are inviting presentations, suggestions, and registrations. Contact BCAMT Conference Organizers, c/o Cariboo College, P.O. Box 3010, Kamloops, B.C. V2C 5N3.

## A Workshop on Statistics and Probability

The Joint Committee on the Curriculum in Statistics and Probability of the American Statistical Association and the National Council of Teachers of Mathematics organized a workshop at Capilano College, North Vancouver, B.C., on February 20 and 21, 1981. The speakers were Jim McBride of the Response Analysis Corporation, Princeton, Jane Davies, Brian Graham, Norman Kreger of B.C. Telephones, Gail Lenoski of Statistics Canada, Jim Nakamoto, Bob Rennie, Jim Swift, and Alf Waterman. Further information can be supplied by Jim Swift, R.R.3, Site E, Nanaimo, B.C.

## Algonquin College Grade 12 Level Pilot Examination

In 1977, the Ontario Association for Mathematical Education recommended a representative mathematics examining board. Subsequently, the Carleton-Ottawa Mathematics Association initiated a project to set up a regional mathematics examining board. On May 24, 1980, F.G.B. Maskell and K.H. Williams, with the co-operation of the Rideau Campus of Algonquin College, Ottawa, administered a test on a voluntary basis to a maximum of 300 students. The candidates were given two hours; those who completed the first section of 100 short questions were encouraged to continue to the seven longer "insight" questions of the second section. It was emphasized the test was not a contest and that nothing hinged upon it. Its purpose was diagnostic, and each candidate was to receive a report listing his areas of weakness. In a sample of 126, 20 displayed mastery and insight, 49 had at least some competence, while the remainder showed evidence of basic weakness.

Enquiries may be directed to F.G.B. Maskell, Mathematics Department, Algonquin College, 200 Lees Avenue, Ottawa, Ontario K1S 0C5.

## Ontario Assessment Instrument Pool

Dr. H.K. Fisher, Ontario Deputy Minister of Education, announced the development of the Ontario Assessment Instrument Pool to provide a resource of assessment materials and techniques in both English and French. The ministry has not prescribed its use for any particular purpose. It involves a variety of school subjects. The first instalment of the Mathematics Pool, for Grades 7 to 10, has already been sent to school boards. Mathematics, Grades 4 to 6, will follow in 1981/82. The first issue of the *Ontario Assessment Newsletter*, just published at OISE, describes the program of field and screening trials. Further newsletters were scheduled for April 15 and September 30, 1981. Direct enquiries to Dr. Les McLean, Head, Education Evaluation Centre, Ontario Institute for Studies in Education, 252 Bloor Street W., Toronto, Ontario M4W 1E6.

## Special Education

Regularly, the *Ontario Mathematics Gazette*, the journal of the Ontario Association for Mathematics Education, uses a number to focus on a particular issue. For example, the December 1980 number ran several articles on calculators. However, the theme of the previous (September) issue is a little more unusual - special education. Douglas Crawford comments on recent assessment studies in U.S.A., Canada, and England. "Crossing the chasm from the clinic to the classroom," a presentation made by Ralph Connelly at a conference on diagnostic and prescriptive mathematics held recently in Vancouver, is summarized here. The Faculty of Education at the University of Western Ontario is housing CHILD (Centre for Helping Individuals with Learning Disabilities), sponsored by a local chapter of the Ontario Association for Children with Learning Disabilities; Joan Simons, Merrill Sitko, and Dan Bachor [phone (519) 679-6023] report on it. Another article deals with a study of special education in the Borough of York. The editor of the *Gazette* is Professor R.S. Smith, Faculty of Education, U.W.O., c/o Althouse College of Education, 1137 Western Road, London, Ontario N6G 1G7.

# Alberta Education Mathematics Achievement Test Results

The names of the 11 students who achieved the highest results in a province-wide Mathematics Achievement Test were announced on March 6, 1981. The test, held in January this year, was administered by Alberta Education to all Grade 12 students enrolled in Mathematics 30.

The highest mark in the province was obtained by Malcolm Hortsong Zung of M.E. LaZerte Composite High School in Edmonton. He will receive an award of $500. The other 10 winners will receive an award of $300 each. The award-winning students are:

| | |
|---|---|
| *Stephen John Gurnett* | Spirit River Secondary School<br>Spirit River, Alberta |
| *Charles Sut Keung Chan* | St. Dominic's School<br>Cold Lake, Alberta |
| *Margaret Ann Wicentowich* | Vegreville Composite High School<br>Vegreville, Alberta |
| *Roland Mah* | Rocky Mountain House Junior-Senior High School<br>Rocky Mountain House, Alberta |
| *Roland Larry Hoppe* | MAC-Alix High School<br>Alix, Alberta |
| *Jean Catherine Miller* | Samuel Crowther School<br>Strathmore, Alberta |
| *So Kam Linda Li* | McCoy High School<br>Medicine Hat, Alberta |
| *Kin David Kuen Tsui* | Harry Ainlay Composite High School<br>Edmonton, Alberta |
| *Eric W. Kwan* | James Fowler High School<br>Calgary, Alberta |
| *James Wong* | Bishop Grandin School<br>Calgary, Alberta |

On the whole, students did not do as well this year as they did in 1978, when the last Mathematics Achievement Test was written. The students scored an average of 48 per cent this year compared to 51 per cent in 1978. The complete test included 75 items of which 37 were repeated from the previous test. However, on the 37 repeated items, students this year scored 3 per cent higher than in 1978. This year, 5 503 students participated in the examination compared to 6 244 in 1978. Those who wrote the examination this year represent 92 per cent of those eligible to do so.

# South West Regional Elects New Officers

The MCATA South West Regional has elected the following executive for 1981/82:

| | | |
|---|---|---|
| President | - | *Jean Poile* |
| Past President | - | *Joe Krywolt* |
| Secretary | - | *M. Jo Maas* |
| Treasurer | - | *Joan Haig* |
| Program Director | - | *Hans Hulstein* |

In reporting on the activities of the regional during 1980/81, Joe Krywolt writes:

> During the past year, our regional council has sponsored and organized a three-day workshop on computers (April 1980), an in-service day (May 1980), and two "hands-on" workshops (March 1981). We have also provided resource people for local professional development activities within our region. Our primary function, though, has been to organize the math sessions at the South Western Annual Convention.
>
> In the coming year, our attention turns to the 21st Annual MCATA Conference which will be held in Lethbridge on October 16 and 17, 1981. Our executive and local representatives are currently hard at work to help make this conference the most memorable one to date. We look forward to meeting you in Lethbridge this fall!

* * * * * * * * * * *

Any other region in the province interested in forming a regional council should contact President Lyle Pagnucco or any other member of the executive listed on page 48 of this journal.

## Canadian Mathematical Society

# 1981 Alberta High School Prize Examination Results

| *Prize* | *$Amt.* | *Student* | *School* |
|---|---|---|---|
| Canadian Mathematical Society Scholarship | 400 | BARAGAR, Arthur | Old Scona Academic H.S.<br>Edmonton, Alberta |
| Nickle Foundation Scholarship | 400 | PHAN, Thong | St. Mary's Community School<br>Calgary, Alberta |
| First runner-up | 150 | ZUNG, Malcom | M.E. LaZerte Composite H.S.<br>Edmonton, Alberta |
| Second runner-up | 150 | HUYNH, Phong | Harry Ainlay Composite H.S.<br>Edmonton, Alberta |

## Special Provincial Prizes

| | | | |
|---|---|---|---|
| Highest Grade 12 student (below first 3) | 75 | BOWMAN, John | Old Scona Academic H.S.<br>Edmonton, Alberta |
| Highest Grade 10/11 student (below first 3) | 75 | KALANTAR, Daniel | Old Scona Academic H.S.<br>Edmonton, Alberta |

## District Prizes

| *District No.* | | | |
|---|---|---|---|
| 1 | 50 | HAWLEY, Michael | George P. Vanier School<br>Donnelly, Alberta |
| 2 | 50 | TUCKEY, David | Lorne Jenken H.S.<br>Barrhead, Alberta |
| 3 | 50 | WICENTOWICH, Margaret | Vegreville Composite H.S.<br>Vegreville, Alberta |
| 4 | 50 | McILRAVEY, Michael | Lindsay Thurber Composite H.S.<br>Red Deer, Alberta |
| 5 | 50 | SINNAMON, Norman | Prairie High School<br>Three Hills, Alberta |
| 6 | 50 | DAVIES, John | Lethbridge Collegiate Institute<br>Lethbridge, Alberta |
| 7(1) | 50 | EWONIAK, Ronald | Ross Sheppard Composite H.S.<br>Edmonton, Alberta |
| 7(2) | 50 | FEYGIN, Gennady | Ross Sheppard Composite H.S.<br>Edmonton, Alberta |
| 8(1) | 50 | CRAIG, Scott | Henry Wise Wood Senior H.S.<br>Calgary, Alberta |
| 8(2) | 50 | WONG, James | Bishop Grandin High School<br>Calgary, Alberta |

424 students from 68 schools in Alberta and the Northwest Territories wrote the 1981 examination. The following students took the first 22 places and are nominated to write the Canadian Mathematical Olympiad:

| *Student* | *School* |
|---|---|
| BARAGAR, Arthur | Old Scona Academic High School, Edmonton |
| ZUNG, Malcom | M.E. LaZerte Composite High School, Edmonton |
| HUYNH, Phong | Harry Ainlay Composite High, Edmonton |
| PHAN, Thong | St. Mary's Community School, Calgary |
| BOWMAN, John | Old Scona Academic High School, Edmonton |
| CRAIG, Scott | Henry Wise Wood Senior High School, Calgary |
| EWONIAK, Ronald | Ross Sheppard Composite High School, Edmonton |
| FEYGIN, Gennady | Ross Sheppard Composite High School, Edmonton |
| KALANTAR, Daniel | Old Scona Academic High School, Edmonton |
| McILRAVEY, Michael | Lindsay Thurber Composite High School, Red Deer |
| LEUNG, Andrew | M.E. LaZerte Composite High School, Edmonton |
| WONG, James | Bishop Grandin High School, Calgary |
| DOETZEL, Randy | St. Thomas Aquinas School, Provost |
| ROEHL, Dean | Harry Ainlay Composite High, Edmonton |
| WLASICHUK, Richard | Jasper Place Composite High School, Edmonton |
| CHOI, Man | James Fowler Senior High, Calgary |
| LIU, Ryan | Bishop Grandin High School, Calgary |
| ZATREPALEK, Kenneth | Henry Wise Wood Senior High School, Calgary |
| TUCKEY, David | Lorne Jenken High School, Barrhead |
| HAWLEY, Michael | George P. Vanier School, Donnelly |
| BRETT, John | Viscount Bennett Jr./Sr. High, Calgary |
| FLEGEL, Michael | Sir Winston Churchill High, Calgary |

*The following students placed 23-34:*

Greg Anglin (Dr. E.P. Scarlett H.S., Calgary); Mark Benning (St. Joseph's Composite H.S., Edmonton); Stephen Doyle (Archbishop Macdonald H.S., Edmonton); Tim Fowlow (Sir Winston Churchill H.S., Calgary); Emil Ochotta (Harry Ainlay Composite H.S., Edmonton); David Pentyliuk (McNally H.S., Edmonton; Larry Prymych (H.A. Kostash H.S., Smoky Lake); Dennis Rouault (Sturgeon Composite H.S., Namao); Ashley Rowland (Sir Winston Churchill H.S., Calgary); Perry Spitzer (Lorne Jenken H.S., Barrhead); Daniel Stephen (Ross Sheppard Composite H.S., Edmonton); Margaret Wicentowich (Vegreville Composite H.S., Vegreville).

*The following students placed 35-50:*

Naved Ali (Archbishop Macdonald H.S., Edmonton); Loren Andruko (Old Scona Academic H.S., Edmonton); Clara Chow (James Fowler H.S., Calgary); John Davies (Lethbridge Collegiate Institute, Lethbridge); Robert Gardner (McNally Composite H.S., Edmonton); Robert Geddes (Camrose Lutheran College, Camrose); Kevin Hildebrand (Lindsay Thurber Composite H.S., Red Deer); Thomas Kalantar (Old Scona Academic H.S., Edmonton); Heather Konrad (Harry Ainlay Composite H.S., Edmonton); Don Koziak (Louis St. Laurent H.S., Edmonton); Michael Lee (Old Scona Academic H.S., Edmonton); Lawrence Liu (Camrose Lutheran College, Camrose); Bruce Ollerenshaw (Sir Winston Churchill H.S., Calgary); Garry Sasseville (Wainwright H.S., Wainwright); Norman Sinnamon (Prairie H.S., Three Hills); Darryl Turner (Ross Sheppard Composite H.S., Edmonton).

# Using Calculators in Pre-College Education:
## Third Annual State-of-the-Art Review*

*by Marilyn N. Suydam*

Over the past several years, the cost of calculators has declined to a relatively stable level. Concurrently, calculator availability has become less and less an issue. Technology has provided prolonged battery life, and some calculators are so small and light they can be carried or worn easily, nullifying additional arguments about their availability. While resistance to their use in schools is still apparent, awareness of potential instructional applications has slowly continued to increase. Heightening this awareness is a significant recommendation from a national association.

### A Recommendation for the 1980s

In April 1980, the National Council of Teachers of Mathematics released *An Agenda for Action: Recommendations for School Mathematics of the 1980s*. One of the eight recommendations addresses concerns presented by computing technology: "Mathematics programs must take full advantage of the power of calculators and computers at all grade levels." The introductory comments present a rationale for this stance:

> Beyond an acquaintance with the role of computers and calculators in society, most students must obtain a working knowledge of how to use them...
>
> The availability of computing aids, including computers and calculators, requires a reexamination of the computational skills needed by every citizen. Some of these computational skills will no longer retain their same importance, whereas others will become more important.
>
> It is recognized that a significant portion of instruction in the early grades must be devoted to the direct acquisition of number concepts and skills without the use of calculators. However, when the burden of lengthy computations outweighs the educational contribution of the process, the calculator should become readily available.

Recommended actions to accomplish the goal include the following:

3.1 All students should have access to calculators and increasingly to computers throughout their school mathematics program.

3.2 The use of electronic tools such as calculators and computers should be integrated into the core mathematics curriculum.

Calculators should be available for appropriate use in all mathematics classrooms, and instructional objectives should include

*The first annual review was prepared in April 1978; the second appeared in May 1979.

the ability to determine sensible and appropriate uses.

Calculators and computers should be used in imaginative ways for exploring, discovering, and developing mathematical concepts and not merely for checking computational values or for drill and practice.

3.3 Curriculum materials that integrate and require the use of the calculator and computer in diverse and imaginative ways should be developed and made available.

Schools should insist that materials truly take full advantage of the immense and vastly diverse potential of the new media...

Educators should take care to choose software that fits the goals or objectives of the program and not twist the goals and developmental sequence to fit the technology and available software.

Teachers of other subjects in which mathematics is applied "should make appropriate use of calculators and computers." Furthermore, teachers and administrators are urged to "initiate interaction with the home to achieve maximum benefit to the student from co-ordinated home and school use of computers and calculators."

Other recommended actions address the needs of teachers, pointing out that colleges need to offer courses on instructional uses of calculators for both preservice and in-service teachers and that certification standards should require such preparation. Professional organizations should provide information through media and meetings of various types.

Thus, the NCTM acknowledges that computational skills are still necessary, but stresses the need to integrate calculator use at all levels, reinforces their usefulness in problem solving, notes the need for imaginative materials, and emphasizes the key component of teacher education.

## Evidence on Availability and Uses of Calculators

The NCTM recommendation accepts the reality of the existence of calculators and computers. Data from the Second Mathematics Assessment of the National Assessment of Educational Progress (Reys et al., 1980) support the fact that many children have access to calculators outside of the classroom: 75 per cent of 9-year-olds, 80 per cent of 13-year-olds, and 85 per cent of 17-year-olds either own their own calculators or have one available to use. Other studies indicate that in some locations this percentage may be even higher; for instance, over 90 per cent of the 220 households surveyed in Florida had at least one calculator (Conner, 1980), and in Indiana a survey of 417 students indicated that ownership or access ranged from 79 per cent for first graders to 100 per cent for sixth graders (Ewbank, 1979). Naturally, however, some studies report lower percentages; for example, only 68 per cent of the Missouri children queried by Reys et al. (1980) had access to calculators.

Data from the many studies* still seeking an answer to the question, "Does use of calculators hurt achievement scores?" continue to support the fact that students who use calculators

*This type of study on achievement comprises about two-thirds of all studies reported. Studies focussing on the development of specific mathematical ideas account for about one-sixth of the studies, while the remainder are surveys. While doctoral students continue to produce at least 50 per cent of the research, ongoing investigations are being conducted by researchers in schools and colleges.

for instruction achieve at least as high or higher scores than students not using calculators, even though the calculator is not used on the test. (In the majority of studies during the past year, no significant differences were reported.) The decrease in time spent on paper-and-pencil practice did not appear to harm the achievement of students who used calculators.

Data from studies on learning mathematics with calculators, as well as evidence from the practical experiences of teachers, are slowly accumulating, indicating that calculators are useful in teaching a variety of mathematical ideas. Reports from Conner (1980) and Moser (1979), for instance, detail some specific ways in which calculators are useful instructional tools.

## Surveys on Beliefs and Attitudes

When beliefs and attitudes are surveyed, however, it becomes obvious that many persons ignore the evidence from research on achievement and learning. Perceptions of the uses and importance of calculators in the mathematics curriculum depend primarily on the audience surveyed. The Priorities in School Mathematics Project (PRISM), conducted in 1979, devoted about 20 per cent of its items to ascertain ways in which educators at all levels from primary through college, parents, and school board members feel about the use of calculators. Educators were much more supportive of increased use of calculators than were lay persons: 54 per cent of the professional samples, but only 36 per cent of the lay samples would increase emphasis on them during the 1980s. Strongest support came from supervisors and teacher educators (85 per cent and 74 per cent, respectively); teachers at all levels had more reservations (support averaged 50 per cent); and parents and school board members gave weak support to increased emphasis - and to almost all uses of calculators except checking answers. The percentage agreeing with various uses of calculators were:

| | *Professional Samples* | *Lay Samples* |
|---|---|---|
| checking answers | 93% | 89% |
| doing a chain of calculations | 89% | - |
| computing area | 78% | - |
| making graphs | 71% | - |
| solving word problems | 70% | 38% |
| solving equations | 70% | - |
| learning why algorithms work | 68% | - |
| doing homework | 66% | 37% |
| developing ideas and concepts | 59% | 49% |
| learning basic facts | 51% | - |
| taking a test | 50% | 22% |

Over 70 per cent of the teachers at all levels endorsed having four-function calculators available. However, 67 per cent of the professional samples and 88 per cent of the lay samples believe that calculator use should be postponed until after paper-and-pencil algorithms are learned. Only 40 per cent of the professional samples and 19 per cent of the lay samples would let slower students use calculators, and putting students who have not learned paper-and-pencil computation by Grade 8 into a calculator course was supported by only 34 per cent (45 per cent of the professional samples and 30 per cent of the lay samples.

Other studies provide data which both compare and contrast with the PRISM data. Cohen and Fliess (1979) reported that over 63 per cent of the teachers they queried favored the use of calculators. In a survey conducted in 1979, Reys and some colleagues interviewed a random sample of 194 classroom teachers in 10 school districts in Missouri. The researchers reported that:

> The overwhelming feeling was that calculators exist, that there are many appropriate places for using them at all levels of the mathematics curriculum, and that the type and extent of this usage should be left up to the discretion of the individual classroom teacher. (Reys et al., 1980)

While 84 per cent of the teachers said that calculators should be available to children in school, only 35 per cent had actually used calculators in mathematics classes (the data ranged from 14 per cent at the primary level to 62 per cent at the senior high level). Another 42 per cent said they would like to use calculators. Teachers who had used them commented that:

> Not only could they work more problems if a calculator was available, but also they actually covered more topics. They also reported dealing more with concept development and less with computation during their mathematics class. (Reys et al., 1980)

It was also reported that:

> Most of the teachers who had not used a calculator in the classroom seemed aware of primarily two uses. One was as a computational device which they saw as defeating the major goals of school mathematics and the other as a tool for students to check the paper-and-pencil computations.... The majority of the teachers were unaware of the instructional potential of the calculator. (Wyatt et al., 1979)

An average of 80 per cent of the teachers felt children should master the four basic arithmetic operations before using calculators. (Interestingly, 76 per cent of the primary teachers held this view, while 89 per cent of the senior high school teachers did.) Indeed, 43 per cent felt that using a calculator would cause students' ability to compute to decline. Teachers generally agreed, however, that slow students or senior high students who had never learned to compute should use a calculator because they would probably never be able to compute otherwise.

Slightly over 50 per cent of the teachers wanted textbooks with activities using calculators. Forty-three per cent favored use of calculators on problem-solving portions of standardized tests.

Four implications were drawn from the study (Wyatt et al., 1979):

1. There is a need for leadership and direction for teachers regarding calculator use in schools.
2. Training in the use of calculators as an instructional tool is needed.
3. Dissemination of current information about calculator usage is needed to dispel many false conceptions.
4. Materials which integrate calculators into the regular mathematics curriculum should be developed and disseminated.

As part of an investigation in which calculators were used in elementary school mathematics instruction, Conner (1980) surveyed parents of children in kindergarten through Grade 5. Percentages favoring what she called "unrestricted" use of calculators as an instructional aid ranged from 13 per cent for the elementary level and 16 per cent for the middle school level to 29 per cent for the high school level. When she asked about "regulated" use, the percentages rose to 83 per cent for the elementary level, 80 per cent for the middle school level, and 81 per cent for the high school level.

Balka (1979) also found that parents were skeptical about the use of calculators in elementary grades. They agreed that calculators could be used along with paper-and-pencil computation, but strongly objected to using calculators in place of paper-and-pencil computation.

Successful integration of calculator uses in the mathematics curriculum will require careful and thorough communication among all concerned groups. Efforts to provide information on how calculators can be used successfully in teaching mathematics without harm to achievement must continue. And parents and other members of the public must receive assurance that necessary computational skills will still be taught. This point is clearly made in the NCTM *Agenda for Action.*

## Development of Instructional Materials

Materials which *integrate* the use of calculators to teach mathematical ideas are still comparatively scarce. Most of the published articles, however, do present ideas for using calculators to promote learning on specific topics, including work with operations, functions, exponents, polynomials, square roots, and problem solving. There appears to be a decrease in the number of books focussed solely on games, and an increase in the number of books which could be used to supplement ongoing instruction.

Two compilations of materials may prove useful to teachers. One is a collection of articles from the *Arithmetic Teacher* and the *Mathematics Teacher* (Burt, 1979); the other is a categorized listing of references on calculators (Suydam, 1979). As has been true ever since calculators appeared in schools, however, there is a continuing need for materials which develop mathematical ideas.

## Concluding Comment

While support from some groups for the use of calculators in schools is low, it is nevertheless changing as people accept the existence of calculators in their lives and in their children's lives. Concern continues to revolve around the issue of when the calculator should be used in relation to instruction on basic facts and algorithms: there is fear that paper-and-pencil computational skills will be lost and achievement scores will decline, despite the continuing reassuring research evidence on this point. Educators need to consider carefully ways of assuring parents that calculators can be used in developing a wide range of mathematical ideas which will promote mathematical achievement.

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# SMTS Position Paper: Calculators (1980)

*The Saskatchewan Mathematics Teachers' Society has drafted a position statement on calculators. The following is their position paper, reprinted in its entirety from* SMTS Newsletter, *Vol. 7, No. 5, November 1980.*

SMTS promotes activities which will help calculators to become an accepted, effective tool in the classroom:

1. Administrators should provide leadership, guidance, and encouragement to promote an atmosphere of acceptance for the use of calculators in mathematics classes.
2. Teachers should be trained to use calculators effectively.
3. Materials that integrate calculators into the regular mathematics curriculum should be developed and disseminated.
4. The entire school community should become familiar with the potential of the calculator for instruction. This should include knowledge of research and reports from schools that have used calculators.

## A. Rationale

Calculators are in and they are never going to be out. They may change form, but they will not go away. They are not a fad that will disappear after a certain number of years due to boredom on the part of the consumer. The decrease in price of calculators is making them accessible to an ever-increasing number of students. The worst response that we, as educators, can make to the use of calculators is to ignore them.

Almost 100 studies on the effects of calculator use have been conducted during the past four or five years. This is more investigations than on almost any other topic or technique for mathematics instruction during this century. The general consensus continues to be that there are no measurable detrimental effects associated with the use of calculators for teaching mathematics.

The SMTS strongly endorses the NCTM position statement on "Calculators in the Classroom" which states:

> The National Council of Teachers of Mathematics encourages the use of calculators in the classroom as instructional aids and computational tools. Calculators give mathematics educators new opportunities to help their students learn mathematics and solve contemporary problems. The use of calculators, however, will not replace the necessity for learning computational skills.
>
> As instructional aids, calculators can support the development and discovery of mathematical concepts. As computational tools, they reduce the time needed to solve problems, thereby allowing the consideration of a wider variety of applications. Furthermore, the use of calculators requires students to focus on the analysis of problems and the selection of appropriate operations. The effective use of calculators can improve student attitudes toward, and increase interest in, mathematics.

Other electronic devices, programmed to generate questions and activities, that provide immediate feedback to students, are not to be confused with calculators or computers. These devices can be used to reinforce computational skills through drill.

## B. History

### Simple Calculators

The electronic pocket calculator has a relatively short history. In 1965, the first electronic calculator was produced. It had about the same size and price as the desk model electromechanical calculator it was designed to replace. Approximately $170 worth of transistors and other electronic components were hand-assembled to make this $1 500 machine. It was faster and quieter than electromechanical calculators.

Progress in electronics led to the integrated circuit and later to the large-scale integrated circuit. A single one-quarter-inch square "chip" could contain at first a few dozen, then hundreds, and then thousands of transistors and related components. The cost per transistor dropped by many factors of 10, and assembly time was also greatly reduced. Eventually, the $170 worth of components in the original electronic calculator were replaced by a few dollars worth of components. Assembly became almost completely automated, so labor costs were greatly reduced.

The pocket calculator became commercially available in the early 1970s. By the mid-'70s, the price of a simple 4-function calculator was about the same as a college-level textbook. These inexpensive calculators quickly became as commonplace as the TV set. Elementary schools began to purchase classroom sets. Their use began to be required in certain high school and college courses. For example, in 1975, Ohio State University began to require their use in introductory mathematics courses. Since then, other colleges have begun to follow suit.

The simple 4-function calculator is a marvelous device. It is designed to perform the operations (functions) of addition, subtraction, multiplication, and division on whole numbers and decimal numbers. To compute the product of 87.5 and 6.93, one depresses the key sequence 8 7 . 5 x 6 . 9 3 = and the answer 606.375 appears immediately in the output display. What could be simpler? Moreover, it is very easy to learn how to use a calculator. A few seconds of training suffices for most adults. This is partially dependent upon their having prior knowledge of the arithmetic operations. But a primary-school-aged child can also easily master these calculator skills.

## C. Characteristics

NOT ALL CALCULATORS ARE THE SAME. You must become familiar with your calculator and the ones the children might have.

Kinds of calculators include:

*algebraic logic calculators* -
process all operations in the order in which they are entered.

*algebraic operating system calculators* -
are programmed to process information according to the order of operations.

*reverse polish notation* -
emphasize ordered pairs and functions. These require that both numbers be entered before the operation is specified. There is a key marked *ent* for enter. Thus, 2X3 is entered as 2 ent 3 X. No = key is necessary.

Calculators are as functional as the person using them. The operator must know what one calculator will or will not do and to do this he must be aware that there are differences in calculators.

## 1. Order of Operation

To become aware of differences, try these:

| | |
|---|---|
| 2 + 3 x 4 = | 3 + 2 x 5 - 1 + 3 x 4 = |
| 2 + 4 - 2 + 3 = | 3 + 2 x 5 + 1 - 7 = |
| 1 - 3 = | 8 + 2 - 7 x 4 = |

Where the order of operations is not simply from left to right, answers may differ - depending on the type of calculator.

## 2. Overloading the Display

Another situation will arise when an answer has more digits than the calculator will display. This *overloads* the machine, and this will be indicated by:

- a flashing display
- an E appearing on the left-hand or right-hand side

Try these: key in - 99 999 999 x 1 =

111 111 x 111 111 =

162 x 50 505 050 =

33 333 333 + 77 777 777 =

(NB - SR-50, T1-59, etc., do not overload with these questions as they can hold more than 8 digits.)

## 3. Division by 0

Enter: any number divided by 0; for example 12 ÷ 0 = .

This is an illegal operation and each calculator has its way of indicating this.

*Some features to look for in a calculator:*

## 4. Floating Decimal Point

Decimal remains at the right of the number until it is entered.

## 5. Shutoff

Some calculators have an automatic shutoff if left unused for a few minutes. Others display a travelling decimal point and then shut off. A great many will remain on and drain the power supply.

## 6. CE Key

Clears erroneous number entries without affecting previous entries. It will not cancel operational entries just pressed. *C key* clears calculator and sets display at 0. *CE/C key* - a combination key which clears the last number entered with one push and clears the entire calculator with two pushes.

## 7. Different Power Sources

Long life silver oxide replaceable batteries are the most costly and time-efficient. Automatic power-down displays and delayed power-off features maximize life of batteries.

## 8. Negative Numbers

Some calculators allow entry of negative numbers by pressing +1- key or CS key.

## 9. Other Keys

Constant, parentheses, square root, per cent, squaring, etc., may appear on the calculator.

## 10. Memory

2 key - stores (STO) the displayed number for later recall (RCL).

4 key - allows functions, usually addition (M+) and subtraction (M-) to be performed on the content of the memory register with retention for later recall (MR). Includes memory clear (MC).

## 11. Displays - two different types:

*LED (light emitting diode)*

- in use longer
- less expensive
- durable (depending on the particular calculator)
- "flashing" or symbols can be read in dark
- use 9-volt battery: relatively short life
- red numerals or blue/green numerals: higher battery drain for blue/green than for red numerals

*LED*

- red numerals not readable from wide angle; blue/green generally readable from wider angle

*LCD (liquid crystal display)*

- more recently on market
- more expensive
- less stable, reportedly (for example, dropping may cause display to shift or lose part of symbol)
- "immediate" display of symbols
- uses silver oxide battery; hundreds of hours of life
- black numerals on gray, yellow: low battery drain

*LCD*

- readable from wide angle

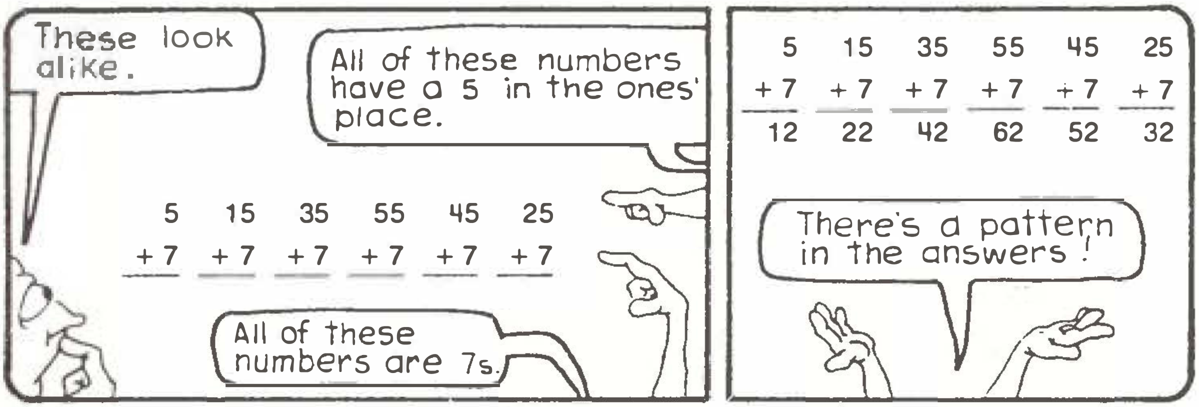
## 12. Per Cent Key

This key changes a number to its equivalent percentage. This is generally viewed as a negative feature, as this is the kind of process a student should do in his head.

## D. Sample Lessons

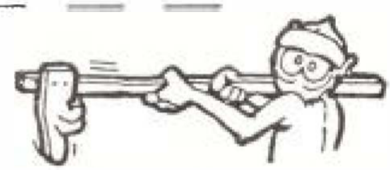
1. Division One

### PATTERN SEARCH*




Use your calculator to solve the following problems. If you see a pattern, try to guess the remaining answers in that row. Then check your guesses on your calculator.

**1.** Enter (+)(2)(=), then (8)(=), (1)(8)(=), and so on.




| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 8 | 18 | 28 | 38 | 48 | 58 | 68 | 78 | 88 | 98 |
| + 2 | + 2 | + 2 | + 2 | + 2 | + 2 | + 2 | + 2 | + 2 | + 2 |

**2.** Enter (+)(9)(=)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 8 | 18 | 28 | 38 | 48 | 58 | 68 | 78 | 98 | 108 |
| + 9 | + 9 | + 9 | + 9 | + 9 | + 9 | + 9 | + 9 | + 9 | + 9 |

**3.** Enter (+)(4)(=)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 7 | 17 | 47 | 67 | 27 | 37 | 57 | 97 | 87 | 107 |
| + 4 | + 4 | + 4 | + 4 | + 4 | + 4 | + 4 | + 4 | + 4 | + 4 |

**4.** Enter (+)(6)(=)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 11 | 31 | 81 | 101 | 91 | 51 | 21 | 41 | 61 | 71 |
| + 6 | + 6 | + 6 | + 6 | + 6 | + 6 | + 6 | + 6 | + 6 | + 6 |

*Refer to Reys, Bestgen et al., *Keystrokes, Calculator Activities For Young Students, Addition and Subtraction* (see bibliography).

2. Division Two

## DOT CONNECTION*

Use your [calculator] to find each answer below. Write the letter of each problem above its answer in the code. Then connect the dots in order according to the code.

| Y | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |

| | | | |
|---|---|---|---|
| (A) 136 ÷ 17 = ___ | (B) 110 ÷ 5 = ___ | (C) 121 ÷ 11 = ___ | (D) 45 ÷ 3 = ___ |
| (E) 80 ÷ 4 = ___ | (F) 432 ÷ 18 = ___ | (G) 39 ÷ 3 = ___ | (H) 330 ÷ 11 = ___ |
| (I) 72 ÷ 12 = ___ | (J) 306 ÷ 18 = ___ | (K) 200 ÷ 40 = ___ | (L) 108 ÷ 4 = ___ |

(M) 91 ÷ 13 = ___

(N) 261 ÷ 9 = ___

(O) 98 ÷ 49 = ___

(P) 184 ÷ 8 = ___

(Q) 504 ÷ 42 = ___

(R) 112 ÷ 7 = ___

(S) 390 ÷ 15 = ___

(T) 92 ÷ 23 = ___

(U) 125 ÷ 5 = ___

(V) 126 ÷ 9 = ___

(W) 50 ÷ 5 = ___

(X) 196 ÷ 7 = ___

(Y) 1 ÷ 1 = ___

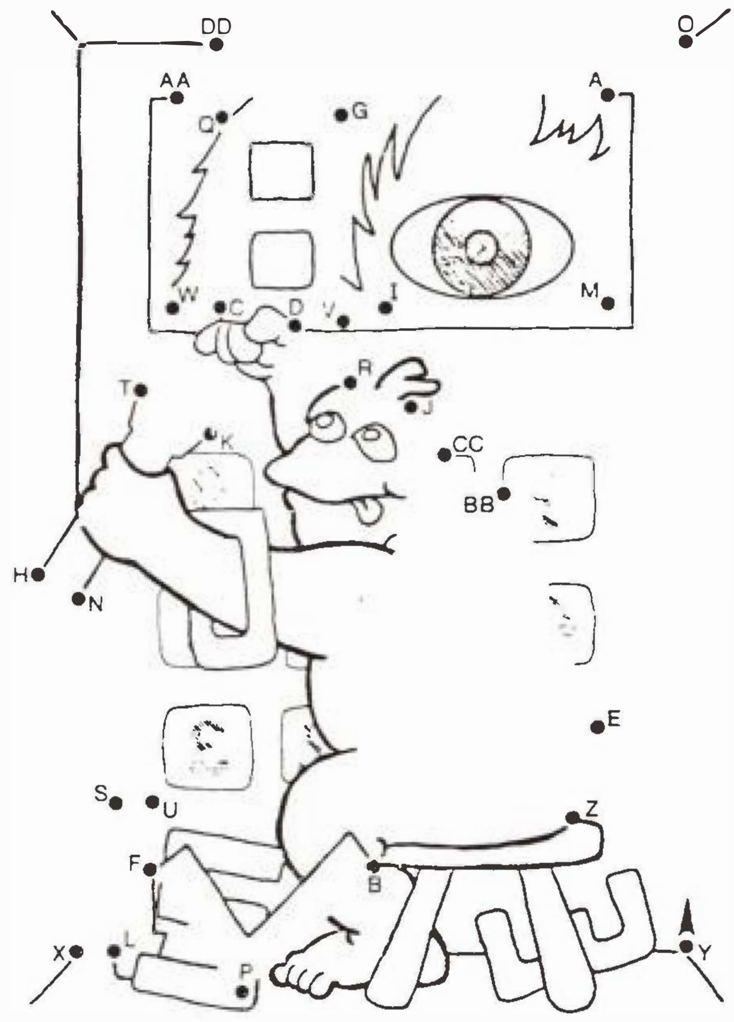
(Z) 126 ÷ 6 = ___

(AA) 261 ÷ 29 = ___

(BB) 646 ÷ 34 = ___

(CC) 108 ÷ 6 = ___

(DD) 42 ÷ 14 = ___




*Refer to Reys, Bestgen et al., *Keystrokes, Calculator Activities For Young Students, Multiplication and Division* (see bibliography).

## 3. Division Three

*Lesson:* PATTERN SEARCH*

*Objectives:*

1. To develop a positive attitude toward working with and developing patterns using the calculator.
2. To give the children practice in finding formulas.

*Directions:*

1. Do the first couple of problems in each set of patterns with the calculator. After doing the first few problems, look at the answers and see if you can predict what the answers for the remaining problems will be.
2. If after doing the first three or four problems you are unable to figure out the pattern, do the next couple using the calculator. Then, do the rest without the use of the calculator, figuring out your answers by just looking at the patterns from the previous answers.

*Problems:*

1. Find a pattern:

8 x 88 =
8 x 888 =
8 x 8888 =
8 x 88888 =
8 x 888888 =
8 x 8888888 =
8 x 88888888 =
8 x 888888888 =
8 x 8888888888 =

2. Complete the pattern:

0! = 1
1! = 1x1 = 1
2! = 1x2 = 2
3! = 1x2x3 = 6
4! = 1x2x3x4 = 24
5! = 1x2x3x4x5 = 120
6! = 1x2x3x4x5x__ = __
7! = 1x2x3x4x5x__x__ = __
8! =
9! =
10! =

3.

1 x 9 =
2 x 9 =
3 x 9 =
4 x 9 =
5 x 9 =
6 x 9 =
7 x 9 =
8 x 9 =
9 x 9 =

4.

1 x 99 =
2 x 99 =
3 x 99 =
4 x 99 =
5 x 99 =
6 x 99 =
7 x 99 =
8 x 99 =
9 x 99 =

5.

1 x 999 =
2 x 999 =
3 x 999 =
4 x 999 =
5 x 999 =
6 x 999 =
7 x 999 =
8 x 999 =
9 x 999 =

6.

1 x 9999 =
2 x 9999 =
3 x 9999 =
4 x 9999 =
5 x 9999 =
6 x 9999 =
7 x 9999 =
8 x 9999 =
9 x 9999 =

7.

0 x 9 + 1 =
1 x 9 + 2 =
12 x 9 + 3 =
123 x 9 + 4 =
1,234 x 9 + 5 =
12,345 x 9 + 6 =
123,456 x 9 + 7 =
1,234,567 x 9 + 8 =
12,345,678 x 9 + 9 =
123,456,789 x 9 + 10 =

8.

(1 x 8) + 1 =
(12 x 8) + 2 =
(123 x 8) + 3 =
(1,234 x 8) + 4 =
(12,345 x 8) + 5 =
(123,456 x 8) + 6 =
(1,234,567 x 8) + 7 =
(12,345,678 x 8) + 8 =
(123,456,789 x 8) + 9 =

9. Use the products of 37 and these multiples of 3 to find the pattern.

3 x 37 =
6 x 37 =
9 x 37 =
12 x 37 =
15 x 37 =
18 x 37 =
21 x 37 =
24 x 37 =
27 x 37 =

10.

$1^2$ =
$11^2$ =
$111^2$ =
$1,111^2$ =
$11,111^2$ =
$111,111^2$ =
$1,111,111^2$ =
$11,111,111^2$ =
$111,111,111^2$ =

11.

9 x 6 =
99 x 66 =
999 x 666 =
9,999 x 6,666 =
99,999 x 66,666 =
999,999 x 666,666 =

---

*Refer to Mauland and Prigge (see bibliography).

## E. Conclusion

It is important that calculators be used for more than checking answers and playing games. Every child should develop skill in estimating and should have algorithms that he/she can use in the absence of a calculator. Students should be allowed to use calculators for classroom tests which assess mathematical ideas rather than computational accuracy. Above all, teachers should realize that the calculator is not a panacea: it cannot resolve all the difficulties in mathematics instruction; teachers must accept the responsibility for teaching children how and when to use calculators, and thus, to be aware of their limitations.

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# Once In a Lifetime?

**by William J. Bruce**
*Professor Emeritus, Department of Mathematics*
*University of Alberta, Edmonton*

The following problem is suitable for beginners of elementary algebra and can be proved at that level. Consider three cases as follows:

A. When will a parent be twice as many years old as any one child of the family?

*Solution:* Let P years be the parent's age when the child is born.
Let C be the child's age in years.
Then P + C is the parent's age when the child is C years old.
If the parent is twice as old as the child, then

$$P + C = 2C.$$

So, $$P = C.$$

Therefore, when a child reaches the age that the parent was when the child was born, the parent will be twice as old in years as the child. And this can occur only once in a lifetime for each child.

B. When will a parent be three times as many years old as any one child of the family?

*Solution:* Proceeding as in case A, we obtain

$$P + C = 3C.$$

So, $P = 2C$, and P must be an even number.

Therefore, this can occur only once in a lifetime for each child *provided* that the age of the parent in years was an even number when the child was born.

C. When will a parent be four times as many years old as any one child of the family?

*Solution:* Here we obtain $P + C = 4C$.

So, $P = 3C$, and P must be divisible by 3.

Therefore, this can occur only once in a lifetime for each child *provided* that the age of the parent in years was a multiple of 3 when the child was born.

Other extensions are left for the reader to explore.

# Pursuing Per Cent Problems

***by Bonnie H. Litwiller and David R. Duncan***
*Mathematics Department*
*University of Northern Iowa*
*Cedar Falls, Iowa*

The following four problems illustrate different applications of the concept of per cent. In each case, a per cent is to be computed. Although the problems may appear similar at first inspection, they are, in fact, quite different.

Let a and b be fixed constants with $a < b$.

1. a is what per cent of b?
2. b is what per cent of a?
3. b is what per cent greater than a?
4. a is what per cent less than b?

To illustrate these numerically, let $a = 4$ and $b = 5$.

1. 4 is 80% of 5. $(80 = \frac{4}{5} \cdot 100)$
2. 5 is 125% of 4. $(125 = \frac{5}{4} \cdot 100)$
3. 5 is 25% greater than 4. $(25 = \frac{(5-4)}{4} \cdot 100)$
4. 4 is 20% less than 5. $(20 = \frac{(5-4)}{5} \cdot 100)$

In general for $a < b$, the answers to the four problems are:

1. If a is $p_1$% of b, then $p_1 = \frac{a}{b} \cdot 100$.
2. If b is $p_2$% of a, then $p_2 = \frac{b}{a} \cdot 100$.
3. If b is $p_3$% greater than a, then $p_3 = [\frac{(b-a)}{a} \cdot 100]$.
4. If a is $p_4$% less than b, then $p_4 = [\frac{(b-a)}{b} \cdot 100]$.

Let us now find relationships among $p_1$, $p_2$, $p_3$, and $p_4$.

A. $p_3 = \frac{(b-a)}{a} \cdot 100$

$= \frac{b(100)}{a} - \frac{a(100)}{a}$

$= \frac{b(100)}{a} - 100$

Therefore, $p_3 = p_2 - 100$

For example, if b is 119% of a, then b is 19% greater than a.

B. $p_4 = (\frac{b-a}{b}) \cdot 100$

$= \frac{b(100)}{b} - \frac{a(100)}{b}$

$= 100 - \frac{a(100)}{b}$

Therefore, $p_4 = 100 - p_1$

For example, if a is 83% of b, then a is 17% less than b.

C. $p_1 \cdot p_2 = (\frac{a}{b} \cdot 100) \cdot (\frac{b}{a} \cdot 100)$

$= 10\ 000$

Therefore, $p_1 = \frac{10\ 000}{p_2}$ and $p_2 = \frac{10\ 000}{p_1}$

For example, if a is 50% of b, then b is 200% of a $(200 = \frac{10\ 000}{50})$; if b is 250% of a, then a is 40% of b $(\frac{10\ 000}{250})$.

D. $p_3 = p_2 - 100$

$= \frac{10\ 000}{p_1} - 100$

$= \frac{10\ 000}{100-p_4} - 100$

$= \frac{10\ 000 - (100-p_4)100}{100-p_4}$

$= \frac{10\ 000 - 10\ 000 + p_4(100)}{100 - p_4}$

Therefore, $p_3 = \dfrac{100\ p_4}{100-p_4}$

E. Solving for $p_4$:

$p_3(100-p_4) = 100\ p_4$

$100\ p_3 - p_3p_4 = 100\ p_4$

$100\ p_3 = p_3p_4 + 100\ p_4$

$100\ p_3 = (p_3 + 100)p_4$

Therefore, $p_4 = \dfrac{100\ p_3}{p_3+100}$

Examples of D and E follow:

If a is 13% less than b, then b is $\dfrac{100(13)}{100-13} = \dfrac{1300}{87}$ or 14.9% greater than a.

If b is 31% greater than a, then a is $\dfrac{100(31)}{100+31} = \dfrac{3100}{131}$ = 23.7% less than a.

One final real world example is given below. The salary of the superintendent of schools is $38 000 while that of a mathematics teacher with 20 years' experience is $18 000. Using the language of per cent, the relationship may be described as follows:

1. The salary of the mathematics teacher is 47.4% of the salary of the superintendent. ($p_1 = 47.4$)

2. The salary of the superintendent is 211.1% of the salary of the mathematics teacher. ($p_2 = 211.1$)

3. The salary of the superintendent is 111.1% greater than the salary of the mathematics teacher. ($p_3 = 111.1$)

4. The salary of the mathematics teacher is 52.6% less than the salary of the superintendent. ($p_4 = 52.6$)

The relations of 3 and 4 may be restated: If the superintendent and mathematics teacher were to exchange salaries, the mathematics teacher would receive a salary increase of 111.1% while the superintendent would receive a salary decrease of 52.6%.

We now verify that relations A through E hold.

A. $p_3 = p_2 - 100$, so 111.1 = 211.1 - 100

B. $p_4 = 100 - p_1$, so $52.6 = 100 - 47.4$

C. $p_1 = \frac{10\ 000}{p_2}$, so $47.4 = \frac{10\ 000}{211.1}$

$p_2 = \frac{10\ 000}{p_1}$, so $211.1 = \frac{10\ 000}{47.4}$

D. $p_3 = \frac{100p_4}{100-p_4}$, so $111.1 = \frac{100(52.6)}{100-52.6}$

E. $p_4 = \frac{100p_3}{p_3+100}$, so $52.6 = \frac{100(111.1)}{111.1+100}$

The reader is to verify these relationships with other sets of data.

---

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# An Advisory Exam in Mathematics

*by Z. M. Trollope*
*Department of Mathematics*
*University of Alberta, Edmonton*

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The Mathematics Department at the University of Alberta gives an examination to the students in its introductory calculus courses early in each fall term. This exam serves to locate those students whose background needs upgrading. A remedial program is set up for the benefit of these students. An increasing number of disciplines now require an introductory calculus course of their students, and results in these courses indicate that an inadequate proficiency with the fundamentals is a real stumbling block for the students.

Our remedial program is of a voluntary nature and of about a five-week duration. This year, the student response was very positive. In the coming year, we hope to supplement the program with computer-based (Plato) material.

We would like the teachers of mathematics in Alberta to be kept informed of the nature of these exams and of our remedial efforts. We welcome any suggestions from them on this procedure.

The 1980 examination is given below, and is followed by its statistics.

---

1. How many of the following numbers are irrational? $\{1.1,\ (-32)^{\frac{1}{5}},\ 1.111\ldots,\ \frac{\sqrt{3}}{\sqrt{12}},\ \pi\}$

   (a) one (b) two (c) three (d) four (e) five

2. $\frac{\sqrt{2}-1}{\sqrt{2}+1} =$ (a) $3+2\sqrt{2}$ (b) $(\sqrt{2}-1)^2$ (c) $\frac{1}{\sqrt{3}}$ (d) $1-2\sqrt{2}$ (e) none of these

3. $2(3+x) \geq 8x + 3(x+2)$ is equivalent to

   (a) $x \leq \frac{1}{9}$ (b) $x \geq 0$ (c) $x \leq \frac{2}{11}$ (d) $x \leq 0$ (e) none of these

4. The solution set of $|2x - 1| = 7$ is

   (a) $\{4\}$ (b) $\{4,-4\}$ (c) $\{3,-3\}$ (d) $\{-4,3\}$ (e) none of these

5. If $\frac{1}{x} + \frac{1}{y} = \frac{1}{R}$, then $x =$

   (a) $\frac{Ry}{y-R}$ (b) $\frac{y}{R-y}$ (c) $\frac{R+y}{Ry}$ (d) $R - y$ (e) none of these

6. $\dfrac{x^3 + 2x^2 - 1}{x + 1} =$

(a) $x^2 + 2x - 1$ (b) $x^2 + x + 1$ (c) $x^2 - x + 1$ (d) $x^2 - x - 1$ (e) none of these

7. $\dfrac{1 + \frac{1}{y} - \frac{20}{y^2}}{1 + \frac{9}{y} + \frac{20}{y^2}} =$ (a) $\frac{1}{9}$ (b) $\frac{y - 5}{y + 5}$ (c) $\frac{y - 4}{y + 4}$ (d) $\frac{y + 4}{y - 4}$ (e) none of these

8. $2x^2 + 3x + c = 0$ has equal real roots if $c =$

(a) $\frac{3}{8}$ (b) $\frac{9}{2}$ (c) $-\frac{3}{4}$ (d) $\frac{9}{8}$ (e) none of these

9. If $g(x) = \frac{1}{2x}$, then $g(g(2)) =$ (a) 4 (b) 2 (c) $\frac{1}{2}$ (d) 1 (e) none of these

10. If $f(x) = 3x^2$, then $\dfrac{f(1+h) - f(1)}{h} =$

(a) $9(h+2)$ (b) $3h$ (c) $3(2+h)$ (d) $3(1 + \frac{2}{h})$ (e) none of these

11. $\dfrac{(-3y)^2 y^{-\frac{2}{3}}}{(2y)^{-1} 3y^{\frac{1}{3}}} =$ (a) $6y^2$ (b) $\frac{3}{2y^2}$ (c) $4^{\frac{1}{3}}(6y)^2$ (d) $6(3^{\frac{2}{3}})y^2$ (e) none of these

12. $(x^{-2}+x^{-4})^{-\frac{1}{2}} =$ (a) $x + x^2$ (b) $\frac{x^2}{x + 1}$ (c) $\frac{x^2}{\sqrt{x^2+1}}$ (d) $x\sqrt{x^2+1}$ (e) none of these

13. If $s = \frac{1}{2}gt^2$, then $\log t =$

(a) $\frac{1}{2}(\log s + \log 2 - \log g)$ (b) $\log\left(\frac{s}{g}\right)$ (c) $\log s - \log 2 - \log g$ (d) $\frac{1}{2}\log(2s-g)$

(e) none of these

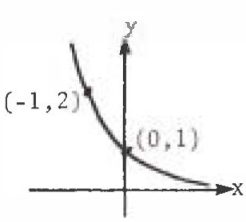
14. If $\log(x+3) = \log x + \log 3$, then $x =$

(a) 0 (b) any positive number (c) $\frac{3}{2}$ (d) $\frac{2}{3}$ (e) none of these

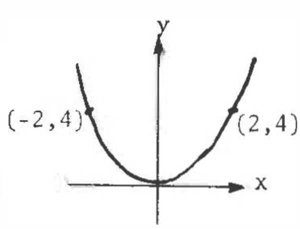
15. If $9 = 4^x$, then $x =$

(a) $\log \frac{9}{4}$ (b) $\frac{\log 4}{\log 9}$ (c) $\log_9 4$ (d) $\log_4 9$ (e) none of these

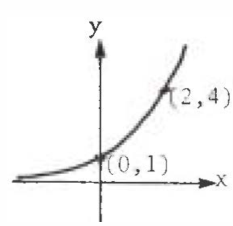
16. Which of the following represents the graph of $y = 2^x$?



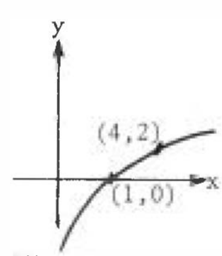

(a)




(b)




(c)



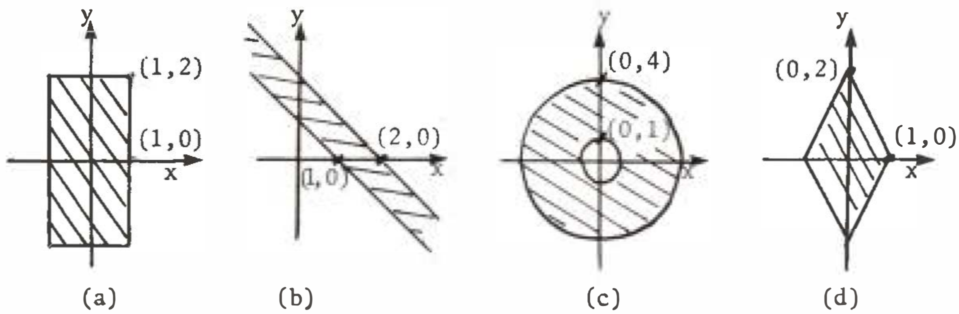

(d)

(e) none of these

17. The lines $2x - y = 1$ and $2y + bx = 3$ are perpendicular if $b =$

(a) 4 (b) 1 (c) -1 (d) -4 (e) none of these

18. Which of the following regions is represented by the inequalities $1 \le x^2 + y^2 \le 4$ ?




(e) none of these

19. The vertex of the parabola $y = x^2 - 2x$ is the point

(a) (2,0) (b) (1,-1) (c) (0,0) (d) (1,1) (e) none of these

20. The points of intersection of the line $3y - x = 25$ and the circle $x^2 + y^2 = 25$ are

(a) (4,3),(-5,0) (b) (-4,-3),(5,0) (c) (-3,4),(5,0) (d) (4,-3),(-5,0)
(e) none of these

21. The radian measure of the angle 150° is

(a) $\frac{-7\pi}{6}$ (b) $\frac{2\pi}{3}$ (c) $\frac{-4\pi}{3}$ (d) $\frac{5\pi}{6}$ (e) none of these

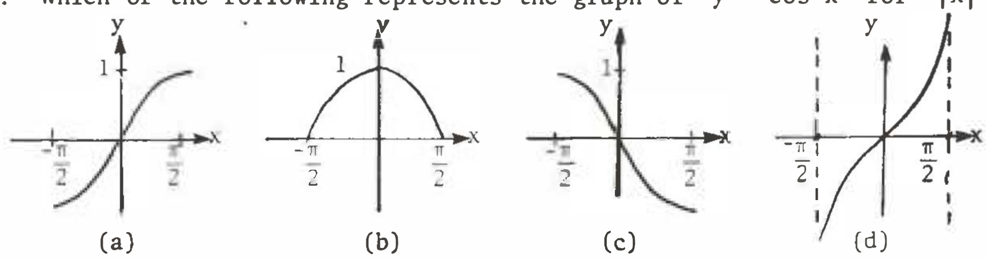
22. $\cos \frac{5\pi}{4} =$

(a) $\frac{5}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{-\sqrt{3}}{2}$ (d) $\frac{1}{2}$ (e) none of these

23. $\tan \theta =$

(a) $\frac{\cos \theta}{\sin \theta}$ (b) $\sec \theta - 1$ (c) $\frac{1}{\cot \theta}$ (d) $\sec \theta + 1$ (e) none of these

24. Which of the following represents the graph of $y = \cos x$ for $|x| \le \frac{\pi}{2}$ ?




(e) none of these

25. A kite string makes an angle of 32° with the (level) ground and 45 feet of string is out. The height of the kite above the ground level is

(a) 45 sin 32° (b) $\frac{\sin 32°}{45}$ (c) 45(1-cos 32°) (d) $\frac{\tan 32°}{45}$ (e) none of these

## Advisory Exam Statistics

Table I gives the percentage of students answering each of the 25 *questions* correctly.

Table II gives the relative frequency (R.F.) and cumulative frequency (C.F.) for each of the possible *scores* (that is, 0 to 25).

TABLE I

| *Question* | *Percentage* |
|---|---|
| 1 | 18 |
| 2 | 27 |
| 3 | 54 |
| 4 | 38 |
| 5 | 35 |
| 6 | 64 |
| 7 | 46 |
| 8 | 25 |
| 9 | 47 |
| 10 | 41 |
| 11 | 36 |
| 12 | 22 |
| 13 | 24 |
| 14 | 09 |
| 15 | 34 |
| 16 | 62 |
| 17 | 35 |
| 18 | 09 |
| 19 | 35 |
| 20 | 44 |
| 21 | 67 |
| 22 | 33 |
| 23 | 37 |
| 24 | 33 |
| 25 | 66 |

TABLE II

| *Score* | *R.F.* | *C.F.* |
|---|---|---|
| 0 | 0.6 | 0.6 |
| 1 | 1.0 | 1.6 |
| 2 | 2.3 | 3.9 |
| 3 | 4.2 | 8.1 |
| 4 | 4.7 | 12.8 |
| 5 | 8.7 | 21.4 |
| 6 | 8.8 | 30.3 |
| 7 | 8.3 | 38.5 |
| 8 | 8.5 | 47.0 |
| 9 | 10.0 | 57.0 |
| 10 | 7.4 | 64.4 |
| 11 | 5.6 | 70.0 |
| 12 | 5.9 | 75.9 |
| 13 | 5.6 | 81.4 |
| 14 | 4.0 | 85.5 |
| 15 | 3.7 | 89.1 |
| 16 | 2.0 | 91.1 |
| 17 | 2.4 | 93.5 |
| 18 | 1.7 | 95.2 |
| 19 | 1.2 | 96.4 |
| 20 | 1.2 | 97.6 |
| 21 | 1.1 | 98.7 |
| 22 | 0.4 | 99.1 |
| 23 | 0.3 | 99.4 |
| 24 | 0.4 | 99.8 |
| 25 | 0.2 | 100.0 |

2 117 students wrote this exam.  The mean score was 9.4.

# ? ? ? Problem Corner ? ? ?

***edited by William J. Bruce and Roy Sinclair***
*University of Alberta, Edmonton*

Problems suggested here are aimed at students of both the junior and senior high schools of Alberta. Solutions are solicited and a selection will be made for publication in the next issue of *delta-K*. Names of participants will be included. All solutions must be received (preferably in typewritten form) within 30 days of publication of the problem in *delta-K*.

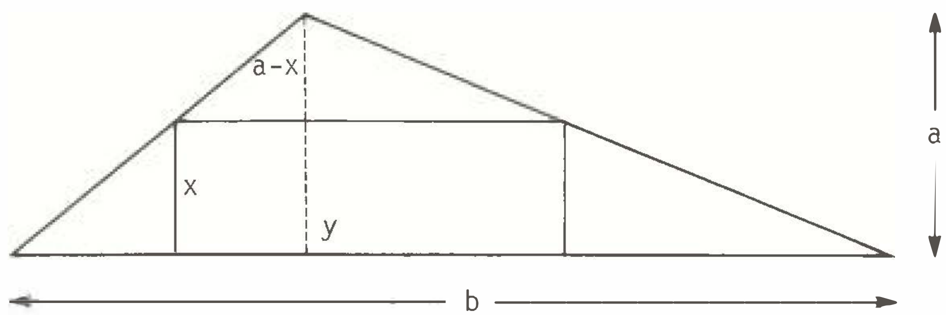
*Mail solutions to:* Dr. Roy Sinclair or Dr. Bill Bruce
Department of Mathematics
University of Alberta
Edmonton, Alberta T6G 2G1

## Problem 5:

*(submitted by William J. Bruce, University of Alberta)*

A rectangle is inscribed in a scalene triangle, as shown, with one side of the rectangle on a side of the triangle and one of its vertices on each of the other sides. Without using calculus, prove that the maximum area of the rectangle is the same no matter on what side of the triangle the one side is located.

NOTE: It has been shown that when the rectangle is drawn with only three of its vertices on the triangle, a rectangle of greater area does not exist.



## Problem 4:

*(submitted by Dr. A. Meir, University of Alberta)*

Let $p$ be a prime number and $\underline{a}$ be a positive integer. Show that $(a-p)^3 + a^3 = (a+p)^3$ cannot be true.

## Solution of Problem 4

*(suggested by Dr. A. Meir)*

Suppose that $(a-p)^3 + a^3 = (a+p)^3$ is true. Then,

$2a^3 - 3a^2p + 3ap^2 - p^3 = a^3 + 3a^2p + 3ap^2 + p^3$

or

$$a^3 - 6a^2p = 2p^3 . \quad (1)$$

This means that $a^3$ must be divisible by p, and since p is prime, $\underline{a}$ must be divisible by p. Say $a = t \cdot p$. Then from (1),

$$t^3p^3 - 6t^2p^3 = 2p^3$$

and, dividing by $p^3$, we get

$$t^3 - 6t^2 = 2$$

or

$$t^2(t-6) = 2$$

So, $t > 6$, otherwise $t - 6$ would be negative or zero. But then, $t^2 > 36$ and $t-6 \geq 1$, so $t^2(t-6) > 36$ and cannot be 2.

# Ideas

**Prepared by M. Bernadine Tabler**
*Assistant Instructor, Indiana University, Bloomington, Indiana*

**and Marilyn Hall Jacobson**
*Title 1 Mathematics Co-ordinator*
*Monroe County Community School Corporation, Bloomington, Indiana*



The *Ideas* this month provide practice for addition, subtraction, multiplication, and division of rational numbers.

## IDEAS For Teachers

Levels: 1, 2

### PLAYING THE TRIANGLE

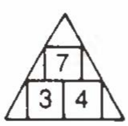
*Objective*

To practise basic addition and subtraction facts.

*Directions*

Review the geometric concept of a triangle. Let the children find examples of triangular shapes in the classroom. Ask the following questions: Have you ever seen someone play a triangle in a band? What does it look like? What does it sound like? Then tell the children that they are going to "play" some "math triangles." Explain that math triangles are numbers that fit together in a special way in the shape of a triangle. Distribute the worksheets. Show the children how to find each number by adding the two numbers in the boxes underneath the number.

For example:




$3 + 4 = 7$

Do triangle A with the children. Have them do B and C. Then ask the children how they could fill in the box between 1 and 3 in triangle D (by subtracting 3 from 6). Have the children finish filling in the rest of the triangles.

*Extensions*

1. Have the children make up some math triangles with four "floors."

   For example:

   16
   7 9
   3 4 5
   2 1 3 2

2. Make up some triangles using larger numbers.

   For example:

   28
   15 13
   8 7 6

3. Have the students investigate the question: Does the order of the numbers on the bottom floor change the number on the top floor?

   For example:
   Try math triangles with these numbers on the bottom floor: 1, 2, 3; 3, 2, 1; and 1, 3, 2.

*Answers*

A.
6
3 3
1 2 1

B.
9
5 4
2 3 1

C.
9
4 5
3 1 4

D.
10
4 6
1 3 3

E.
9
5 4
3 2 2

F.
7
3 4
1 2 2

## IDEAS For Teachers

Levels: 3, 4

### TRYING TRIANGLES

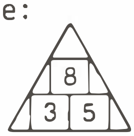
*Objective*

To practise adding and subtracting with one-digit and two-digit numbers.

*Directions*

Ask the students to fill in the rest of the boxes in each triangular array. To find each number they must add the two numbers in the boxes right underneath the number.

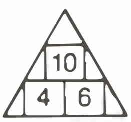
For example:




$3 + 5 = 8$

Have them do the second "floor" in triangle A. Check their results to see that they have the right idea. Then ask the students how they can find the number when one of the boxes underneath the number is empty.

For example:



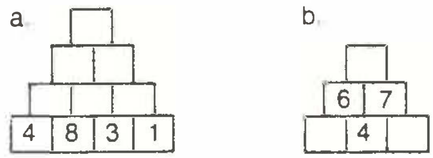

$10 - 4 = 6$

Have the students complete the rest of the triangles.

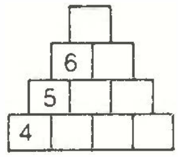
*Extensions*

1. Have the students make up their own triangles for their friends to try. Have them design both types: (a) those with all the numbers on the bottom floor given, and (b) those where the given numbers are scattered.

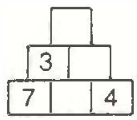
   For example:




2. Have students investigate the question: How many squares do you need to have filled in to complete a particular triangular array? For example, would everyone get the same array of numbers if they completed the following triangle?




3. Have students investigate the question: Could you make partially completed triangular arrays that would be impossible to complete? For example, could you complete the following triangle? (Note that this opens the discussion to the idea of negative numbers.)




*Answers*

(With many of the triangles, checking the top number only would be adequate.)

A.
36
19 17
10 9 8
4 6 3 5

B.
128
58 70
24 34 36
9 15 19 17
2 7 8 11 6

C.

60
30 30
16 14 16
10 6 8 8
7 3 3 5 3

D.

203
90 113
23 67 46

**IDEAS For Teachers**
Levels: 4, 5

## PYRAMID POWER

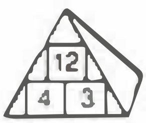
*Objective*

To practise multiplication and division.

*Directions*

Review the geometric concept of a pyramid, and the fact that on each face of a pyramid there is a triangle. Distribute the worksheets. Explain that to find each number on the face of the pyramid, they must multiply the two numbers in the boxes right underneath the number.

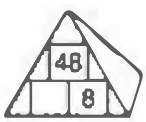
For example:




$4 \times 3 = 12$

Ask the students how they can find the missing number when one of the boxes underneath the number is empty.

For example:




$48 \div 8 = 6$

Have the students complete the rest of the pyramid faces.

*Extension*

Have students make up their own pyramids for their friends to try.

*Answers*

(With many of the pyramids, checking the top number only would be adequate.)

A.

0
0 0
32 0 0
8 4 0 0
2 4 1 0 8

B.

3436
48 72
4 12 6
1 4 3 2

C.

48000
240 200
12 20 10
3 4 5 2

D.

1452
33 44
3 11 4

**IDEAS For Teachers**
Levels: 7, 8

## MORE (PYRAMID) POWER TO YOU

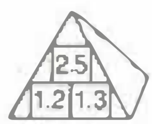
*Objective*

To practise adding and subtracting decimals and common fractions.

*Directions*

Explain to the students how the boxes in the pyramid faces are to be filled in. To find each number, they must add the two numbers in the boxes right underneath the number.

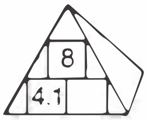
For example:




$1.2 + 1.3 = 2.5$

Ask students how they can find the missing number when one of the boxes underneath is empty.

For example:




$8 - 4.1 = 3.9$

Have the students fill in the rest of the numbers.

*Extensions*

1. Use the same worksheet, but have students multiply (or divide) to get the missing numbers. That is, each number is obtained by multiplying the numbers in the boxes underneath the number.
2. Have students make up their own pyramids for their friends to try.

*Answers*

**A.**

**40.98**

**23.5   17.48**

**16.3   7.2   10.28**

**11.3   5.0   2.2   8.08**

**0.7   3.6   1.4   0.8   0.08**

**B.**

$\frac{48}{12}$

$\frac{20}{12}$   $\frac{28}{12}$

$\frac{3}{4}$   $\frac{11}{12}$   $\frac{17}{12}$

$\frac{1}{2}$   $\frac{1}{4}$   $\frac{2}{3}$   $\frac{3}{4}$

**C.**

$\frac{24}{8}$

$\frac{13}{8}$   $\frac{11}{8}$

$\frac{3}{4}$   $\frac{7}{8}$   $\frac{1}{2}$

$\frac{1}{8}$   $\frac{5}{8}$   $\frac{2}{8}$   $\frac{2}{8}$

D.

$4\frac{17}{10}$

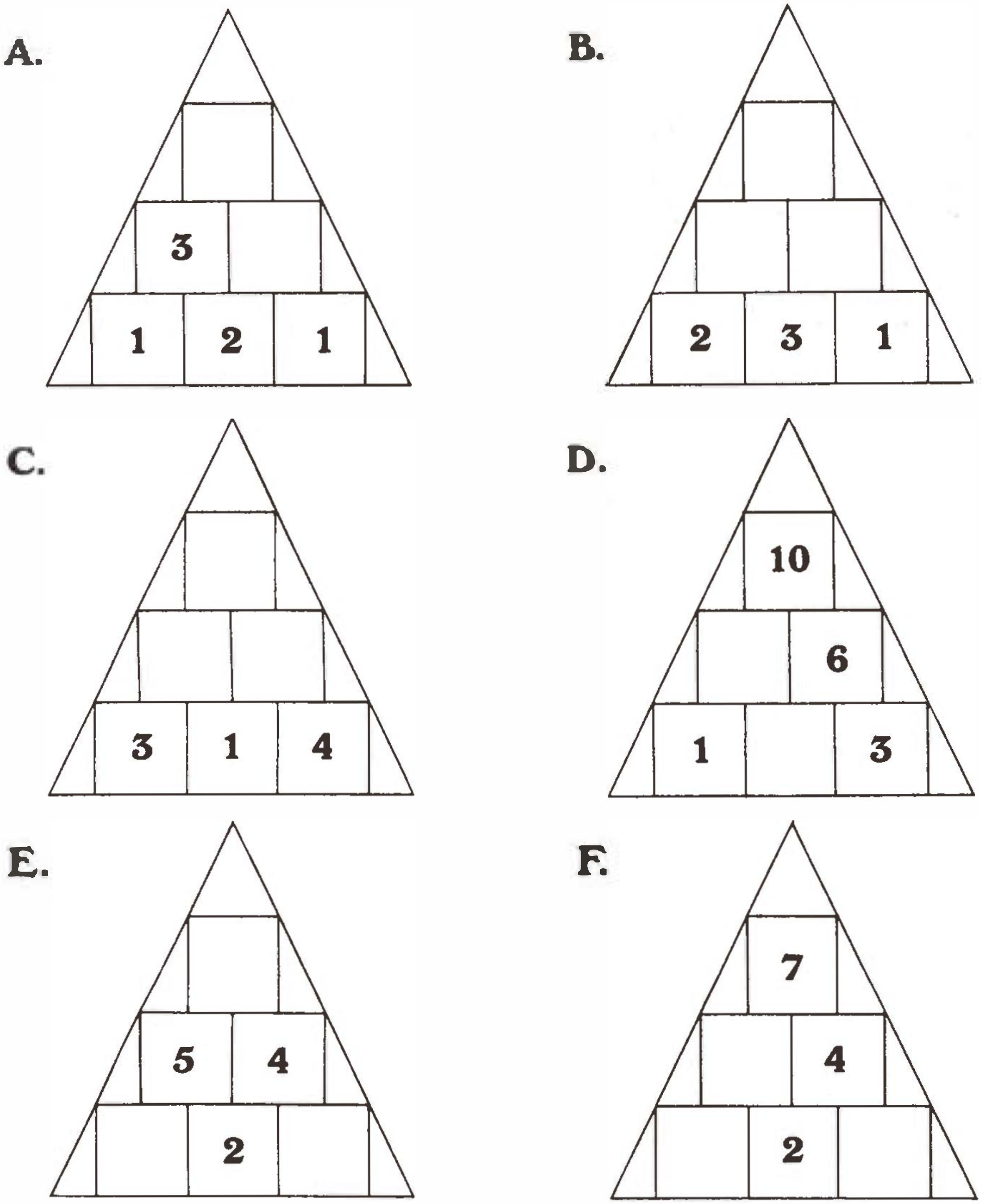
$4\frac{3}{5}$   $\frac{11}{10}$

$4$   $\frac{3}{5}$   $\frac{1}{2}$

IDEAS

Name ____________________

# Playing the Triangle




**Make some more math triangles of your own.**

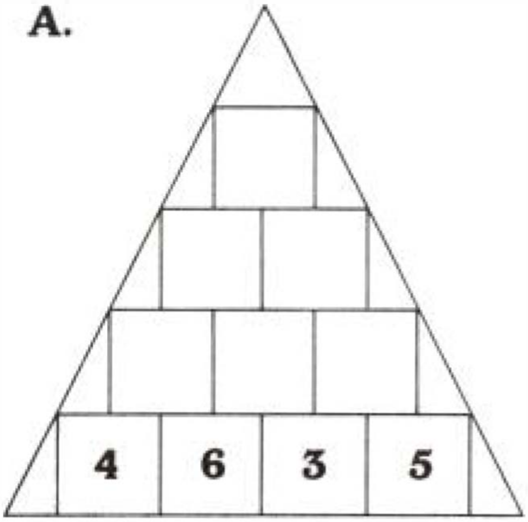
From the *Arithmetic Teacher*, December 1980

IDEAS

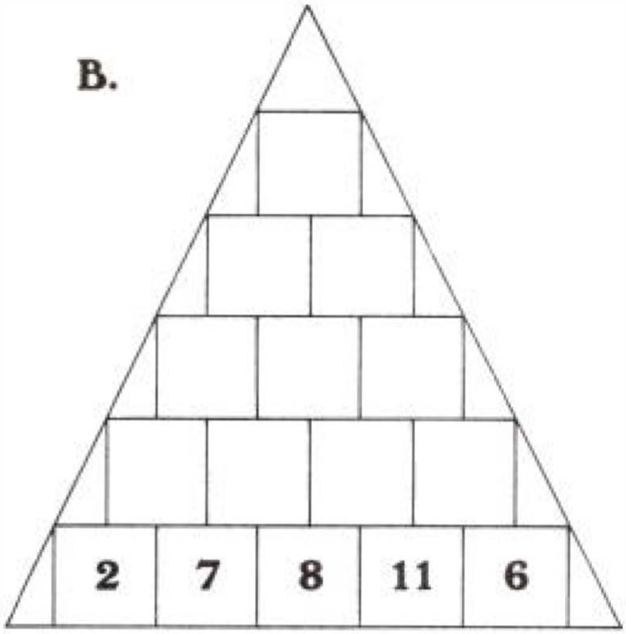
Name ____________________

# Trying Triangles

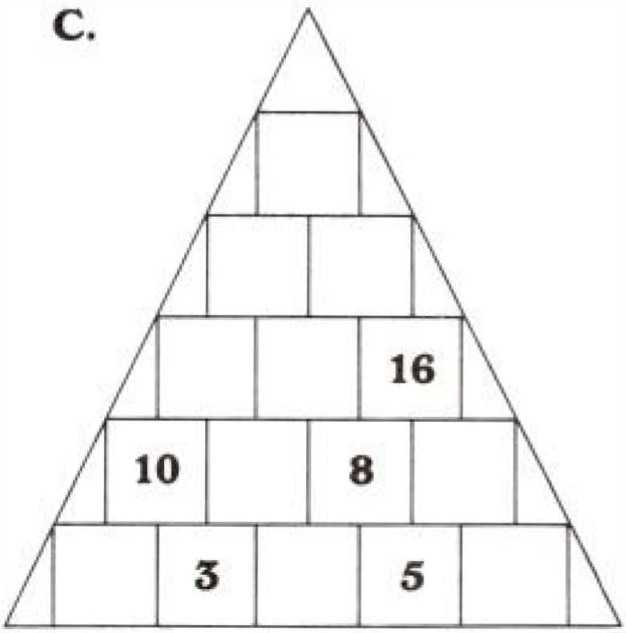
A.



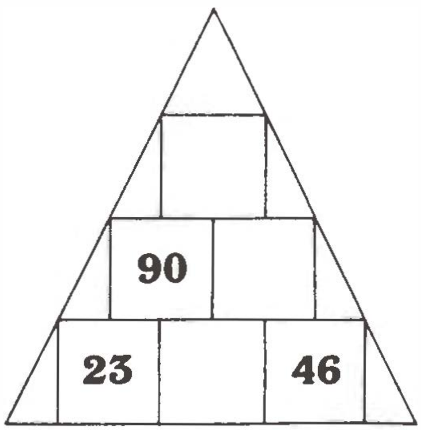

B.




C.




D.

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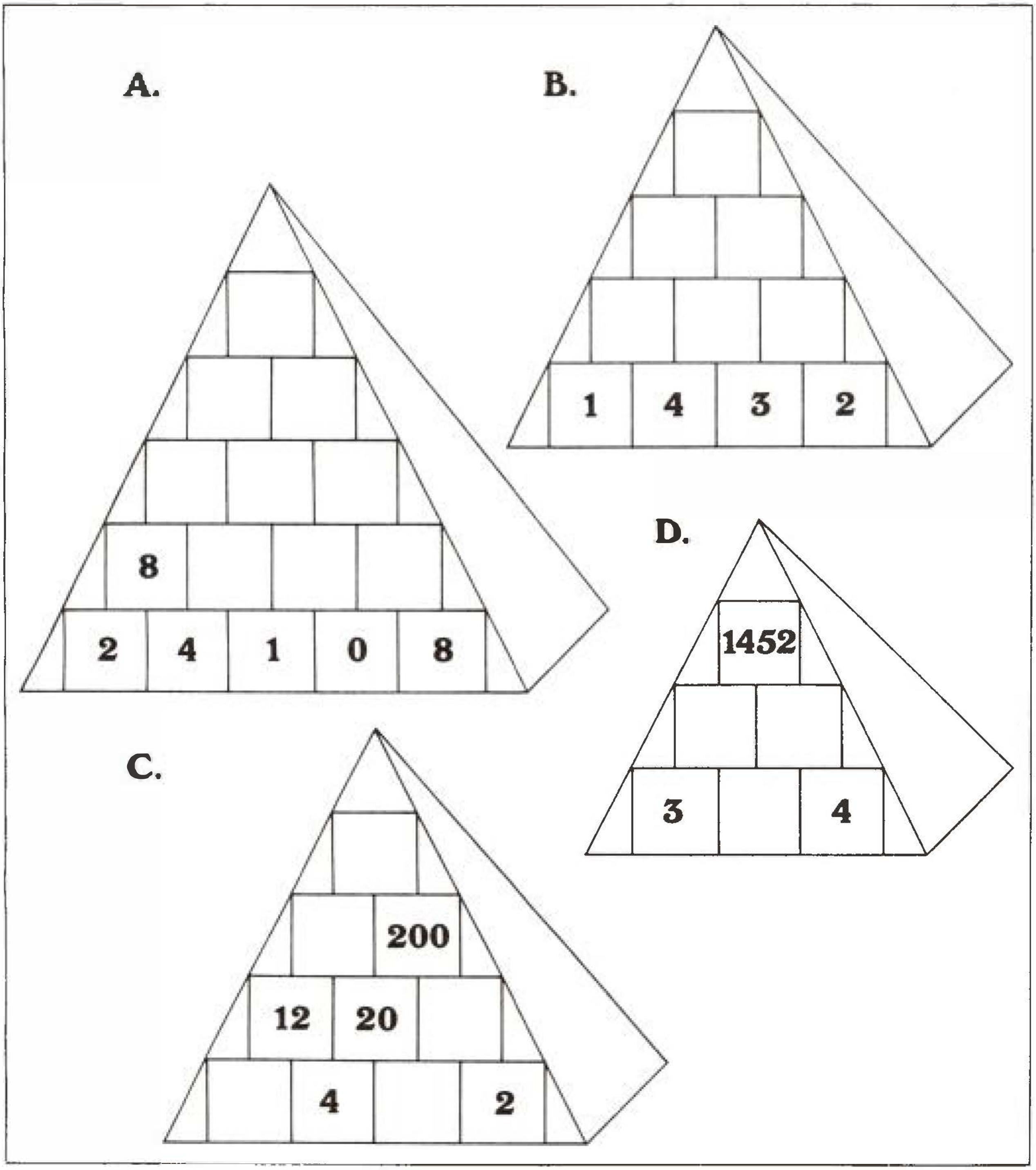
# Pyramid Power

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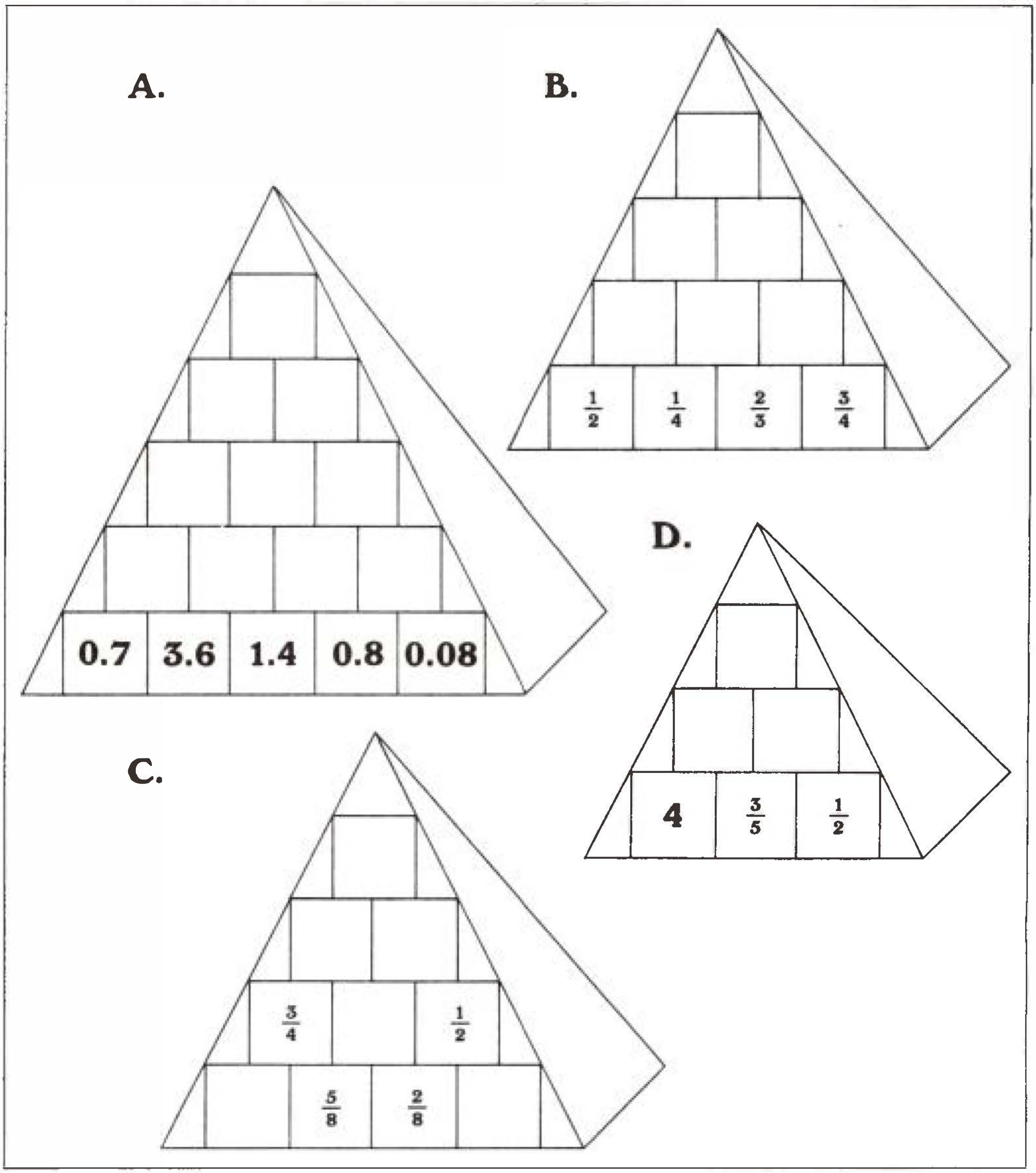
Name ______________________

# More (Pyramid) Power to You

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