? ? ? Problem Corner ? ? ?

edited by William J. Bruce and Roy Sinclair
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Problems suggested here are aimed at students of both the junior and senior high schools of Alberta. Solutions are solicited and a selection will be made for publication in the next issue of *delta-K*. Names of participants will be included. All solutions must be received (preferably in typewritten form) within 30 days of publication of the problem in *delta-K*.

Mail solutions to: Dr. Roy Sinclair or Dr. Bill Bruce
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Problem 6:

(submitted by Roy Sinclair, University of Alberta)

A person has 31 dominoes, each 2 cm by 4 cm, and a checkerboard, each of whose squares are 2 cm by 2 cm. Is it possible to place all the dominoes on the board so as to leave only a pair of diagonally opposite squares uncovered?

Solution to Problem 2

Problem

(submitted by William J. Bruce, University of Alberta)

Clearly $1 = 1^2$ is a perfect square, but 11 and 111 are not. Consider all numbers $11111 \cdots 1 = S$, in which all digits are unity, and prove or disprove that, except for $s = 1$, no such number is a perfect square.*

*EXTENSION (Proof not to be submitted for publication.)

The theorem is true for any number that can be written in the form $100m + 10 + 1$ (m a positive integer). Also, true for $100m + k + 1$ when k is *not* divisible by 4.

Solution

(suggested by Dr. A. Meir, University of Alberta)

Since the given number is odd, it must be the square of an odd number, say $2K + 1$. Then we have

$$(2K + 1)^2 = 4k^2 + 4k + 1 .$$

If we write $11111\cdots1 = 100m + 10 + 1$, we must have

$$4k^2 + 4k + 1 = 100m + 10 + 1$$

or

$$4k^2 + 4k = 100m + 10 ,$$

which is not possible because the left member is divisible by 4, whereas, the right member is not. Thus $11111\cdots1$ cannot be a perfect square except for 1.

Solution to Problem 3

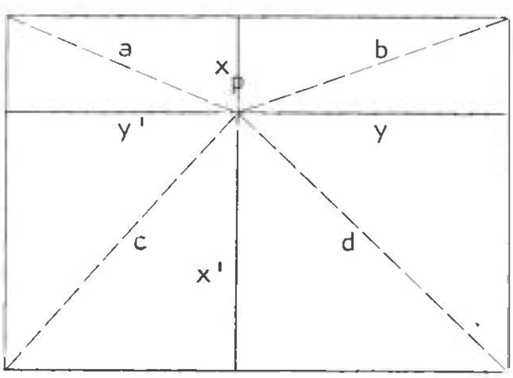
Problem

(submitted by William J. Bruce, University of Alberta)

Point P is located in a rectangular region such that its distances from three of the vertices of the rectangle are given by "a ft.," "b ft.," and "c ft." Let "d ft." be the unknown distance to the fourth vertex and find a relation among the four distances, "a," "b," "c," and "d," so that whenever any three are known, the fourth can be computed.

Solution

(suggested by William J. Bruce)



From the figure

$$a^2 = x^2 + y'^2$$

$$d^2 = x'^2 + y^2$$

$$b^2 = x^2 + y^2$$

$$c^2 = x'^2 + y'^2$$

$$a^2 + d^2 = (x^2 + y^2) + (x'^2 + y'^2)$$

$$= b^2 + c^2$$