activities

Spirolaterals

Richard Brannan and Scott McFadden
Lane Education Service District, Eugene, OR 97402

Teacher's Guide

Grade Level: 7 - 9.

Materials:

One set of the activity sheets and several sheets of grid paper for each student; a set of transparencies of the activity sheets and grids for the teacher.

Objectives:

Students will discover the following about order-3 spirolaterals on square grid paper.

1. Each order-3 spirolateral can be classified as one of three types depending on whether the sum of the two smaller numbers is equal to, less than, or greater than the largest number.
2. The number of small squares in the "hole" of type *B* and in the "overlap" of type *C* can be predicted without drawing the spirolateral by squaring the difference between the largest number and the sum of the two smaller numbers.

Procedure:

1. Distribute activity sheets 1 and 2.
2. Using the overhead projector, demonstrate how to draw an order-3 spirolateral as students follow along and draw the spirolateral.
3. Have students finish activity sheets 1 and 2.
4. Distribute activity sheet 3. Have students fill in the table with their own and classmates' numbers. Have students discover the relationships.

Supplementary Activities:

1. For a given order, find the number of loops through the numbers needed to bring a spirolateral back to its starting point. See the article by Schwandt (1979) for suggestions.
2. Similar investigations are possible on isometric grid paper using order-2 spirolaterals.

REFERENCES

Gardner, Martin. "Mathematical Games." *Scientific American*, February 1974.

Olds, Frank C. "Spirolaterals." *Mathematics Teacher* 66 (February 1973): 121-24.

Schwandt, Alice. "Spirolaterals: Advanced Investigations from an Elementary Standpoint." *Mathematics Teacher* 72 (March 1979):166-69.

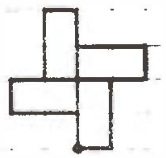
(Answers on page 32)

To draw a 1,2,3 spirolateral: Trace *one* unit *up*, *two* units *right*, *three* units *down*, *one* unit *left*, *two* units *up*, and so on. Continuing in a clockwise direction, you should finish at the starting point. The first five steps are shown below. The completed spirolateral is shown to the right. This is called an *order-3* spirolateral because *three* numbers are used as the lengths of the segments.

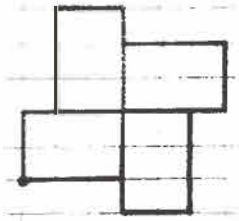
Figure ***A***
1,2,3 spirolateral
(Type ***A***)

The completed 1,2,3 spirolateral shows four rectangles fitting snugly around a middle point. Call this *Type A*.

1. A 3.2.1 spirolateral and a 2.5.3 spirolateral are shown below. Trace each one. Is each one a Type *A*?_____

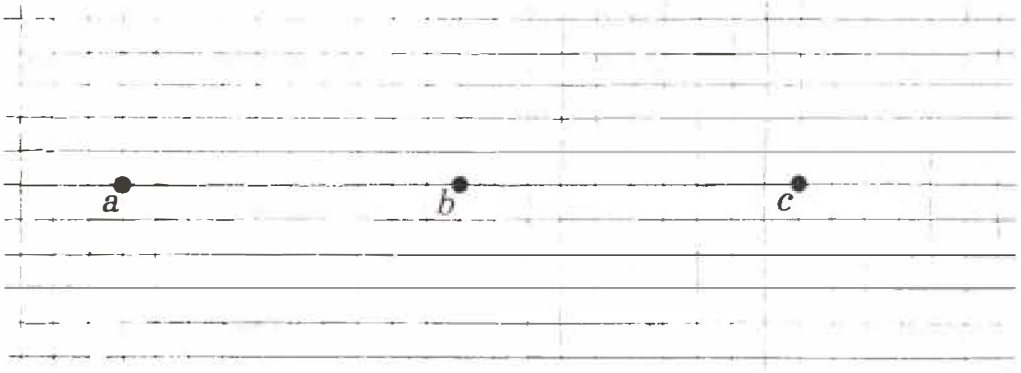


3,2,1 spirolateral



2,5,3 spirolateral

2. Draw these order-3 spirolaterals: a) 1.3,4; b) 2.1,3; c) 5.1,4. The starting point for each is shown. Is each one a Type *A*?_____



An order-3 spirolateral showing four rectangles with a "hole" in the middle is called a *Type B* spirolateral (see Figure *B* for an example). An order-3 spirolateral showing four rectangles that "overlap" in the middle is called a *Type C* spirolateral (see Figure *C* for an example).

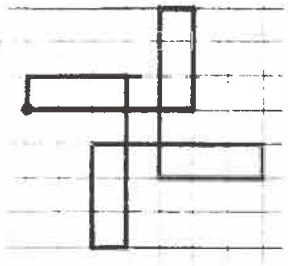


Figure *B*
1,3,5 spirolateral
(Type *B*)

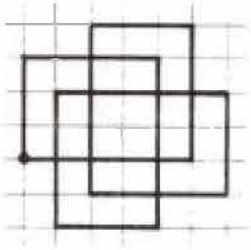
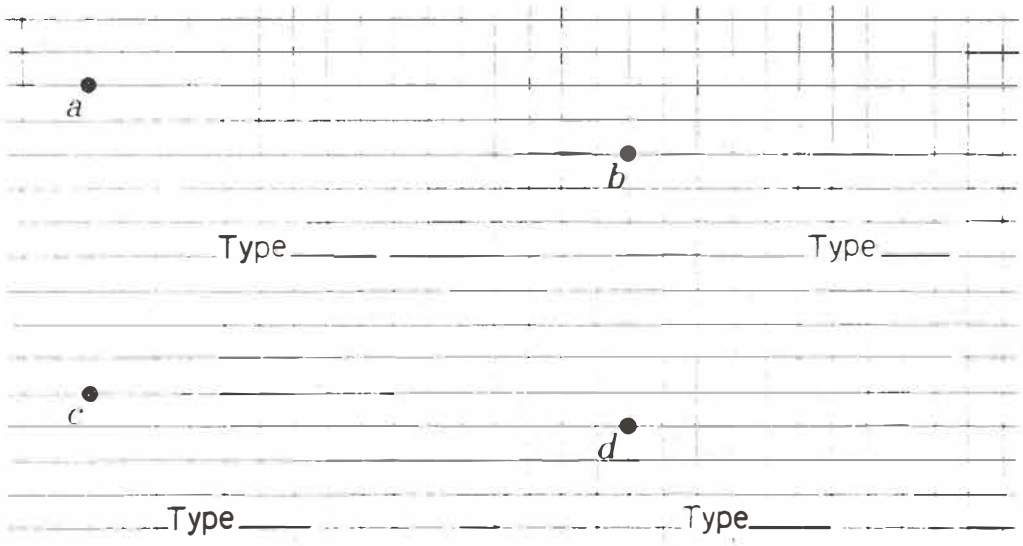


Figure *C*
3,4,5 spirolateral
(Type *C*)

3. On the grid below, draw these order-3 spirolaterals. Starting points are shown. Label each spirolateral as Type *B* or Type *C*.

a) 1,2,6 b) 2,4,5 c) 1,2,4 d) 4,2,4



4. On a sheet of grid paper, create six order-3 spirolaterals of your own. Label each as Type *A*, *B*, or *C*.

5. You have drawn many order-3 spirolaterals. Each was a Type *A*, *B*, or *C*. Write the three numbers of each spirolateral in the appropriate column.

   Three are done for you. Add others that your classmates have done.

	Type	
A	*B*	*C*
1,2,3	1,3,5	3,4,5

6. Look at the numbers in column A. Without drawing, predict how you can tell if an order-3 spirolateral is Type *A*. _____

   _____

7. Look at the numbers in column B. Without drawing, predict how you can tell if an order-3 spirolateral is Type *B*. _____

   _____

8. Look at the numbers in column C. Without drawing, predict how you can tell if an order-3 spirolateral is Type *C*. _____

   _____

9. Without drawing, what type is each of these order-3 spirolaterals?

   a) 10,15,5 ____ b) 21,22,23 ____ c) 100,32,25 ____

   d) 4,21,17 ____ e) 1,1,1 ____ f) 25,16,9 ____

10. Again investigate the numbers in columns B and C. Also look at the spirolaterals drawn with the numbers. Find a way to predict the number of small squares in the "hole" or in the "overlap."

    _____

    _____