ISSN 0319 - 8367

delta-k

THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

Volume XXI, Number 1

September 1981

Mathematics Council, ATA 21st ANNUAL CONFERENCE

October 16 and 17, 1981

University of Lethbridge/ Lethbridge Lodge

An Agenda for Action

Recommendations for School Mathematics of the 1980s

The National Council of Teachers of Mathematics recommends that—

1. problem solving be the focus of school mathematics in the 1980s;
2. basic skills in mathematics be defined to encompass more than computational facility;
3. mathematics programs take full advantage of the power of calculators and computers at all grade levels;
4. stringent standards of both effectiveness and efficiency be applied to the teaching of mathematics;
5. the success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing;
6. more mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population;
7. mathematics teachers demand of themselves and their colleagues a high level of professionalism;
8. public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society.

The National Council of Teachers of Mathematics, Inc.
1906 Association Drive, Reston, Virginia 22091

Printed in the United States of America
ISBN 0-87353-166-3

delta-k

THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

Volume XXI, Number 1 *September 1981*

Contents

	2	Mathematics Council Executive, 1981-82
George Cathcart	3	Editorial
	4	Leadership Conference on Statistics and Probability
	5	MCATA Annual Conference
Bill Skelly and *Byron Skretting*	6	Vauxhall Teachers Try Different Ideas
Emil Dukovac	8	Vectors and Art
Merwin J. Lyng	10	A Perspective of Mathematics Education in the People's Republic of China
Marilyn Hall Jacobson and M. *Bernadine Tabler*	14	Ideas with a Halloween Theme: 'Tricks or Treats' and 'Haunted House'
William J. Bruce and *Roy Sinclair*	19	Problem Corner
Richard Brannan and *Scott McFadden*	21	Activities: Spirolaterals
George W. Bright	25	Ideas: Addition, Subtraction, Multiplication (for levels 1 to 8)
J. W. Roberts	31	Similar Triangles with a New Slant

EDPRESS

delta-K is published by The Alberta Teachers' Association for the Mathematics Council. Editor: Dr. George Cathcart, Department of Elementary Education, University of Alberta, Edmonton T6G 2G5. Editorial and Production Services: Communications staff, ATA. Opinions of writers are not necessarily those of either the Mathematics Council or The Alberta Teachers' Association. Please address all correspondence regarding this publication to the editor. *delta-K* is indexed in the Canadian Education Index.

123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789

MATHEMATICS COUNCIL EXECUTIVE, 1981-82

123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789

PRESIDENT
Gary Hill — Res. 327-5601
310 Laval Blvd. — Bus. 329-1243
Lethbridge T1K 3W5

PAST PRESIDENT
Lyle Pagnucco — Res. 271-0259
14135 Parkside Dr. SE — Bus. 276-5521
Calgary T2J 3X3

VICE-PRESIDENT
Bruce Stonell — Res. 346-7814
Box 5002 — Bus. 343-5262
3rd Floor, West
Provincial Building
4920 - 51 Street
Red Deer T4N 5Y5

SECRETARY
Dr. Arthur Jorgensen — Res. 723-5370
Box 2069 — Bus. 723-5515
Edson TOE OPO

TREASURER
Brian Chapman — Res. 782-3551
Box 1525 — Bus. 782-3812
Lacombe TOC 1SO

Monograph EDITORS
Sid Rachlin — Res. 284-4029
Faculty of Education — Bus. 284-6164
University of Calgary
Calgary T2N 1N4

Ron Cammaert — Res. 223-4948
Box 1771 — Bus. 223-2902
Taber TOK 2GO

ATA STAFF ADVISOR
C.E. Connors — Bus. 453-2411 ext. 249
Barnett House
11010 - 142 Street
Edmonton T5N 2R1

PEC LIAISON REPRESENTATIVE
Norval A. Horner — Res. 456-2744
15235 - 117 Street — Bus. 973-3301
Edmonton T5M 3V7

delta-K editor, directors, and representatives for 1981-82 are still to be appointed.

SOUTH WEST REGIONAL

President
Jean Poile — Res. 328-6121
1705 Ashgrove Road — Bus. 328-4532
Lethbridge T1K 3M2

Secretary
M. Jo Maas — Res. 553-4848
Box 484 — Bus. 737-3963
Fort Macleod TOL OZO

Treasurer
Joan Haig — Res. 328-3824
1115 - 8 Avenue S — Bus. 328-9606
Lethbridge T1J 1P7

Program Director
Hans Hulstein
1409 Waterton Road
Lethbridge T1K 4Z6

Editorial

Best Wishes For a Successful School Year!

It is important to get off to a good start at the beginning of the year. To assist you in making a good beginning, this issue of *delta-K* contains a larger number of puzzles, problems, and interesting activities than most other issues. Hopefully each of you will be able to use at least a few of the ideas. If you have an interesting puzzle, game, activity, or problem you have found successful, why not share it with your colleagues around the province. I would be happy to publish it in *delta-K*.

Robert Holt, Chairman of the Election Committee, reports that only one nomination was received for each of the elected positions on the MCATA executive. The following nominees have been declared elected for 1981-82:

President	*Mr. Gary Hill*, Lethbridge
Vice-President	*Mr. Bruce Stonell*, Red Deer
Secretary	*Dr. Art Jorgensen*, Edson
Treasurer	*Mr. Brian Chapman*, Lacombe

Congratulations to each of these. They have each contributed in a significant way to MCATA in the past and we all look forward to their continuing leadership.

Others who have contributed in a significant way to MCATA are now leaving the executive. These include Past President Robert Holt and Vice-President Judy Steiert. Of course, President Lyle Pagnucco moves into the Past President's office. We are assured of their continued interest and support. Since directors, representatives, and the *delta-K* editor are appointed annually, I cannot be sure of those from this group who will be back until after the next executive meeting.

Throughout this and other issues of *delta-K* you have noticed little puzzles or write-ups of available mathematics material. These are used for fillers to complete a page. Can you help me out with some "fillers," brief descriptions of a teaching device, game or activity, a puzzle, problem, a mathematical joke, pun, or cartoon, etc.? Your contribution will be acknowledged. Thanks!

- *George Cathcart*

AMERICAN STATISTICAL ASSOCIATION – NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

JOINT COMMITTEE
ON THE CURRICULUM IN STATISTICS AND PROBABILITY

LEADERSHIP CONFERENCE

"Statistics and Probability in the Classroom"

1776 Holiday Inn, Williamsburg, Virginia

December 2-6, 1981

As part of its work toward implementing the recommendations of the AGENDA FOR ACTION, the ASA/NCTM Joint Committee on the Curriculum in Statistics and Probability is organizing a leadership conference along the same lines as the Instructional Affairs Committee's Calculator Conference held last December in Las Vegas.

The conference is directed toward supervisors, consultants, teacher educators, teachers, and others who are involved with in-service training. Our aim is to provide right-to-copy materials, advice, and assistance for those who want to conduct in-service training for teachers in their own districts. There will be material for all grade levels and courses which will include a variety of topics such as sampling, simulation, probability, exploration of data, using newspapers and magazines as sources of data, resources, and project work. Materials will emphasize the practical nature of the subject and ways of developing the strong link between statistics and society will be explored.

A special feature of the conference will be workshops that tap the expertise that can be expected to be present among the participants. In these idea workshops, groups will collaborate on the development of ideas for display. The ideas will then be printed and circulated after the conference, and a network will be established to further the distribution of ideas, suggestions, and materials.

One of the greatest benefits of conferences of this nature is the contacts that are made and renewed. This aspect of the conference will, perhaps, be enhanced by the unique atmosphere of the Christmas season in Williamsburg and will give the conference an extra dimension.

If you want to make sure that you are kept informed of the details of the conference and are sent a registration brochure, please write to:

Dr. Ena Gross
Division of Teacher Education
3086 Oliver Hall
Virginia Commonwealth University
Richmond, VA 23284

Mathematics Council, ATA
21st Annual Conference

Friday ~~evening,~~ October 16,
and Saturday, October 17, 1981
University of Lethbridge/Lethbridge Lodge

Three Major Themes:

- Problem Solving
- Gifted Students
- Computers and Technology

Keynote Speaker:

Earl Ockenga
Price Laboratory School, Cedar Falls, Iowa

Accommodation:

Please make your own arrangements for accommodation.
Possible hotels and rates (subject to change):

- Lethbridge Lodge – $39 single; $43 double
- Travelodge – $32 single; $38 double
- Lethbridge Inn – $28 single; $32 double
- El Rancho – $24 single; $32 double

Conference Registration:

	Before Sept. 25	*After Sept. 25*
MCATA Members	$30	$35
Non-Members*	40	45
Student Members	10	10
Student Non-Members*	16	16

*includes membership

To preregister, complete the registration form which was sent earlier to members and schools or send your cheque (payable to *Mathematics Council, ATA*) to Mrs. Joan Haig, 1115 - 8 Avenue S., Lethbridge T1J 1P7.

Vauxhall Teachers Try Different Ideas

Bill Skelly writes:

In our Math 30 class, we have been doing work on finding factors of polynomials by synthetic division. I have found a way of limiting the number of possible factors of a polynomial under certain conditions.

Here is an example: Find the ratio of 4/12 = 1/3 and ± 1/3 are possible zeros.

$12x^3 - 4x^2 - 3x + 1$

$$\begin{array}{r|rrrr} -\frac{1}{3} & 12 & -4 & -3 & 1 \\ & & -4 & -0 & 1 \\ \hline & 12 & 0 & -3 & 0 \end{array}$$

$p(x) = (x - 1/3) \quad (12x^2 - 3)$

$12x^2 = 3$

$x = 1/3, \; x^2 = 3/12 = 1/4$

$x = \pm 1/2$

Take the second coefficient and put it over the first, and the fourth over the third, and you get the same ratio.

This formula works only under the following conditions:

1. there must be 4 terms
2. they must appear in descending order
3. it does not matter if a term is missing
4. and a zero is a ± of the ratio of the coefficient
5. the coefficients must be in a ratio to each other:

$$\frac{\text{second coefficient}}{\text{first coefficient}} = \frac{\text{fourth coefficient}}{\text{third coefficient}}$$

We tried this for several other polynomials which abide by these conditions and they also work.

... and Byron Skretting writes:

In our Math 30 class, we have been doing conic sections. In doing the circle, we were given a formula for finding the centre when the equation for the circle was in general form, $\left(\frac{-D}{2}, \frac{-E}{2}\right)$.

After moving on to the ellipse, we couldn't find any formula for finding the centre of the ellipse when the equation was written in general form.

We started off with an equation for a particular ellipse with the centre (5,3) in standard form and converted it to general form.

$$\frac{(x - 5)^2}{25} + \frac{(y - 3)^2}{16} = 1$$

$$16(x-5)^2 + 25(y-3)^2 = 400$$

$$16(x^2-10x+25)+25(y^2-6y+9) = 400$$

$$16x^2-160x+400+25y^2-150y+225 = 400$$

$$16x^2+25y^2+{}^-160x+{}^-150y+225 = 0$$

$$A=16 \quad C=25 \quad D={}^-160 \quad E={}^-150 \quad F=225$$

I then noticed that if you take ($\frac{-D}{2A}$, $\frac{-E}{2C}$), you would get the centre of the ellipse.

$$x=\frac{-D}{2a}=\frac{160}{2x16}=\frac{160}{32}=5$$

$$y=\frac{-E}{2C}=\frac{150}{2x25}=\frac{150}{50}=3$$

centre = (5,3)

We tried this for several other ellipses and it seems to work for all ellipses.

EDITOR'S NOTE: I appreciate Bill and Byron taking the time to share their ideas with us. May their example serve as an encouragement to others to share ideas and suggestions through delta-K.

In addition to your ideas, I would also be pleased to publish any reactions to the above methods. If you prefer to correspond directly with either Bill or Byron, their address is Vauxhall Junior-Senior High School, Box 618, Vauxhall, Alberta TOK 2KO.

Vectors and Art

Emil Dukovac

Kapuskasing District High School, Kapuskasing, Ontario

This material was used in a sample of 100 14-year-old students in Grade 9.

Materials: compass, ruler, grid paper.

Procedure:

1. On Diagram 1:
   (a) Draw the co-ordinate axes.
   (b) Draw a circle, centre at the origin, radius 8 units.
   (c) Plot the following ordered pairs and label them all A: A(8,0), A(0,8), A(7,4), A(4,7).

2. On Diagram 2:
   Reflect the ordered pairs labelled A in the x-axis; label them A.

3. On Diagram 3:
   Reflect all the ordered pairs marked A in the y-axis; label them A.

4. On Diagram 4:
   (a) Draw the vectors $\vec{AB} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$; A will be the initial point of $\vec{AB}$; B will be the terminal point of $\vec{AB}$.
   (b) Label the origin C(0,0); draw the vector $\vec{CD} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

5. On Diagram 5:
   Draw a series of parallelograms labelled ABDC; do this by joining all the segments labelled CA and DB.

6. How do you see the picture?

7. Note: the students do the above development on one graph.

Follow-Up Activities

1. Discuss symmetry in the origin.

2. Art Project -

   Use vectors to create an art piece. Give equal marks for (a) originality, (b) aesthetic qualities, (c) mathematical explanation.

3. Pythagorus -

Teacher: "Does A(4,7) really lie on the circle?"

Student: "No, it is off just a bit; I can tell by looking at my diagram."

Teacher: "How can we prove that A(4,7) does not lie on the circle?"

Student: "Find the length of CA."

Teacher: "How shall we find the $|CA|$?"

Student: "Use Pythagorus."

Solution:

$CA^2 = 4^2 + 7^2$

$CA^2 = 16 + 49$

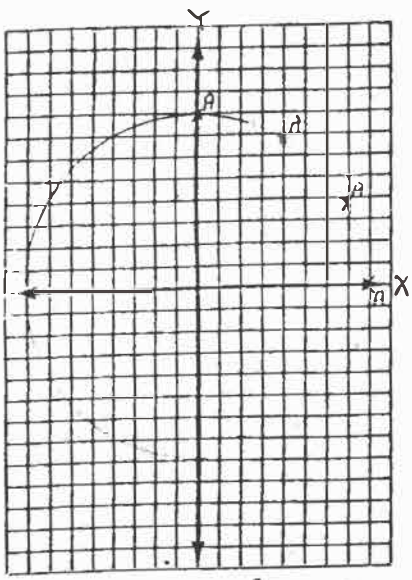
$CA^2 = 65$

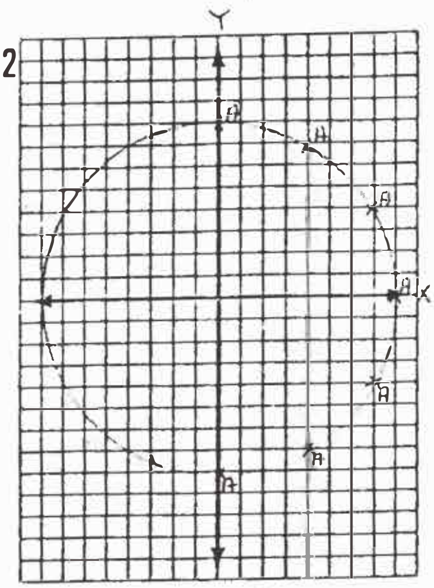
$|CA| = \sqrt{65}$

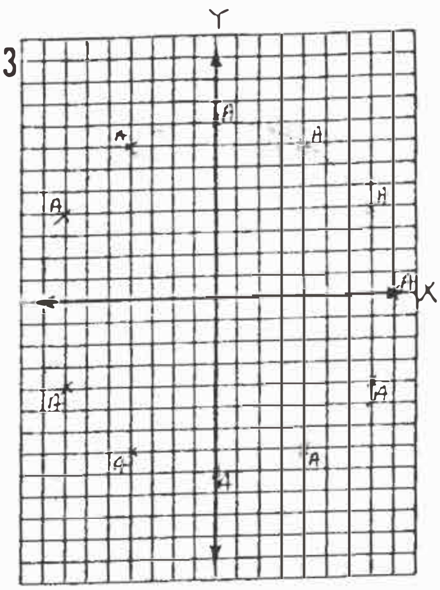
but the radius of the circle is $8 = \sqrt{64}$

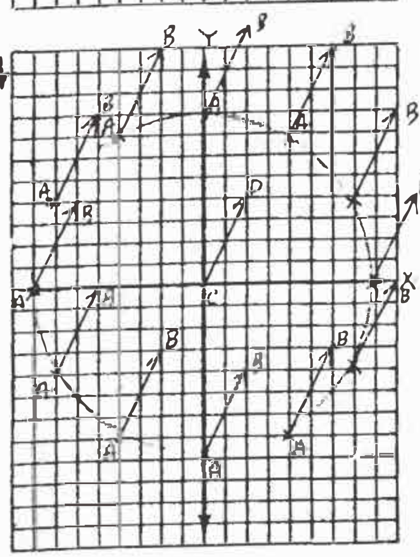
$\sqrt{65} > \sqrt{64}$

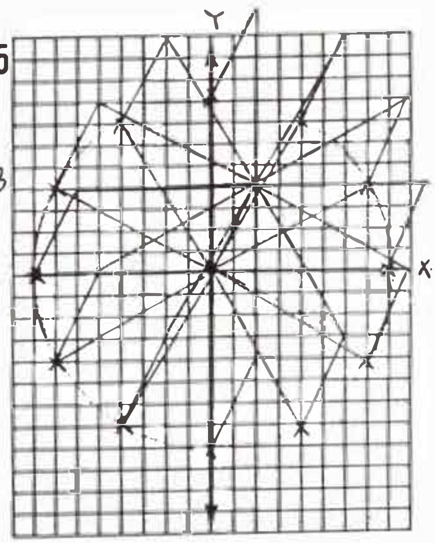
$\therefore$ A(4,7) is outside the circle.

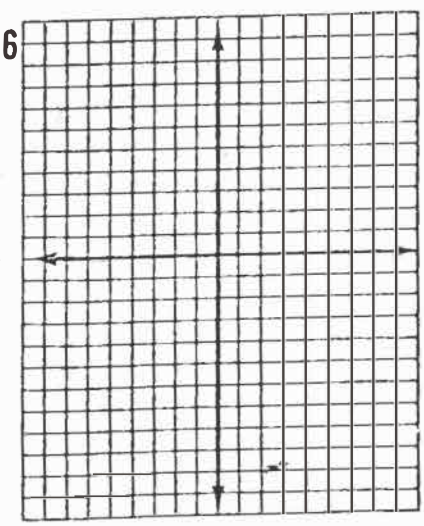
1


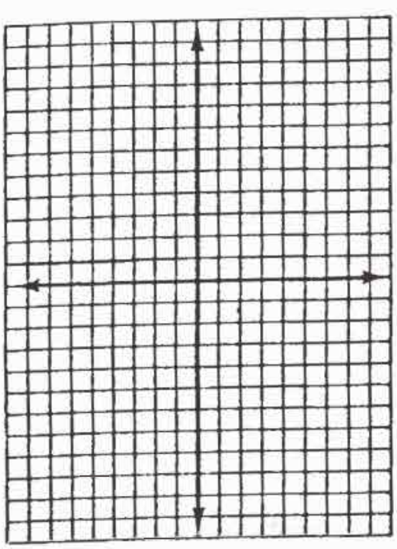
2


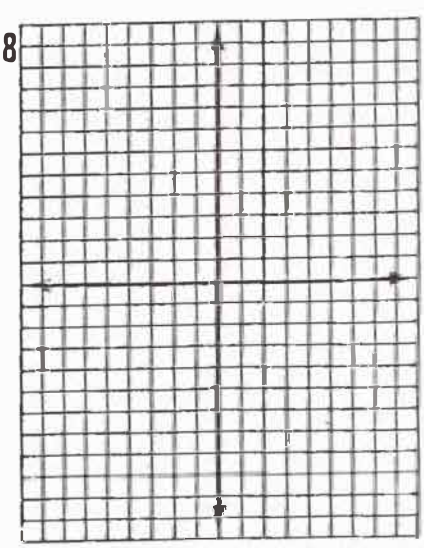
3


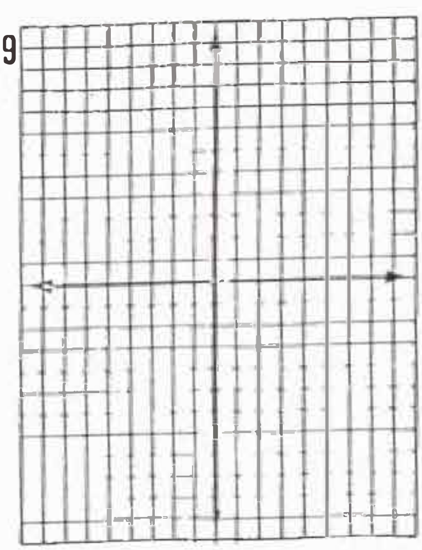
4


5


6


7


8


9


A Perspective of Mathematics Education In the People's Republic of China

Merwin J. Lyng
Mayville State College, Mayville, North Dakota

This article is based on a presentation given at the Calgary meeting of the NCTM, October 13, 1979.

The Present School System

In contemporary China, a student's formal general education lasts ten years. Some children between the ages of three and seven may attend kindergartens where instructional emphasis is on socialization and the acquisition of basic knowledge through informal means. Children begin primary school at age seven. The period of schooling lasts five years. Classes meet five and one-half days per week for ten months a year. The average school day lasts from 7:30 a.m. to 4:30 p.m., with a two-hour break for lunch within which the children may return home for their meal. A typical primary school program may include the following subjects:

- Politics - 2 classes/week
- Chinese Language - 11 classes/week
- Mathematics - 6 classes/week
- Physical Education - 2 classes/week
- Music - 1 class/week
- Art, including calligraphy - 1 class/week
- General Knowledge - 2 classes/week
- Agricultural Work - 1 week/term
- Industrial Work - 1 week/term

The Middle School

In graduating to the five-year middle school, a student encounters a program that is fairly heavy in the study of science:

- Chinese Language and Literature - 5 classes/week for 5 years
- Mathematics - 5-7 classes/week for 5 years
- Politics - 3 classes/week for 5 years
- Physics - 2 classes/week for 3 years
- Chemistry - 2 classes/week for 3 years
- Biology - 2 classes/week for 3 years
- History
- Geography
- Foreign Language (Russian, English, Japanese)
- Fine Arts
- Physical Training
- Productive Work

Middle school also meets six days a week. Class periods range from 45 to 50 minutes each, and a typical school day will consist of either six or seven periods of study. During the school year, approximately 30 per cent of student time is spent in productive work experiences either in the school itself or in the countryside or a factory.

The following is a brief outline of the current middle school mathematics curriculum.

First year -
Operations on rational numbers; operations on polynomials; equations in one and two variables.

Second year -
Algebra is continued during the first semester; second semester - plane geometry, rectangular co-ordinates, trigonometry, and some solid geometry.

Third year -
Geometry - regular polygons, circles, spheres, and conic sections; surveying; blueprint reading.

Fourth year -
Functions - linear, quadratic, logarithmic, exponential, and trigonometric.

Fifth year -
Sequences and limits; analytic geometry; polar co-ordinates; and parametric equations.

Mathematics Teachers

Mathematics teachers have two classes daily with each usually numbering from 45 to 55 students. The rest of the day is spent in preparation of lessons, evaluation of homework, preparation and grading of examinations, helping students, supervision, reporting to parents, and visits to the home. They may participate in self-study groups aided by more experienced teachers to extend their knowledge of mathematics or the teaching of mathematics. They may give after-school lectures for more talented students. Sometimes they take trips to factories and farms to learn about applications which would make school studies more relevant and meaningful. Experiences are shared with other teachers in the neighborhood. Teachers are evaluated on political attitude, ability to "use their brain," health, ability to assist students with problems, and ability to organize the best students for helping others.

Shanghai Mathematics Teaching Materials Compiling Group

The group was organized in 1976 to review and to develop the mathematics curriculum for the primary and middle schools. Teachers from primary school, middle school, and college levels were selected for the group, which is also concerned with methods of teaching and training of teachers. According to the group, students entering the middle school are expected to: (1) calculate with whole numbers, integers, fractions, and decimals, (2) recognize and "manage" simple geometric figures, (3) use simple algebraic expressions up to the third degree, and (4) solve simple applied problems. The group is working on an eight-volume series for the last four years of the middle school. Teachers are expected to volunteer suggestions for the content of the eight volumes. After completion, the volumes will be tried by members of the group, then revised according to the opinions of the teachers and students. The following topics will be covered in the volumes:

- linear equations
- inequalities
- systems of equations
- operations on polynomials and rational expressions
- powers, roots, and radicals
- distance formula
- quadratic equations
- logarithmic operations
- geometric forms; parallel lines
- congruence of triangles
- properties of parallelograms
- measurement
- circle properties
- solution of right triangles
- solid geometry
- trigonometric ratios
- linear functions
- logarithmic functions
- exponential functions
- trigonometric functions

- quadratic curves (second-degree relations in two variables)
- polar co-ordinates
- parametric equations
- complex numbers and variables
- plane vectors
- calculus

Children's Palaces

Outside of formal school programs, children and young people may strengthen their knowledge either directly or indirectly by participating in a variety of after-school extracurricular activities. These activities are conducted on school grounds and/or at local district centres called Children's Palaces. An excellent variety of self-study and enrichment materials for children is available in Chinese bookstores.

Peking University

- Founded in 1898.
- 20 departments - 10 science, 7 liberal arts, 3 foreign languages.
- 2,700 teachers.
- 7,000 students; once had 10,000 students; 130 foreign students from 37 countries; Mao attended (1918, 1920) and worked in the library.
- Mao set forth all educational lines: education should serve utilitarian quality and be productive morally, mentally, and physically (mentally, physically, and ideologically).
- Gang of Four wanted the uneducated worker rather than the educated elite. The university was, in effect, closed during the reign of the Gang of Four.
- Admission standards are now different than before the reign of the Gang of Four; some students are admitted directly from the middle (high) school, others come from the country or the factory.
- Degree will take four years in the future, with some specialties taking longer.
- In 1976, 38 per cent of the students were women.
- Students attend free, plus stipend, plus free medical care.
- Classes meet six days per week. Seven weeks are for festival and winter and summer vacations. Five weeks per year are spent in agriculture, military, or industry.
- 140 mathematics teachers with 150 students in computational (applied and abstract) mathematics and information theory. Calculus is more intuitive.
- Library is two years old. It has 24,000 square metres of floor space. It contains 3.1 million volumes.

Peking Teaching University

- Purpose is to train teachers for the middle school.
- Founded in 1902.
- 15 departments (no background given).
- 1,000 teachers and research workers.
- 3,000 students (three times that of preliberation days).
- 15,000 students in correspondence and supplementary training courses.
- Library collection over two million books.
- Since the liberation in 1949, many changes have taken place at the university. The spirit of the university is also changing (no details provided).

- Three courses required of all students:
  - History of Chinese Communist Party
  - Political Economy
  - Production (I believe this is correct; my tape was not clear.)
- During the past few years, they have been emphasizing the combination of theory and practice.
- Every year the students spend a certain period of time (not specified) in the factories, in communes, or in practice with the army units.
- University also runs several small factories where they combine teaching work, student work, and production work.
- Senior students have to practice (I assume practice teaching) in the middle school.
- In recent years the University has run spare time (correspondence) courses for in-service teachers.
- University was damaged by the Gang of Four disrupting the educational work.

Futan University (Shanghai)

- 2,200 faculty, 4,000 students, 14 departments, 1,100 teaching faculty. 1.3 million volume library, two 22-week terms with two-week review period before exams plus two-week period working in the country, factory, or military (18 weeks of classes).
- Students come from all of China - determined by Peking.

The University Mathematics Program in the People's Republic of China

First year -
- Algebra
- Analytic Geometry (plane and solid)
- Mathematical Analysis (calculus)
- General Physics
- Foreign Language
- Politics

Second year -
- Analysis
- Physics
- Differential Equations
- Complex Variable
- Higher Algebra (one-half year)

Third year -
- Real Variable
- Probability and Statistics
- Numerical Analysis
- Computer Programming

Fourth year - electives from:
- Modern Algebra
- Differential Geometry
- Topology
- Logic

Ideas

Prepared by Marilyn Hall Jacobson
Title I Mathematics Co-ordinator
Monroe County Community School Corporation, Bloomington, Indiana

and M. Bernadine Tabler
Assistant Instructor, Indiana University, Bloomington, Indiana

The *Ideas* for this month consist of two problem-solving posters with a Halloween theme. Some of the problems can be easily solved and have only one solution. Others will take students more time, especially if they find all the possible solutions.

IDEAS For Teachers — Levels 1–4

HAUNTED HOUSE

Objective

To use information from a chart to help solve problems.

Directions

Put the poster on the bulletin board. For younger children, review chart and go over the questions that are to be answered. Provide a place near the poster for the students to hand in their answers.

Answers

1. Fairview Elementary School
2. Four hours
3. One and one-half hours
4. Seventy cents
5.

Dimes	*Nickels*	*Pennies*
1	0	0
0	2	0
0	1	5
0	0	10

6. Ghosts 11 and 12
7.

Ghosts	*Children*
0	4
2	3
4	2
6	1
8	0

IDEAS For Teachers — Levels 4–8

TRICKS OR TREATS

Objective

To provide experience in problem solving.

Directions

Put the poster on the bulletin board. Encourage the students to attempt to solve the problems. Provide a place near the poster for the students to hand in their solutions. Assigning one or two problems a day would be a reasonable assignment.

Answers

1.

Pennies	*Nickels*	*Dimes*
20	0	0
15	1	0
10	2	0
10	0	1
5	3	0
5	1	1
0	4	0
0	2	1
0	0	2

2.

Nickels	*Dimes*	*Quarters*
0	14	0
3	10	1
6	6	2
9	2	3

3. 12 snakes and 18 wings
4. 14 snakes and 21 wings
5.

Packs of lizard ears	*Packs of bat ears*
9	0
6	2
3	4
0	6

6.

Packs of lizard ears	*Packs of bat ears*
10	0
5	4
0	8

7. Pumpkins #2 and #5.

Tricks

1. Wanda Witch will cast a spell for 20¢. Show the ways that Wanda can be paid exactly 20¢.

2. Dried black snakes cost $1.40. Wendell Warlock paid for one snake with quarters, dimes and nickels. He used 14 coins. How many of each coin did he use?

3. Wanda is going to use six frogs. How many snakes and wings will she need?

4. Walter
seven
many
wings

Recipe

Witch's Br

For every
use four d
snakes an

From the *Arithmetic Teacher*, October 1980

r Treats

g to use
How
and
need?

5. Wilma Witch bought a total of 18 ears. How many of each kind could she have bought?

6. Walter Warlock spent $4.00 for ears. What could he have bought?

7. Which two pumpkins are carved exactly alike?

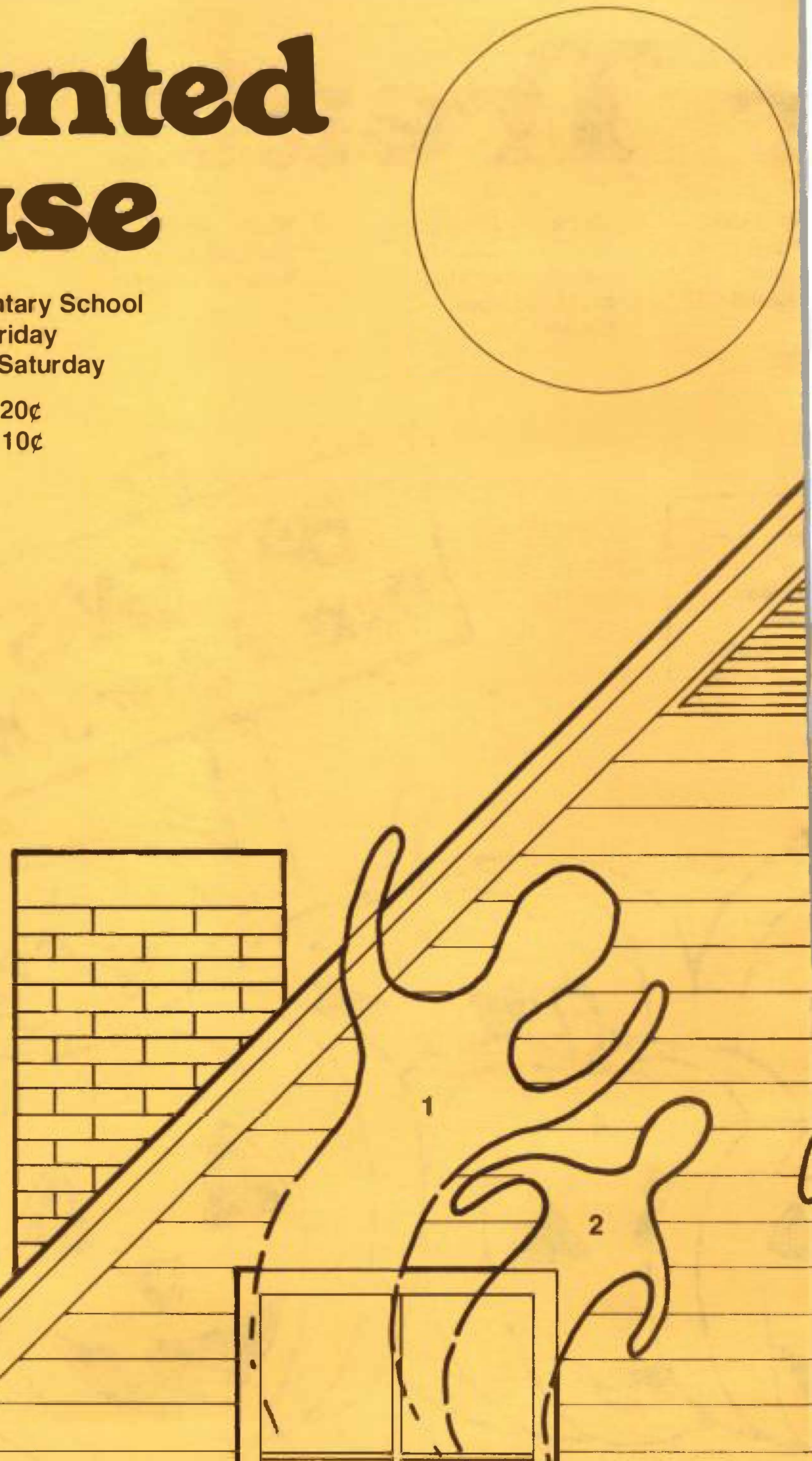
og legs
ack
at wings.

Haunted House

Place: Fairview Elementary School
Open: 4:00-6:00 p.m. Friday
11:00-3:00 p.m. Saturday

Admission: Children 20¢
Ghosts 10¢

1. Where is the Haunted House?
2. How many hours will it be open on Saturday?
3. If you go to the Haunted House at 4:30 p.m. on Friday, what is the longest you can stay?
4. How much would it cost three ghosts and two children to get in?
5. Show the ways that a ghost can pay exactly 10¢ to get in.
6. Which two ghosts are shaped exactly alike?
7. It cost 80¢ for some ghosts and children to get in. How many of each could there be?



From the *Arithmetic Teacher*, October 1980

14
15
10
11
13
12
6
8
9
7
4
3
5

? ? ? Problem Corner ? ? ?

edited by William J. Bruce and Roy Sinclair
University of Alberta, Edmonton

Problems suggested here are aimed at students of both the junior and senior high schools of Alberta. Solutions are solicited and a selection will be made for publication in the next issue of *delta-K*. Names of participants will be included. All solutions must be received (preferably in typewritten form) within 30 days of publication of the problem in *delta-K*.

Mail solutions to: Dr. Roy Sinclair or Dr. Bill Bruce
Department of Mathematics
University of Alberta
Edmonton, Alberta T6G 2G1

Problem 6:

(submitted by Roy Sinclair, University of Alberta)

A person has 31 dominoes, each 2 cm by 4 cm, and a checkerboard, each of whose squares are 2 cm by 2 cm. Is it possible to place all the dominoes on the board so as to leave only a pair of diagonally opposite squares uncovered?

Solution to Problem 2

Problem

(submitted by William J. Bruce, University of Alberta)

Clearly $1 = 1^2$ is a perfect square, but 11 and 111 are not. Consider all numbers $11111 \cdots 1 = S$, in which all digits are unity, and prove or disprove that, except for $s = 1$, no such number is a perfect square.*

*EXTENSION (Proof not to be submitted for publication.)

The theorem is true for any number that can be written in the form $100m + 10 + 1$ (m a positive integer). Also, true for $100m + k + 1$ when k is *not* divisible by 4.

Solution

(suggested by Dr. A. Meir, University of Alberta)

Since the given number is odd, it must be the square of an odd number, say $2K + 1$. Then we have

$$(2K+1)^2 = 4k^2 + 4k + 1 \ .$$

If we write $11111\cdots1 = 100m + 10 + 1$, we must have

$$4k^2 + 4k + 1 = 100m + 10 + 1$$

or

$$4k^2 + 4k = 100m + 10 \ ,$$

which is not possible because the left member is divisible by 4, whereas, the right member is not. Thus $11111\cdots1$ cannot be a perfect square except for 1.

Solution to Problem 3

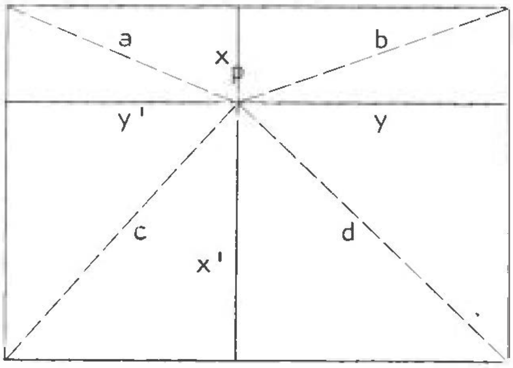
Problem

(submitted by William J. Bruce, University of Alberta)

Point P is located in a rectangular region such that its distances from three of the vertices of the rectangle are given by "a ft.," "b ft.," and "c ft." Let "d ft." be the unknown distance to the fourth vertex and find a relation among the four distances, "a," "b," "c," and "d," so that whenever any three are known, the fourth can be computed.

Solution

(suggested by William J. Bruce)



From the figure

$$a^2 = x^2 + y'^2$$

$$d^2 = x'^2 + y^2$$

$$b^2 = x^2 + y^2$$

$$c^2 = x'^2 + y'^2$$

$$a^2 + d^2 = (x^2 + y^2) + (x'^2 + y'^2)$$

$$= b^2 + c^2$$

activities

Spirolaterals

Richard Brannan and Scott McFadden
Lane Education Service District, Eugene, OR 97402

Teacher's Guide

Grade Level: 7 - 9.

Materials:

One set of the activity sheets and several sheets of grid paper for each student; a set of transparencies of the activity sheets and grids for the teacher.

Objectives:

Students will discover the following about order-3 spirolaterals on square grid paper.

1. Each order-3 spirolateral can be classified as one of three types depending on whether the sum of the two smaller numbers is equal to, less than, or greater than the largest number.
2. The number of small squares in the "hole" of type *B* and in the "overlap" of type *C* can be predicted without drawing the spirolateral by squaring the difference between the largest number and the sum of the two smaller numbers.

Procedure:

1. Distribute activity sheets 1 and 2.
2. Using the overhead projector, demonstrate how to draw an order-3 spirolateral as students follow along and draw the spirolateral.
3. Have students finish activity sheets 1 and 2.
4. Distribute activity sheet 3. Have students fill in the table with their own and classmates' numbers. Have students discover the relationships.

Supplementary Activities:

1. For a given order, find the number of loops through the numbers needed to bring a spirolateral back to its starting point. See the article by Schwandt (1979) for suggestions.
2. Similar investigations are possible on isometric grid paper using order-2 spirolaterals.

REFERENCES

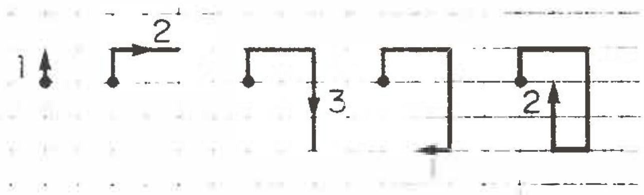
Gardner, Martin. "Mathematical Games." *Scientific American*, February 1974.

Olds, Frank C. "Spirolaterals." *Mathematics Teacher* 66 (February 1973): 121-24.

Schwandt, Alice. "Spirolaterals: Advanced Investigations from an Elementary Standpoint." *Mathematics Teacher* 72 (March 1979):166-69.

(Answers on page 32)

To draw a 1,2,3 spirolateral: Trace *one* unit *up*, *two* units *right*, *three* units *down*, *one* unit *left*, *two* units *up*, and so on. Continuing in a clockwise direction, you should finish at the starting point. The first five steps are shown below. The completed spirolateral is shown to the right. This is called an *order-3* spirolateral because *three* numbers are used as the lengths of the segments.



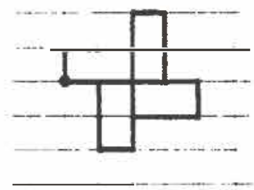
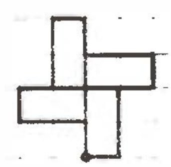


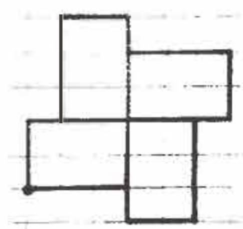
Figure ***A***
1,2,3 spirolateral
(Type ***A***)

The completed 1,2,3 spirolateral shows four rectangles fitting snugly around a middle point. Call this *Type A*.

1. A 3,2,1 spirolateral and a 2,5,3 spirolateral are shown below. Trace each one. Is each one a Type *A*?__________

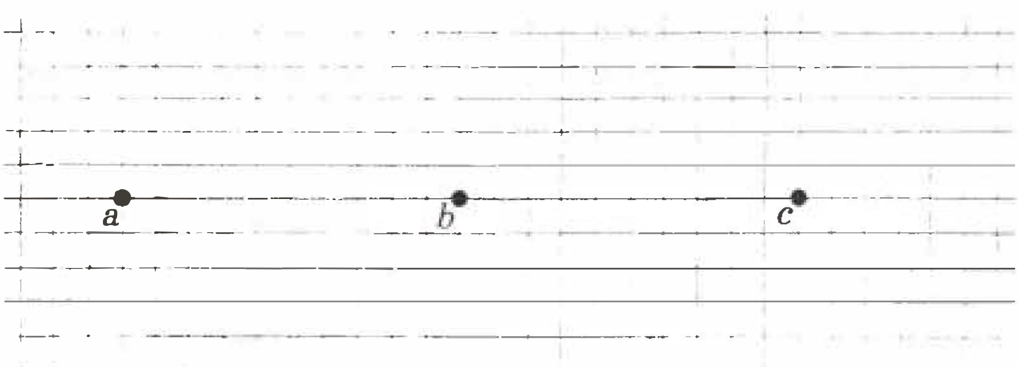


3,2,1 spirolateral



2,5,3 spirolateral

2. Draw these order-3 spirolaterals: a) 1,3,4; b) 2,1,3; c) 5,1,4. The starting point for each is shown. Is each one a Type *A*?__________



An order-3 spirolateral showing four rectangles with a "hole" in the middle is called a *Type B* spirolateral (see Figure *B* for an example). An order-3 spirolateral showing four rectangles that "overlap" in the middle is called a *Type C* spirolateral (see Figure *C* for an example).

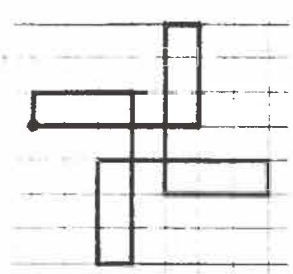


Figure *B*
1,3,5 spirolateral
(Type *B*)

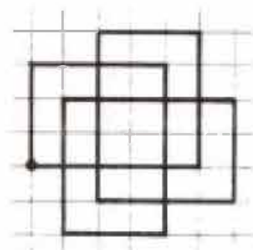
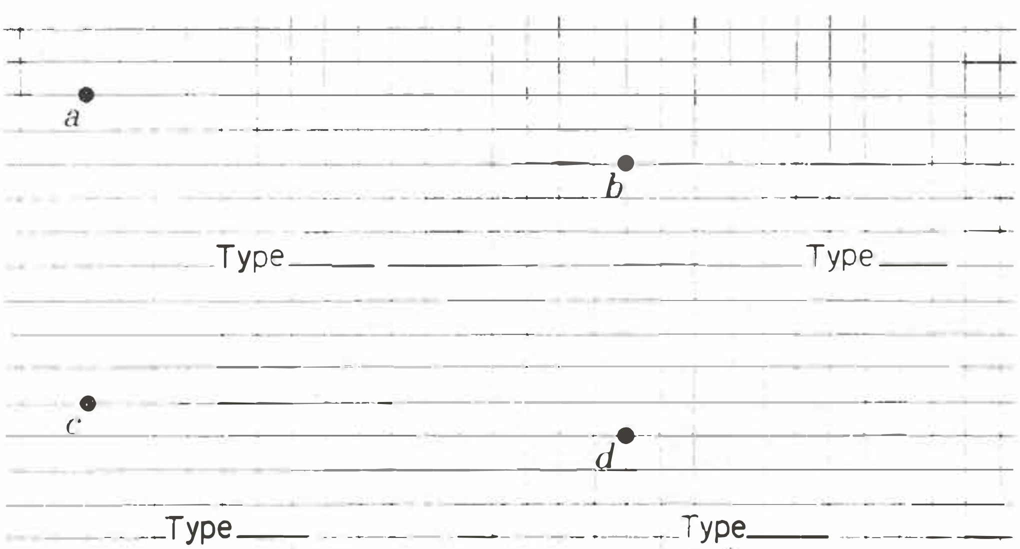


Figure *C*
3,4,5 spirolateral
(Type *C*)

3. On the grid below, draw these order-3 spirolaterals. Starting points are shown. Label each spirolateral as Type *B* or Type *C*.

   a) 1,2,6 b) 2,4,5 c) 1,2,4 d) 4,2,4



4. On a sheet of grid paper, create six order-3 spirolaterals of your own. Label each as Type *A*, *B*, or *C*.

5. You have drawn many order-3 spirolaterals. Each was a Type *A*, *B*, or *C*. Write the three numbers of each spirolateral in the appropriate column.

   Three are done for you. Add others that your classmates have done.

	Type	
A	*B*	*C*
1,2,3	1,3,5	3,4,5
______	______	______
______	______	______
______	______	______
______	______	______
______	______	______
______	______	______
______	______	______

6. Look at the numbers in column A. Without drawing, predict how you can tell if an order-3 spirolateral is Type *A*. ______________________

   ____

7. Look at the numbers in column B. Without drawing, predict how you can tell if an order-3 spirolateral is Type *B*. ______________________

   ____

8. Look at the numbers in column C. Without drawing, predict how you can tell if an order-3 spirolateral is Type *C*. ______________________

   ____

9. Without drawing, what type is each of these order-3 spirolaterals?

   a) 10,15,5 ____ b) 21,22,23 ____ c) 100,32,25 ____

   d) 4,21,17 ____ e) 1,1,1 ____ f) 25,16,9 ____

10. Again investigate the numbers in columns B and C. Also look at the spirolaterals drawn with the numbers. Find a way to predict the number of small squares in the "hole" or in the "overlap."

    ____

    ____

Ideas

Prepared by George W. Bright
Northern Illinois University, DeKalb, Illinois

IDEAS For Teachers
Levels: 1, 2

Objective

To practice basic addition and subtraction facts.

Directions

1. Give each student a copy of the worksheet.
2. Read the directions to the students and let them go to work. If you think students will have trouble writing their answers in the proper boxes, you may want to complete the top row of each square with the class as a whole.
3. Tell them that if all numbers in the boxes in the last square are not the same, they should check their work.
4. Although this worksheet is published in the October issue of the *Arithmetic Teacher*, you may want to wait until later in the year to use it.

Going further:

1. Help students recognize that every number in the last square is a 2 because the difference of the numbers that are added is 3 - 1 = 2.
2. If students can add three numbers, ask them to add the numbers in each row and column around the sides of each square. For each square, all four sums are the same.
3. For a related activity, see "Mystic Squares for Primary Grades" by Leland Moon, Jr., and Sharon Pearl Moon in the December 1976 issue of the *Arithmetic Teacher*.

IDEAS For Teachers
Levels: 2, 3, 4, 5

Objective

To practice addition and subtraction.

Directions

1. Give each student a copy of the worksheet.
2. Be sure the students understand the directions printed in the ovals before they begin work. It is important that students understand that answers are to be written in the same relative positions in each square.
3. Tell students, if they do not see it by themselves, that if all numbers in the last square are not the same, they should check their work.

Going further:

1. Each square is a magic square. The sums of the numbers in each

row (→), each column (↓), and each diagonal (↙ or ↘) of each square are the same. Ask students to find each of these magic sums. Answers: 15,57 = 15 + 3(14),42 = 15 + 3(9),15 = 57 - 42.

2. Ask the students to add corresponding numbers in the two middle squares. The result is a new magic square with a magic sum of 99 = 57 + 42.

IDEAS For Teachers

Levels: 4, 5, 6

Objective

To practice basic multiplication facts and simple addition.

Directions

1. Give each student a copy of the worksheet.
2. Be sure the students understand the directions before they begin work; it is important that they understand that answers are to be written in the same relative positions in each square.
3. Tell students to add the numbers in each row, column, and diagonal in each square. Each square should have its magic sum.

Going further:

1. Ask students to explain the relationship between the numbers in the first and last squares. (In the last square each number is 10 times the corresponding entry in the first square; for example, 20 = 2(10) = 2(7 + 3) = 2(7) + 2(3) = 14 + 6. This result is based on the distributive property of multiplication over addition.)
2. The magic sums of the four magic squares are 15, 105 = 15(7),45 = 15(3), and 150 = 15(10) = 15(7 + 3).

3. For more information on magic squares see "Guided Discovery with Magic Squares," by Thomas P. Atkinson in the April 1975 issue of the *Arithmetic Teacher*. The reference list is very good.

IDEAS For Teachers

Levels: 6, 7, 8

Objective

To practice multiplication and addition of decimals.

Directions

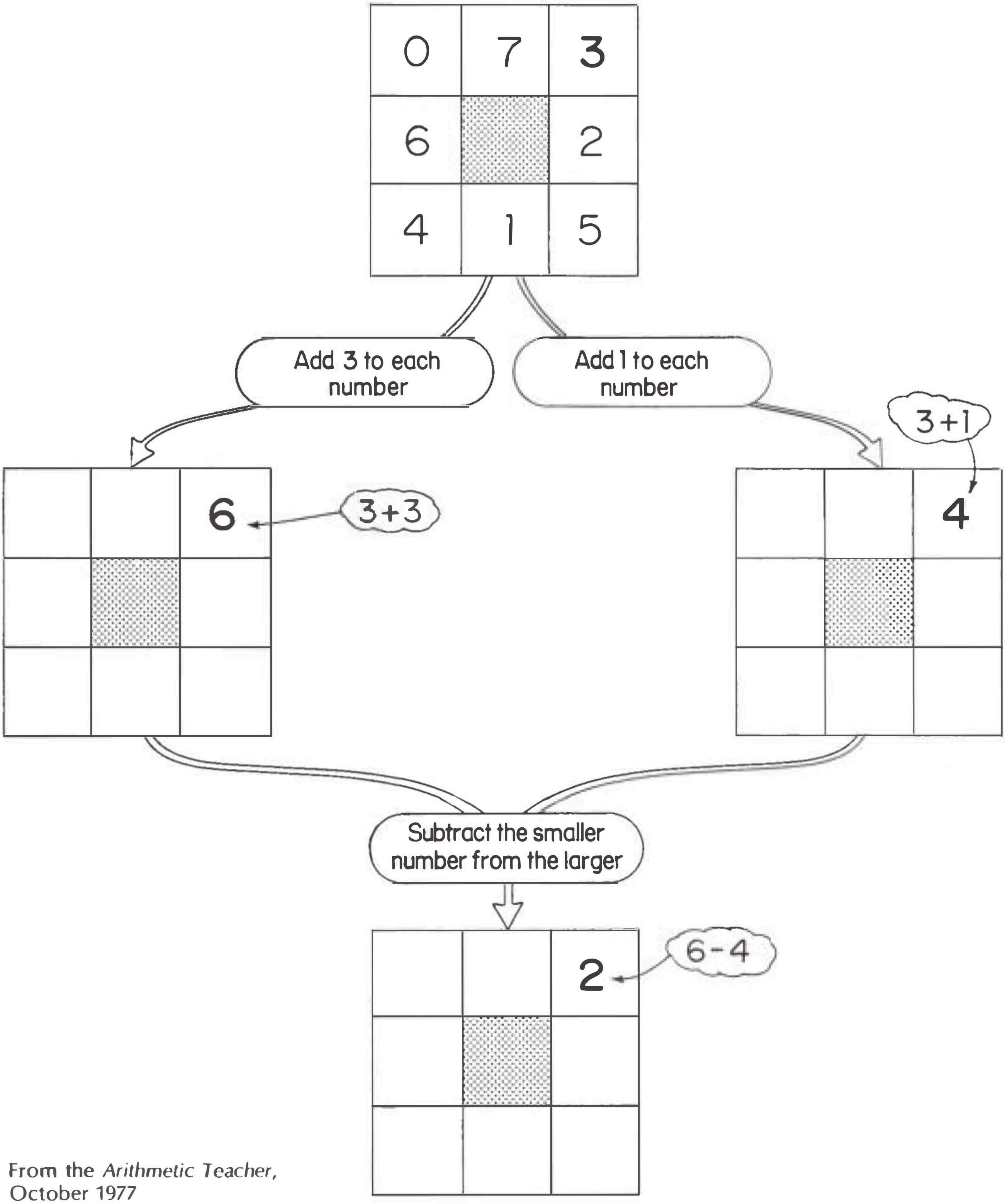
1. Give each student a copy of the worksheet.
2. Let the students read the directions and begin work. It is important that students understand that answers are to be written in the same relative positions in each square.
3. Tell students to find the magic sum for each square.

Going further:

1. Help students see that each entry in the last square is .8 more than 10 times the corresponding entry in the first square; for example, 4.8 = .4(10) + .8. That is, in the last magic square, the whole number part of each entry is the same digit that appeared as the decimal part of the corresponding entry in the first magic square. The decimal part of each entry is .8, which is the amount added in the right-hand path. For example, 4.8 = 3.6 + 1.2 = .4(9) + (.4 + .8)= .4(9) + .4(1) + .8 = .4(10) + .8 = 4.0 + .8. This result is based on the associative property of addition and distributive property of multiplication over addition.
2. The magic sums of the four magic squares are 1.5,13.5 = 1.5(9),3.9 = 1.5 + 3(.8), and 17.4 = 13.5 + 3.9 or 17.4 = 1.5(10) + 3(.8).

IDEAS

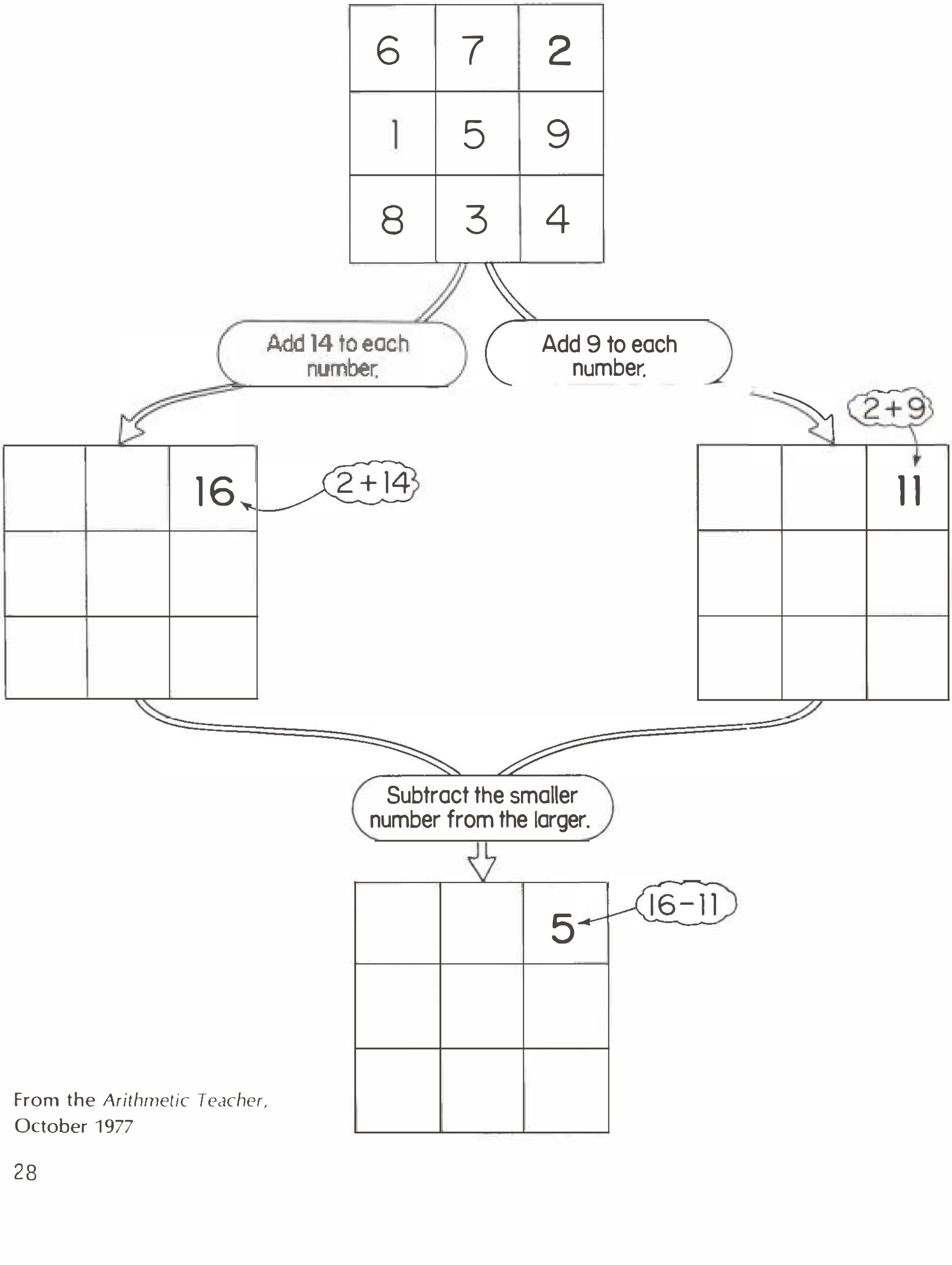
Name ____________________



From the *Arithmetic Teacher*, October 1977

IDEAS

Name ____________________



From the *Arithmetic Teacher,*
October 1977

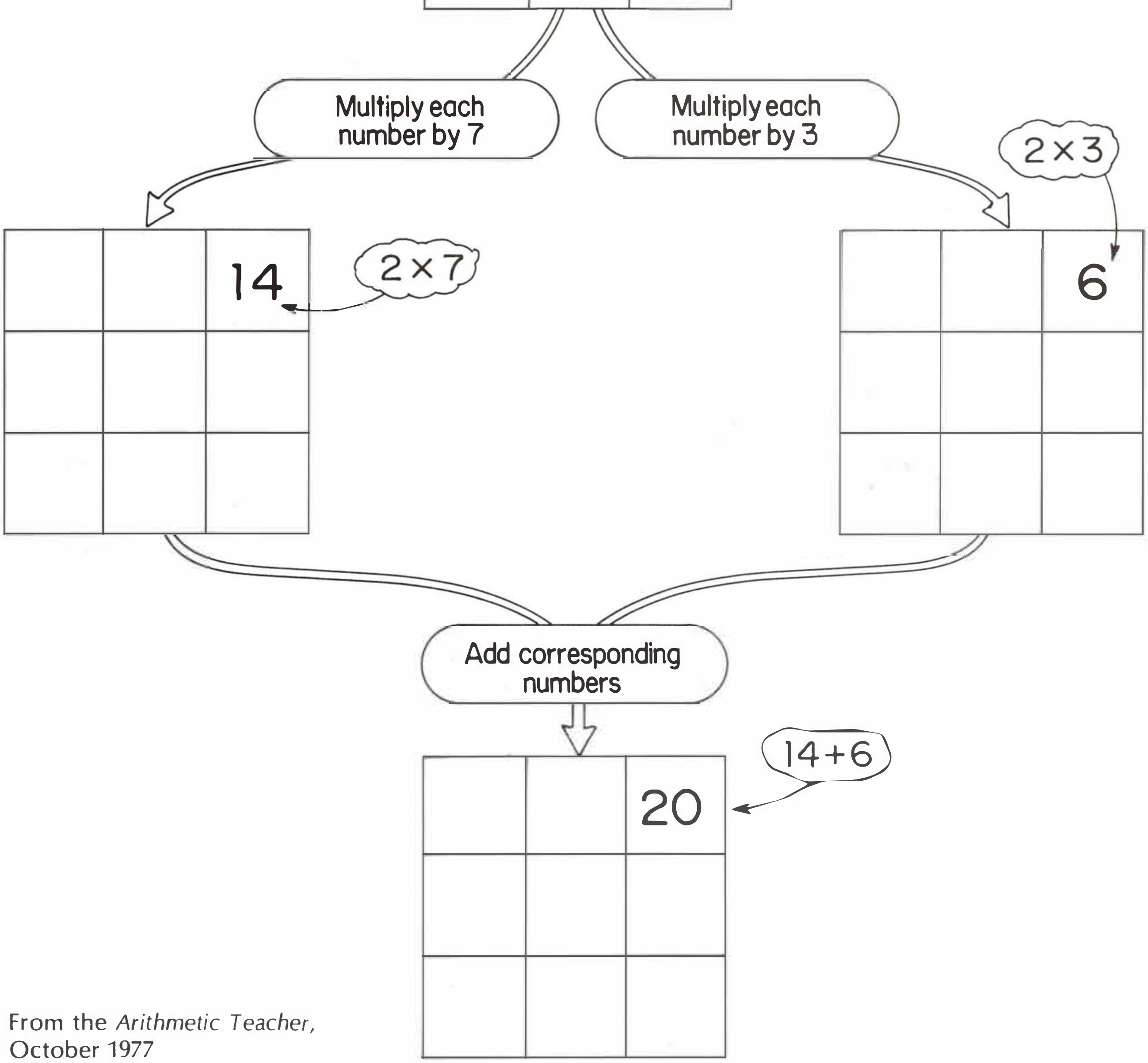
IDEAS

Name ______________________________

This is a magic square.

6	7	2
1	5	9
8	3	4

The magic sum is ____ .



From the *Arithmetic Teacher*, October 1977

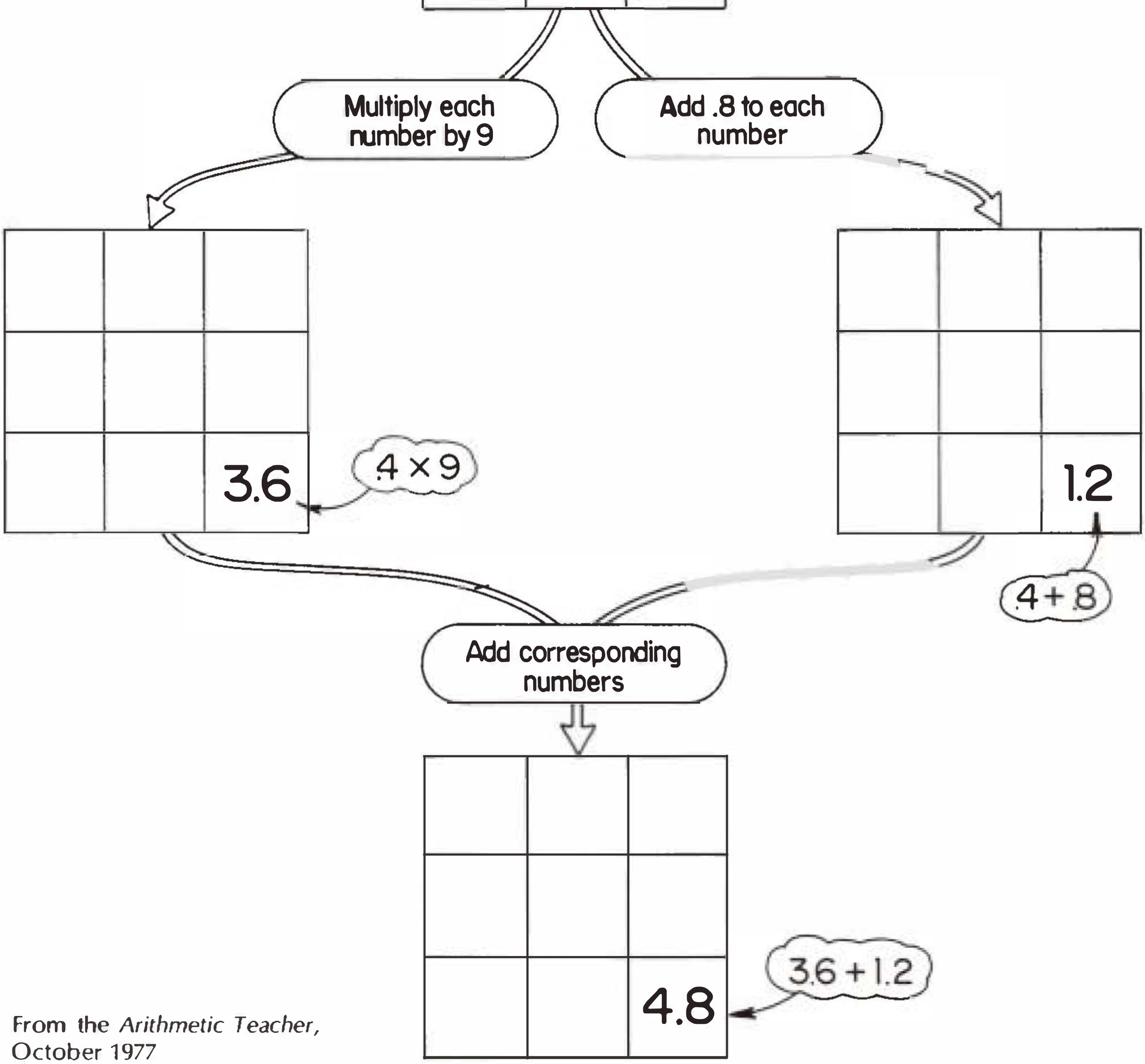
IDEAS

Name ______________________________

This is a magic square.

.6	.7	.2
.1	.5	.9
.8	.3	.4

The magic sum is ___.



From the *Arithmetic Teacher*, October 1977

Similar Triangles With a New Slant

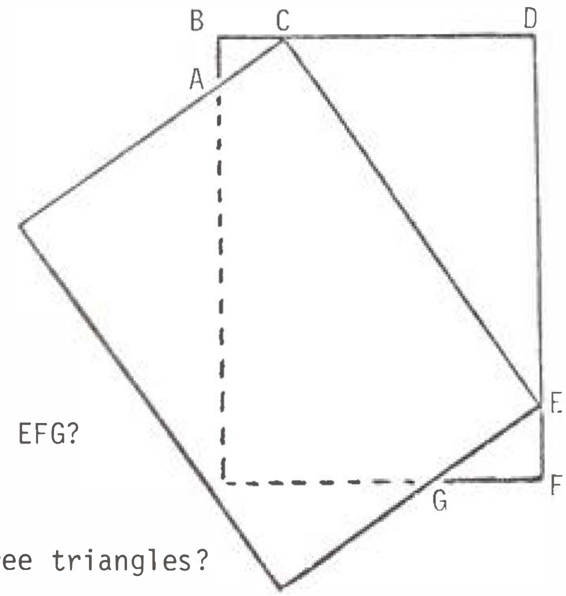
J.W. Roberts

Port-aux-Basques Integrated School Board

Reprinted from Teaching Mathematics, *Vol. 8, No. 2, March 1981.*

The following investigation is proposed based on the premise that mathematics teachers are keen to collect ideas and activities that serve to enrich the topics they teach

Consider two sheets of paper approximately 21 cm by 27 cm. Overlap the two sheets of paper so that the two right-hand corners of the top sheet touch the top and right-hand sides of the bottom sheet as shown in the figure at the right.



The resulting triangles ABC, CDE, and EFG may be the focus of some discussion. The following is a list of some of the features of the diagram that you may wish to investigate further:

1. What kind of triangles are ABC, CDE, and EFG?
2. Prove: Δ ABC ~ Δ CDE ~ Δ EFG.
3. Is there a 30-60-90 position for all three triangles?
4. Is there a 45-45-90 position for all three triangles?
5. What happens when B and C coincide?
6. What happens when E and F coincide?
7. At what position is $\overline{AG} \parallel \overline{CE}$?
8. Is there a position for congruent triangles?
9. Are there any new features to the above questions if the two sheets were square?
10. Investigate the possibility of there being a constant ratio between the perimeters or the areas of the triangles.

The above are just a sample of possible questions that could be asked about this situation. Perhaps you can come up with some others.

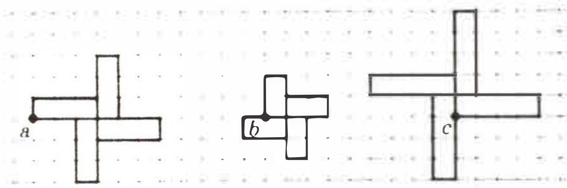
activities

(continued from page 21)

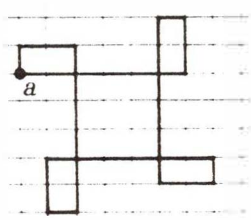
Answers:

1. Yes

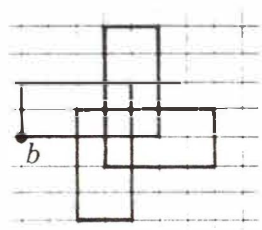
2. Yes



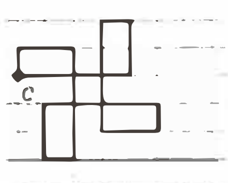
3.



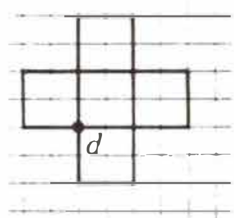
Type *B*



Type *C*



Type *B*



Type *C*

6. The sum of the two smaller numbers is equal to the largest.

7. The sum of the two smaller numbers is less than the largest.

8. The sum of the two smaller numbers is greater than the largest.

9. a) A b) C c) B
   d) A e) C f) A

10. Square the difference between the largest number and the sum of the two smaller numbers.

PRINTED AT
BARNETT HOUSE