Shaping Up Geometry, K-12

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Seventeen-year-old students have difficulty with those geometry problems which demand even routine application of properties of shapes. Moreover, when asked about the importance of different areas of study, students at all levels put geometry topics at or near the bottom of the list. These data from the 1977 National Assessment of Educational Progress come at the end of a decade of great interest and activity in geometry. The interest and activity are reflected in NCTM publications such as the *1973 Yearbook* and numerous articles in the *Arithmetic Teacher* and the *Mathematics Teacher*; in elementary school textbooks where the quantity and variety of geometry material has increased; in the several interesting geometry textbooks for elementary school teachers which have been published, including a particularly nice one by O'Daffer and Clemens; and in articles and experimental texts for secondary geometry including one on transformational geometry by Coxford and Usiskin.

Why are there still major problems with geometry?

One difficulty seems to be that geometry instruction lacks purpose and direction. The ten years of interest and activity produced creative geometry activities and attractive geometry material, but it failed to produce a clear picture of the goals of the geometry curriculum. It failed to give teachers and students a sense that there is an important body of geometric facts and skills. Too often geometry has been subordinated to arithmetic and algebra which are generally accepted to be important. Even when geometry is taught, facts and skills tend not to be the focus. In elementary school and middle school, geometry is often used as a change of pace and there is little follow-up to confirm achievement. In secondary school the formal aspects of the course tend to dominate. The focus tends to be on the meaning and utility of theorems.

This article explores the nature and importance of geometry, and it suggests a purpose and direction for geometry instruction. Some attention is given to the implementation of the suggestions, although most of the needed ideas and materials already exist and are readily available.

WHAT GOOD IS GEOMETRY? Geometry can be defined as the study of space experiences. Since most of our physical activities are space experiences, the potential importance of geometry is clear. The study of space experiences can include the following:

- identifying, naming, describing, and drawing shapes
- analyzing the particular properties of important shapes that make them important
- analyzing the changes in shapes which result in congruent shapes and in similar shapes

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- applying the results from the above to real problems.

To analyze what knowledge and skills our students should be acquiring in the study of space experiences, it is useful to make a distinction between the geometry demands of everyday life and the demands encountered in college and in the practice of certain disciplines. Every student should

- *learn the names of common shapes*. (NAEP results suggest that this goal is being met fairly well at present.)
- *learn to describe, visualize, and draw shapes*. (There is no need to quote statistics to convince most people that adults and children alike lack skills with three-dimensional drawing and visualization. Yet these skills are so often in demand in planning home or office space, in giving or understanding directions, and in just moving from place to place.)
- *learn what makes each shape useful*. (Triangles are rigid and consequently are a critical source of structural stability; all polygons can be triangulated, so that knowledge about triangles can be used to generate knowledge about any polygon; triangles, rectangles, and certain other shapes tessellate and so are useful for covering plane surfaces; all points on a circle are the same distance from the centre, hence the invention of the wheel; circles and spheres have optimal contents/boundary/ratios, a fact which has important implications for packaging; straight lines on the plane share with great circles on spheres the property of being the shortest distance between points.)
- *learn the fact that the ratios of the measures of corresponding elements of similar figures are equivalent*. (This fact is the key to scale drawing which is fundamental to any activity which involves planning and building from plans.)
- *learn to use all of the above to solve actual problems*. (We all know that it is entirely different to know a fact than it is to have the experience and confidence required to recognize its uses and to actually use the fact. Moreover, it is entirely different for a teacher to tell students that geometry is useful than it is for students to actually experience its usefulness.)

In addition to the above knowledge and skills, students who are going to college or who are going into certain technical fields need to learn analytic geometry. Analytic geometry is the bridge from geometry to the rest of mathematics. Expressing geometric relationships numerically makes it possible for the tools of algebra and calculus to be applied to geometric problems. Of great recent importance is the fact that analytic geometry makes it possible to analyze geometric problems on a computer. Today, many capable students come to college with very weak backgrounds in analytic geometry, especially three-dimensional analytic geometry.

Enough of complaining. What can be done to improve things?

What To Do?

Fortunately, things can be improved without making radical changes. In the elementary school and middle school, the students should gain useful knowledge and develop intuition and experience with all of the important plane and solid shapes. For each shape the student should learn its name, how to draw it, where it occurs in the world, why it occurs there (i.e., what special properties it has that cause it to occur), and the student should gain some experience using the shape.

Name of Shape	Drawing	Occur-rences	Proper-ties	Applica-tions

This matrix shows the kinds of learning that should take place for each of the important shapes. Here are some illustrations of entries for some of the cells of the matrix:

Occurrences, properties, and applications of straight lines: Have students look at footpaths that have been worn in the grass where sidewalks do not provide a minimal path between points, and have them discuss why the paths are where they are and are the shape they are.

Occurrences, properties, and applications of triangles: Have students try to stabilize an unstable shape (either one made of cardboard and brads or a real one). Have them find real world examples of the use of triangles to stabilize shapes.

Occurrences, properties, and applications of circles: Have students gather around to see something on the floor. Then ask them why they formed a circle. Ask them what special properties of circles make theater-in-the-round a good idea. Ask them to find other real world examples where the facts that all points on a circle are equidistant from the centre and that the circle is the plane shape with maximal area/perimeter ratio are used.

Drawing, occurrences, properties, and applications of spheres, circular cylinders, and rectangular prisms: Have students draw and make a cylinder, a rectangular prism and a sphere. Ask students why milk cartons are rectangular, why some water towers are spherical, and why others are cylindrical. Ask students to notice what products are packaged in what shape containers and why.

For such activities to be effective, it is important for the teacher to have a clear idea of what specific facts and skills the students should learn. It is also important that the teacher verify that the students have, indeed, learned the skills.

In addition to the study of shapes, students in the elementary school should begin preparation for the study of analytic geometry. Map reading and map making can provide valuable experience with associating locations with pairs of numbers (e.g., 3rd Street and 4th Avenue), pairs of words (e.g., at the corner of Elm and High), and pairs with a word and a number (e.g., 602 East Smith). Activities with room numbering in multifloor buildings can also help students see the relationship between location and numbers.

High school geometry presents a very special set of considerations. Most high school geometry students find that a large fraction of their time is spent attempting to prove theorems. Little of their time is spent determining what the theorem says about shapes in the world. For example, how many students recognize that the rigidity of triangles is an immediate consequence of the theorem which says that two triangles must be congruent if their sides are congruent? Further, how many students have stopped to ponder the fact that there is no such theorem for polygons with more than three sides and that this has implications for the rigidity of non-triangular shapes? The problem is that the high school geometry course is attempting to fulfill two different sets of objectives. On the one hand, it is attempting to teach students about shapes. On the other hand, it is introducing students to deductive reasoning. I propose that these two objectives of the course be dealt with one at a time rather than together.

PROPOSED HIGH SCHOOL GEOMETRY COURSE

History of geometry and introduction to deductive logic.	Properties of two-dimensional and three-dimensional shapes and their application.

In the proposed course, half of the first semester would be a history of geometry, with emphasis on the importance of Euclid's *elements* both as an intellectual landmark and as a compendium of knowledge about shapes. Formal deduction would be introduced in the context of a subset of Euclidean geometry with "stronger" axioms that are chosen to produce interesting theorems fairly easily. (This "local" axiomatic approach is introduced in a chapter by Seymour Schuster in the 1973 NCTM Yearbook entitled "Geometry in the Mathematics Curriculum.") This first quarter of the course would end with a brief study of Descartes and of the role of analytic geometry in mathematics. The remaining three quarters of the course would study the properties of plane and solid shapes. Each property would be introduced and verified in the most edifying and meaningful way. Some properties would be proved synthetically, others analytically. Some properties would be discovered experimentally. Others would merely be established by plausibility argument. The emphasis would be on meaning, on intuition, on drawing, on applications, and on analytic geometry where possible.

Such a course as this would introduce the important idea of deductive thought without encumbering the entire course with proof. More time could be spent analyzing shapes and applying properties and less time memorizing theorems. The course would have greater utility for the college-bound student as well as the non-college-bound student. Its perceived relevance might even attract more students.

What are the Chances?

There are reasons to believe that the time may be right for the changes suggested here. At all levels in the curriculum there is a renewed interest in problem solving and applications. This interest is compatible with an increase in emphasis on the occurrences and applications of properties of shapes and has resulted in resources for applications (MAA-NCTM Source-book) and has also influenced textbooks. Moreover, it has created a climate in which such an increase might be accepted. It is also possible that the increasing presence of calculators and computers in the schools will cause teachers to look for new clusters of meaningful mathematics objectives. Geometry might well fill the need. Even before curriculum changes are adopted, much can be gained at any level by additional focus and follow-through on standard objectives related to the properties of shapes, and by making some effort to relate those properties to their applications in the real world.