## How to Draw an Octagonal Figure Quickly

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The following method, commonly used by a carpenter, is well within the understanding of students of elementary plane geometry.

Method: 1. Draw both diagonals of a square to intersect in the centre $C$ as shown in the accompanying diagram.
2. With each corner of the square as centre and with radii equal to the length of a semi-diagonal, draw arcs to intersect all sides of the square.
3. The points of intersection in (2) are the corners of the required octagon. (See proof below.)


Proof: From the figure, $B$ is a point of bisection of a side of the square. By construction, $\triangle$ ACG is isosceles with $\mathrm{AG} \equiv \mathrm{CG} . \angle \mathrm{AGC}=45^{\circ}$, so $\angle \mathrm{ACG}=\angle \mathrm{CAG}=67 \frac{1}{2}^{\circ} . \angle \mathrm{BCG}=\angle \mathrm{HCB}=45^{\circ}$, so $\angle \mathrm{ACB}=\angle \mathrm{ACD}=22 \frac{1}{2}^{\circ}$. Thus $\angle C A D=67 \frac{1}{2}^{\circ}$ also. Therefore, by a.s.a., $\triangle$ 's $A B C$ and $A D C$ are congruent. So $\mathrm{AB} \equiv \angle \mathrm{AD}$. But $\mathrm{AF}=2 \mathrm{AB}$ and $\mathrm{AE}=2 \mathrm{AD}$, by construction. Hence $A F \equiv A E$, as required for an octagon.
Note: If $s$ units is the length of a side of the square and $h$ units is the length of a side of the octagon, it is easy to obtain $h=s \tan 22 \frac{1_{2}^{\circ}}{}{ }^{\circ}$ and, from the appropriate half-angle trigonometric identity, that $h=s(\sqrt{2}-1)$.

