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# delta-k

THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

Volume XXI, Number 2

December 1981




## Special Issue on Geometry

delta-k

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*Volume XXI, Number 2* *December 1981*

# Contents



EDPRESS

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# From the Editor's Desk

## *A Joyous Christmas and a Happy New Year to Each of You*

Christmas is celebrated in different ways, not only in different cultures, but within our own society. Whatever your view of Christmas, I trust that Christmas 1981 will be a happy time for you.

For many of us, the New Year holiday is a stimulus to review the past and contemplate the future. In the past year MCATA has published four issues of *delta-k*. Our last monograph was on reading in mathematics and was edited by John Percevault. During 1980-81, MCATA supported the South West Regional, the High School Prize Examination, the Junior Mathematics Examination, a Calgary Junior High Prize, and a Calgary Mathematics Night.

The Donald H. Hinde Memorial Award was established for outstanding mathematics students in the Lacombe Junior High School where Don taught.

The 1981 (21st) Annual Meeting was held in Lethbridge on October 16 and 17 with over 300 in attendance.

Looking ahead to 1982, there are four more issues of *delta-k* in the planning stage. My goal for the new year is to make *delta-k* more of a journal of the members. I very much want you to contribute, be it a letter to the editor, a reaction to an issue or curriculum decision, a description of a teaching strategy, or a longer article about mathematics or mathematics education.

Two monographs are in the works and should be out in 1982. One on problem solving is being edited by Sid Rachlin (Calgary), and the other is on computers in education. Ron Cammaert (Taber) is editing the latter.

The 1982 Annual Meeting is being planned to be held in Edmonton next fall. Rod Anderson is organizing this meeting.

Suggestions please!! What kind of services would you like to see MCATA provide in 1982? We are elected or appointed to serve you. How can we do it? Drop a note to President Gary Hill or any member of the executive listed on page 28 of this journal.

Yours for a successful 1982,

*George Cathcart*

## In this Issue

The first mathematical experiences a new-born infant has are geometric. Every day of our lives, from birth to death, we touch, see, and interact with geometric figures in a variety of ways.

This issue of *delta-k* focuses on geometry. Al Olson talks about the "no-things" we need to teach in geometry. Donald Kerr suggests some essential components of a geometry curriculum, and Alan Hoffer summarizes the thought levels in geometry proposed by the van Hieles from the Netherlands. Bill Bruce outlines a procedure for constructing an octagon.

The Problem Corner contains a problem related to geometry. The seven pages of activities contain geometric activities adapted by your editor from a variety of sources.

## Long-Time MCATA Executive Members Retire

John Percevault from the University of Lethbridge and Robert Holt from the Edmonton Public School Board have retired from the MCATA executive after many years of dedicated service.

During the past nine years John served as Faculty of Education representative, edited a recent monograph on reading in mathematics, and assisted in the organization of the 21st Annual Meeting in Lethbridge.

Bob has served in a number of important capacities including president, vice-president, and past president. Bob was also chairman of the Program Committee for the 1973 NCTM meeting in Edmonton.

The input both John and Bob gave to decisions made by the executive was always appreciated. We wish John and Bob well as they continue their professional careers and thank them for their valuable contributions to MCATA.

## First Winners of the Donald H. Hinde Memorial Award

The first students in the Lacombe Junior High School to receive the Donald H. Hinde Memorial Award were announced recently. The winners were:

Grade 7 - Todd Peterson
Grade 8 - Jim Dixon
Grade 8 - Shane Milne
- Leslie Newman

CONGRATULATIONS!

# MCATA 21st Annual Meeting

**Lethbridge, October 16-17**

The 21st Annual Meeting of the MCATA was held in Lethbridge on October 16 and 17. Over 300 people participated in a variety of workshops and sessions that dealt with a variety of topics and issues. Sessions on problem solving were conspicuous by their abundance. Other sessions dealt with diagnosis, calculators, curriculum revisions, and other topics. Earl Ockenga from Cedar Falls, Iowa was the keynote speaker Friday evening and at the luncheon on Saturday. His emphasis was on making problem solving meaningful and interesting.

### What Do You Think?

This is the first time since the Jasper Meeting that the MCATA Annual has been held in what some might consider a non-central location. The main reason for this was to make our services available and more visible to members who might have had a more difficult time in the past to attend our Annual Meeting. The attendance, the best we have had for some time, seems to justify the decision. What do you think? Should the executive consider locations like Medicine Hat, Grande Prairie, and others for future meeting sites? Next year's meeting (Fall 1982) will be in Edmonton. Your reactions to the above questions will help us decide sites for 1983 and beyond.

# President's Report, 1980-81

Once again we have arrived at a time of excitement and annual change for the Mathematics Council of The Alberta Teachers' Association. There seems to be a unique excitement when members of our organization meet at our annual conference to rekindle old friendships and embark on new ones. This year's conference in particular offers an opportunity to make new friendships. For the first time in MCATA history the Annual Conference is being sponsored by the Southern Alberta Regional, an affiliate of MCATA, which operates out of communities in and around Lethbridge. Although this regional is the result of the efforts of several people, I feel Gary Hill (Lethbridge) has been a key figure and motivating force that has brought it into being.

The conference organizing committee and chairman Gary Hill have been able to make this the best conference ever by arranging a number of excellent sessions covering topics of problem solving and the state of the art of technology.

In keeping with the theme of this year's conference, a monograph on problem solving (edited by Dr. S. Rachlin from the University of Calgary) was released in 1980-81, and a new one commissioned for 1981-82 (edited by Ron Cammaert). The new monograph will include submissions from a wide variety of people involved in computing technology and educational computing. In addition to our monograph publications, Dr. George Cathcart, as editor of *delta-k*, has helped to keep us well informed.

Finally, let me take this opportunity to thank all of those people who have contributed to MCATA this past year, and let us look ahead to the enthusiastic leadership of Gary Hill in 1981-1982.

*Lyle Pagnucco*
President

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Presented at the 21st Annual Business Meeting

# Financial Statement for the Year Ending June 30, 1981

## Combined General and Barnett House Account

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**Receipts**

| | |
|---|---|
| Memberships | 4688.50 |
| MCATA Publications | 113.00 |
| Annual Conference | 8793.00 |
| S/A Interest | 688.38 |
| NCW Regional | 393.87 |
| 1980-81 ATA Grant | 4034.00 |
| Miscellaneous | 10.00 |
| | 18 720.75 |
| Balance: July 1, 1980 | |
| General Account | 7491.96 |
| Barnett House Account | 950.62 |
| | 8442.58 |
| | 27 163.33 |

**Expenditures**

| | |
|---|---|
| Executive Meetings | 1101.19 |
| 1980 Annual Conference | 6375.35 |
| 1981 Annual Conference | 331.40 |
| NCTM Annual Conference | 579.63 |
| Regina Name-of-Site | 100.00 |
| Specialist Council Seminar | 42.00 |
| AACE Conference | 315.05 |
| Supplies | 85.54 |
| Telephone | 857.90 |
| NCTM Affiliation | 50.00 |
| *delta-k* Honorarium | 100.00 |
| Math Exams & Contests | |
| High School Prize Exam | 500.00 |
| Calgary Jr. High | 100.00 |
| Calgary Math Night | 100.00 |
| Printing: | |
| General | 479.58 |
| *delta-k* | 1414.33 |
| Monograph #5 | 653.00 |
| Monograph #6 | 1501.66 |
| Postage: | |
| General | 378.92 |
| *delta-k* | 432.23 |
| Monograph #6 | 389.06 |
| Envelopes | 197.18 |
| Miscellaneous | 93.00 |
| | 16 177.02 |
| Balance: July 1, 1981 | |
| General Account | 10 986.31 |
| | 27 163.33 |

*Brian Chapman*
Treasurer

# Revised Budget, 1981-82

| Receipts | | Expenditures | |
|---|---|---|---|
| Memberships | 4500.00 | Executive Meetings | 2000.00 |
| MCATA Publications | 100.00 | NCTM Annual | 700.00 |
| ATA Grant | 4500.00 | Editor's Honorarium | 300.00 |
| | 9100.00 | Math Contests and Exams | 800.00 |
| | | Donald Hinde Memorial Award | 1100.00 |
| | | Supplies | 100.00 |
| | | Telephone | 700.00 |
| | | NCTM Affiliation | 50.00 |
| | | *delta-k* | 2500.00 |
| | | Monograph #7 | 3000.00 |
| | | Monograph #8 | 3000.00 |
| | | General Printing | 500.00 |
| | | General Postage | 400.00 |
| | | Regional Grants | 100.00 |
| Add Deficit | 6500.00 | Miscellaneous | 350.00 |
| | 15 600.00 | | 15 600.00 |

# Idiot Quiz – For Fun

1. A ship standing in the locks extends 14 m above the water line. The water rises in the locks at a rate of 2 m a minute. After 3 minutes, how far does the ship still extend above the water?

2. How many three-cent stamps in a dozen?

3. If you went to bed at 8 in the evening, and set the alarm for 9 in the morning, how much sleep would this permit you to have?

4. Do they have a first of July in England?

5. Why can't a man living in Edmonton, Alberta, be buried east of the Manitoba border?

6. How many birthdays does the average man have?

7. If you had only one match, entered a room with a kerosene lamp, an oil burner, and a wood stove, which would you light first?

8. If a doctor gives you three pills and tells you to take one every half hour, how long would they last?

9. Some months have 31 days, some 30. How many have 28?

10. You are driving a bus. At the first stop you pick up three people, the second you pick up 9 people; at the third stop 4 people get off and you pick up 13 people at the next stop. How old is the bus driver?

11. I have two Canadian coins totaling 55 cents. One is not a nickel. What two coins do I have?

12. A farmer had 19 sheep. All but 9 died. How many does he have left?

13. Divide thirty by halves and add ten. What is the total?

14. An airplane flying from Canada to the U.S. crashes on the border. Where do they bury the survivors?

15. Two people playing checkers played 5 games. Each person won the same number of games with no ties. Explain.

16. Take 2 apples from 6 apples. How many apples do you have?

17. An archeologist claims he found a gold coin dated back to 46 B.C. What is the total value of the coin?

18. A woman gives a beggar 50 cents. The woman is the beggar's sister, but the beggar is not the woman's brother. How come?

19. How many animals of each species did Moses take aboard the ark?

20. Is it legal for a man to marry his widow's sister in Newfoundland.

# Plus + + +

*The following material is reprinted from* Plus + + +, *a short magazine informing mathematics educators across Canada about important events, research, curriculum development, and items of national interest.*

## Proceedings of the Education Day Seminar of CMS

The Canadian Mathematical Society sponsored an instructional seminar as a part of its Winter Meeting on December 12, 1980 in Vancouver, B.C. This seminar was intended to give some of the flavor and nature of activities undertaken by applied mathematicians, particularly in British Columbia universities. Copies of the proceedings entitled "Modern Developments in the Uses of Mathematics," Technical Report No. 15, August 1981, edited by Dr. George Bluman, may be obtained at a cost of $17.00 each from:

The Institute of Applied Mathematics and Statistics
University of British Columbia
Vancouver, B.C. V6T 1Y4

## 1981 Winter Meeting of the Canadian Mathematical Society

The Canadian Mathematical Society will again sponsor an education section at its winter meeting in Victoria, B.C. on December 11-13, 1981. We hope to have a report on the second International Mathematics Study (IEA), as B.C. will have a lot of its data and Ontario will have selected its samples.

## Canadian Mathematics Education Study Group 1981 Meeting

CMESG held its 1981 meeting at the University of Alberta, Edmonton, Alberta on June 5-9. The keynote speakers were Doctors Jeremy Kilpatrick, University of Georgia, and Kenneth Iverson, I.P. Sharp Associates, Toronto. Copies of the proceedings of this conference are available from:

Dr. Joe Hillel, Secretary-Treasurer
Department of Mathematics
Concordia University
7141 Sherbrooke St. W
Montreal, Quebec H4B 1R6

## For the Learning of Mathematics – An International Journal of Mathematics Education

An international journal of mathematical education has been recently started by the FLM Publishing Association under the auspices of the Canadian Mathematics Education Study Group. This publication aims to promote enquiry and research at all levels of mathematics and mathematics education. For further information, write to:

Dr. David Wheeler, Editor
Department of Mathematics
Concordia University
7141 Sherbrooke Street W
Montreal, Quebec H4B 1R6

## Reading in Mathematics

"Reading is an essential skill in all subject areas - a skill which should be developed in those subjects. Can we, the teachers of mathematics, assume that skills developed in reading automatically transfer to the reading of mathematics? Or should we teach to ensure the transfer of reading skills?"

These are the questions addressed in Math Monograph No. 6: *Reading in Mathematics*, edited by John Percevault and published by The Alberta Teachers' Association. Seventeen papers contain quite a thorough account of the issues, both theoretical and practical, of the relation of reading skills to use of notation, reasoning, and solving of problems. They are sorted into five parts: I. Reading in the content areas; II. Survey papers; III. Practical considerations; IV. Teaching ideas; V. Bibliography. This useful volume can be obtained by forwarding $5.00 to The Alberta Teachers' Association, 11010 - 142 Street, Edmonton, Alberta T5N 2R1.

## The Beatty Essay Contest

The submissions to the third Samuel Beatty Mathematics Essay contest have been judged. Unfortunately, there were no essays of the same standard of those in the first two contests, so this year no first and second prizes are being awarded. A third prize of $75 goes to Sallianne Deck of Sarnia Northern Collegiate Institute and Vocational School for her essay, *Six numbers: Add any two - perfect square*. Honorable mention is made of the essays of Helen Earnshaw and Christopher Gaspar, both of Sarnia Northern C.I. & V.S., David Vinke of Glencoe District H.S. and Lindley Bassarath of Kirkland Lake C. & V.I.

A junior prize of $50 is awarded to Stephen Smith, in Grade 8 at the Royal Orchard School, Thornhill, Ontario for his essay, *A study of Pythagorean triplets*. Vivian Verelas, in Grade 10 at Ajax High School, gets honorable mention.

Although all the competitors in the third contest were from Ontario, the contest is actually open to school students all across Canada. The deadline for submissions to the fourth contest will be March 31, 1982. For the rules and suggested topics, write to Prof. E. J. Barbeau, Department of Mathematics, University of Toronto, Toronto, Ontario M5S 1A1. Those who would like, in addition, a book containing the winning essays from the 1979 and 1980 contests are requested to send a cheque for $3, payable to the University of Toronto.

## The 22nd International Mathematical Olympiad 1981 – Washington, D.C.

For the first time, Canada sent a team to the IMO, held this year in the United States. Travel expenses to and from the host country were covered by the Canadian Mathematical Society and grants of $1000 each from the Samuel Beatty Fund and the Ministry of Education of Ontario. The eight team members, accompanied by Professor G. J. Butler of the University of Alberta and Professor E. J. Barbeau of the University of Toronto, first attended a training session at the University of Saskatchewan and then spent 12 days at Rutgers and Georgetown Universities for the competition which involved 185 students from 27 countries. David Ash (Ontario) and George Gonthier (Quebec) received first prizes for perfect papers. Second prizes went to John Chew (Ontario) and Cary Timar (Ontario), and a third prize went to Julian West (B.C.). Also on the team were Arthur Baragar (Alberta), David Bernier (Quebec), and John Bowman (Alberta). The total score for the Canadian team was seventh highest after U.S.A., West Germany, U.K., Austria, Bulgaria, and Poland. Copies of the question paper and individual scores can be had from E. J. Barbeau, University College, University of Toronto, Toronto, Ontario M5S 1A1.

## The 23rd International Mathematical Olympiad 1982

Hungary has tentatively offered to host next year's Olympiad. If Canada is to send a team, we need money. Those who are willing to provide a donation or to help find money are requested to get in touch with Professor Barbeau of the University of Toronto. Professor Butler or Professor Barbeau are willing to consult with teachers about the training of students who are potential team members. Our team did us proud - let us respond!

## About the Leapfrogs Group

Leapfrogs Group is a network of users and producers of books/materials for use in middle and secondary schools. Membership is open to all who wish to join and there is no membership fee. When you register you will be sent a list of books and materials available in the current "wave" of publications. You are then invited to maintain your membership by placing an order from at least one of the two bi-annual lists. For further information write to:

Leapfrogs Group, Cold Harbour
Newton Street,
Cyres, Exeter, Devon, U.K.

## Gender and Mathematics – Canadian Report

The ERIC Centre is planning to publish an International Review on Gender and Mathematics. If you know of any Canadian publications, researches, action programs, . . . on this subject, please write to:

Dr. Roberta Mura
Faculty of Education
Laval University
Quebec City, P.Q. G1K 7P4

## Professional Dates to Remember

- 60th Annual Meeting of the National Council of Teachers of Mathematics
  April 14-17, 1982
  Toronto, Ontario

  For further information write to: NCTM
  1906 Association Drive
  Reston, Virginia 22091

- 21st Northwest Mathematics Conference
  November 5-7, 1982
  Vancouver, B.C.

  For further information write to: Mr. Gary Phillips
  4024 West 35th Avenue
  Vancouver, B.C. V6N 2J3

# Teach Nothing About Geometry

***Alton T. Olson***
*University of Alberta*

Contrary to a likely interpretation of the title, I am not advocating the deletion of geometry from the mathematics curriculum. In fact, I am quite concerned about the near future of geometry in the curriculum and would not wish to see its position eroded any more than it is. I am concerned because the coming emphasis on and enthusiasm for computer literacy and microcomputer applications could easily push geometry further into the background, simply because geometry doesn't lend itself easily to microcomputer uses.

To return to the title, I am advocating the teaching of nothing about geometry in the sense of "no-thing." Perhaps it is obvious, but I wish to emphasize that "no-thing" implies that we are not talking about a "thing." It is generally held that geometry instruction ought to include practice in space visualization, skills for organizing knowledge about space, attitudes favorable to local space exploration, and so on. But these are no-things which are about things. They are procedural skills, attitudes, or the seeing of relationships. The notion that no-things can be about things is crucial here, since the distinction between things and no-things is frequently the essence of arguments about the value of using geometric activities in the classroom. As an example, the "seeing of geometric relationships" might be acknowledged as an important mathematical goal, but nonetheless be slighted because it lacks a certain concreteness; for example, it is difficult to define as a teaching objective and is certainly difficult to test. Nonetheless, there is a growing body of research indicating the existence of certain generalized skills and abilities that are important in problem-solving and applications. In this paper I am suggesting that we ought to recognize these no-things of geometric activities and acknowledge their importance by insisting on their inclusion in the mathematics curriculum.

To further illustrate some of the points that I have been trying to make, I will describe and use a family of geometric activities. (Incidentally, these activities can easily be put into a game format if desired.) The activities will be defined and references will be made to the no-things of geometry that they illustrate.

## The Game of "Turn a Pattern" (TAP)

(This is adapted from Marion Walter's *Boxes, Squares and Other Things*.) I will begin with a discussion of the rules for the two-dimensional version of the game:

1. Use line segments of the same length.
2. The line segments must be placed end-to-end with a right angle at every joint.
3. Play the game first with two line segments, and then with three, four, five segments, and so on.
4. The objective is to generate as many "different" patterns as possible in each case.

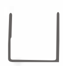
### Discussion

The following no-things would probably be exemplified in the activity above:

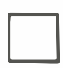
1. Two-dimensional space visualization skills would be exercised.
2. Inevitably, the process of defining "different" for Rule 4 above would include some no-things. For example, devising a rational decision rule for calling patterns different would probably be included.

(Are the three-segment patterns above different?)

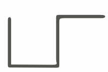
3. Systematic methods for generating all possible patterns might emerge naturally or could be encouraged.
e.g. from



we can obtain either



or



4. Systematic record-keeping could also be practised so that number patterns might be explored.

These are just a few of the possible important no-things that could emerge in such a geometric activity.

This game has an obvious extension into three dimensions. One additional rule forbidding more than two sticks to come from each joint is necessary here. The rest of the rules are the same.

## Conclusion

Some additional no-things could emerge in this setting:

1. Three-dimensional space visualization skills would be exercised particularly when combined with some of the three-dimensional space transformations.
2. By permitting the variation of rules it would be possible to set up natural comparisons between different systems.
3. The enjoyment of experiencing and exploring the familiar space around us could be enhanced. Most importantly, this can be done without the need for much formal knowledge of geometry.
4. That questions can be raised and problems posed is a recognition skill that would probably emerge naturally in these activities.

The activities and statements above are only suggestive of the importance of the no-things of geometry. Other activities and discussion points could be devised to illustrate these notions equally as well.

As a final note, I would like to make a paradoxical plea that we recognize the possibility that teaching the no-things of geometry may be the most important thing that we can do in geometry.

## Reference

Walter, M. I. *Boxes, Squares, and Other Things*. National Council of Teachers of Mathematics, 1970.

# Shaping Up Geometry, K-12

**_Donald R. Kerr Jr._**
_Indiana University_

Seventeen-year-old students have difficulty with those geometry problems which demand even routine application of properties of shapes. Moreover, when asked about the importance of different areas of study, students at all levels put geometry topics at or near the bottom of the list. These data from the 1977 National Assessment of Educational Progress come at the end of a decade of great interest and activity in geometry. The interest and activity are reflected in NCTM publications such as the *1973 Yearbook* and numerous articles in the *Arithmetic Teacher* and the *Mathematics Teacher*; in elementary school textbooks where the quantity and variety of geometry material has increased; in the several interesting geometry textbooks for elementary school teachers which have been published, including a particularly nice one by O'Daffer and Clemens; and in articles and experimental texts for secondary geometry including one on transformational geometry by Coxford and Usiskin.

*Why are there still major problems with geometry?*

One difficulty seems to be that geometry instruction lacks purpose and direction. The ten years of interest and activity produced creative geometry activities and attractive geometry material, but it failed to produce a clear picture of the goals of the geometry curriculum. It failed to give teachers and students a sense that there is an important body of geometric facts and skills. Too often geometry has been subordinated to arithmetic and algebra which are generally accepted to be important. Even when geometry is taught, facts and skills tend not to be the focus. In elementary school and middle school, geometry is often used as a change of pace and there is little follow-up to confirm achievement. In secondary school the formal aspects of the course tend to dominate. The focus tends to be on the meaning and utility of theorems.

This article explores the nature and importance of geometry, and it suggests a purpose and direction for geometry instruction. Some attention is given to the implementation of the suggestions, although most of the needed ideas and materials already exist and are readily available.

WHAT GOOD IS GEOMETRY? Geometry can be defined as the study of space experiences. Since most of our physical activities are space experiences, the potential importance of geometry is clear. The study of space experiences can include the following:

- identifying, naming, describing, and drawing shapes
- analyzing the particular properties of important shapes that make them important
- analyzing the changes in shapes which result in congruent shapes and in similar shapes

Reprinted with permission from *Math Lab Matrix*, No. 14, Fall 1980.

- applying the results from the above to real problems.

To analyze what knowledge and skills our students should be acquiring in the study of space experiences, it is useful to make a distinction between the geometry demands of everyday life and the demands encountered in college and in the practice of certain disciplines. Every student should

- *learn the names of common shapes.* (NAEP results suggest that this goal is being met fairly well at present.)
- *learn to describe, visualize, and draw shapes.* (There is no need to quote statistics to convince most people that adults and children alike lack skills with three-dimensional drawing and visualization. Yet these skills are so often in demand in planning home or office space, in giving or understanding directions, and in just moving from place to place.)
- *learn what makes each shape useful.* (Triangles are rigid and consequently are a critical source of structural stability; all polygons can be triangulated, so that knowledge about triangles can be used to generate knowledge about any polygon; triangles, rectangles, and certain other shapes tessellate and so are useful for covering plane surfaces; all points on a circle are the same distance from the centre, hence the invention of the wheel; circles and spheres have optimal contents/boundary/ratios, a fact which has important implications for packaging; straight lines on the plane share with great circles on spheres the property of being the shortest distance between points.)
- *learn the fact that the ratios of the measures of corresponding elements of similar figures are equivalent.* (This fact is the key to scale drawing which is fundamental to any activity which involves planning and building from plans.)
- *learn to use all of the above to solve actual problems.* (We all know that it is entirely different to know a fact than it is to have the experience and confidence required to recognize its uses and to actually use the fact. Moreover, it is entirely different for a teacher to tell students that geometry is useful than it is for students to actually experience its usefulness.)

In addition to the above knowledge and skills, students who are going to college or who are going into certain technical fields need to learn analytic geometry. Analytic geometry is the bridge from geometry to the rest of mathematics. Expressing geometric relationships numerically makes it possible for the tools of algebra and calculus to be applied to geometric problems. Of great recent importance is the fact that analytic geometry makes it possible to analyze geometric problems on a computer. Today, many capable students come to college with very weak backgrounds in analytic geometry, especially three-dimensional analytic geometry.

Enough of complaining. What can be done to improve things?

## What To Do?

Fortunately, things can be improved without making radical changes. In the elementary school and middle school, the students should gain useful knowledge and develop intuition and experience with all of the important plane and solid shapes. For each shape the student should learn its name, how to draw it, where it occurs in the world, why it occurs there (i.e., what special properties it has that cause it to occur), and the student should gain some experience using the shape.

| Name of Shape | Drawing | Occur-rences | Proper-ties | Applica-tions |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |

This matrix shows the kinds of learning that should take place for each of the important shapes. Here are some illustrations of entries for some of the cells of the matrix:

*Occurrences, properties, and applications of straight lines:* Have students look at footpaths that have been worn in the grass where sidewalks do not provide a minimal path between points, and have them discuss why the paths are where they are and are the shape they are.

*Occurrences, properties, and applications of triangles:* Have students try to stabilize an unstable shape (either one made of cardboard and brads or a real one). Have them find real world examples of the use of triangles to stabilize shapes.

*Occurrences, properties, and applications of circles:* Have students gather around to see something on the floor. Then ask them why they formed a circle. Ask them what special properties of circles make theater-in-the-round a good idea. Ask them to find other real world examples where the facts that all points on a circle are equidistant from the centre and that the circle is the plane shape with maximal area/perimeter ratio are used.

*Drawing, occurrences, properties, and applications of spheres, circular cylinders, and rectangular prisms:* Have students draw and make a cylinder, a rectangular prism and a sphere. Ask students why milk cartons are rectangular, why some water towers are spherical, and why others are cylindrical. Ask students to notice what products are packaged in what shape containers and why.

For such activities to be effective, it is important for the teacher to have a clear idea of what specific facts and skills the students should learn. It is also important that the teacher verify that the students have, indeed, learned the skills.

In addition to the study of shapes, students in the elementary school should begin preparation for the study of analytic geometry. Map reading and map making can provide valuable experience with associating locations with pairs of numbers (e.g., 3rd Street and 4th Avenue), pairs of words (e.g., at the corner of Elm and High), and pairs with a word and a number (e.g., 602 East Smith). Activities with room numbering in multifloor buildings can also help students see the relationship between location and numbers.

High school geometry presents a very special set of considerations. Most high school geometry students find that a large fraction of their time is spent attempting to prove theorems. Little of their time is spent determining what the theorem says about shapes in the world. For example, how many students recognize that the rigidity of triangles is an immediate consequence of the theorem which says that two triangles must be congruent if their sides are congruent? Further, how many students have stopped to ponder the fact that there is no such theorem for polygons with more than three sides and that this has implications for the rigidity of non-triangular shapes? The problem is that the high school geometry course is attempting to fulfill two different sets of objectives. On the one hand, it is attempting to teach students about shapes. On the other hand, it is introducing students to deductive reasoning. I propose that these two objectives of the course be dealt with one at a time rather than together.

PROPOSED HIGH SCHOOL GEOMETRY COURSE

| | |
|---|---|
| History of geometry and introduction to deductive logic. | Properties of two-dimensional and three-dimensional shapes and their application. |

In the proposed course, half of the first semester would be a history of geometry, with emphasis on the importance of Euclid's *elements* both as an intellectual landmark and as a compendium of knowledge about shapes. Formal deduction would be introduced in the context of a subset of Euclidean geometry with "stronger" axioms that are chosen to produce interesting theorems fairly easily. (This "local" axiomatic approach is introduced in a chapter by Seymour Schuster in the 1973 NCTM Yearbook entitled "Geometry in the Mathematics Curriculum.") This first quarter of the course would end with a brief study of Descartes and of the role of analytic geometry in mathematics. The remaining three quarters of the course would study the properties of plane and solid shapes. Each property would be introduced and verified in the most edifying and meaningful way. Some properties would be proved synthetically, others analytically. Some properties would be discovered experimentally. Others would merely be established by plausibility argument. The emphasis would be on meaning, on intuition, on drawing, on applications, and on analytic geometry where possible.

Such a course as this would introduce the important idea of deductive thought without encumbering the entire course with proof. More time could be spent analyzing shapes and applying properties and less time memorizing theorems. The course would have greater utility for the college-bound student as well as the non-college-bound student. Its perceived relevance might even attract more students.

## What are the Chances?

There are reasons to believe that the time may be right for the changes suggested here. At all levels in the curriculum there is a renewed interest in problem solving and applications. This interest is compatible with an increase in emphasis on the occurrences and applications of properties of shapes and has resulted in resources for applications (MAA-NCTM Source-book) and has also influenced textbooks. Moreover, it has created a climate in which such an increase might be accepted. It is also possible that the increasing presence of calculators and computers in the schools will cause teachers to look for new clusters of meaningful mathematics objectives. Geometry might well fill the need. Even before curriculum changes are adopted, much can be gained at any level by additional focus and follow-through on standard objectives related to the properties of shapes, and by making some effort to relate those properties to their applications in the real world.

# Thought Levels in Geometry

***Alan R. Hoffer***
*University of Oregon*

Thought levels in geometry were proposed by two teachers in the Netherlands - Pierre and Dina van Hiele. They were concerned with the difficulty that their students had in learning geometry. After several years of teaching, the van Hieles recognized that levels of thought and language are obstacles to student learning, because the students who are reasoning at a lower level are unprepared to perform at a higher level. The higher level questions are too difficult for the students.

Consider a hurdle race as an analogy. To finish the race one must jump over one hurdle at a time sequentially. (High jump or pole vault competition are also analogies.) It makes no sense to try to jump the last hurdle first and then go back and jump the beginning hurdles. That is just what we do in our geometry programs for students.

Many students are cheated out of adequate sequential geometric experiences in the elementary school and junior high school grades. Then, after one year of algebra, they are thrust unprepared into a high school geometry course and are expected to write formal proofs of theorems.

Here are the thought levels as originally described by the van Hieles.

## Level 0 – Recognition

The behavior at this level is primarily visual. Students may recognize pictures of triangles, squares, rectangles as such but they are not actively aware of properties of the figures. It is interesting that even in the junior high school grades many students think there are "good triangles" and "bad triangles." They think that a good triangle must be equilateral, with its base parallel to the floor.

Students at this level think of shapes as a whole. They are not aware of many properties of the figures.

## Level 1 – Analysis

The students now begin to analyze properties of the figures. As examples, they realize that the opposite sides and possibly even the diagonals of a rectangle are congruent; that a cube has six square faces; that the diagonals of a rhombus are perpendicular; that the base angles of an isosceles triangle are congruent.

Students at this level are able to work with parts of a figure. They are not aware of interrelationships between figures. For example, at this level squares, rectangles, and parallelograms all seem to be very different.

## Level 2 – Ordering

The students begin to logically order figures, to understand interrelationships between figures, and to realize why it is necessary to describe things accurately. For

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example, because they know definitions of figures, students will realize that every square is a rhombus, that every rectangle is a parallelogram.

Students at this level are able to work with concise definitions and to apply elementary logic rules. They are able to classify figures. They are not able to explain (or prove) why certain things are true. For example, at this level students will have difficulty explaining why the opposite sides of a parallelogram are congruent, why a quadrilateral with congruent pairs of opposing sides must be a parallelogram, why the base angles of an isosceles triangle are congruent.

### Level 3 – Deduction

The students begin to understand the significance of deduction and the role of postulates, theorems, and proofs.

Students at this level understand the difference between postulates and theorems. They are able to develop a chain of statements to connect a given hypothesis with a conclusion. They can work with given information from a figure and deduce conclusions whether or not the figure is drawn accurately. (They are working logically rather than visually.) They do not understand the foundations of geometry, why there are different geometries, or even how the SAS postulate connects with distance and angle measures in Euclidean geometry.

### Level 4 – Rigor

This most advanced level is rarely reached by high school students. At this level students understand the importance of precision in dealing with foundations and interrelationships between structures. For example, students know how the Euclidean parallel postulate relates to the existence of rectangles and why in non-Euclidean geometry rectangles do not exist.

### Proper Preparation

For a student to function adequately at one of the advanced levels, he or she must have mastered large chunks of the prior levels. All too often students who are forced to work at a level for which they are not prepared behave like parrots who are imitating the teacher or the textbook without understanding. They depend on memory to get them through. Most high school geometry courses operate at level 3, while students have not fully rounded out level 0. It is no wonder that geometry is a disliked subject. We force many students into failure situations.

*The Future Directions of Geometry* must involve this awareness of thought levels to better organize learning experiences for students of all ages. In particular, we would do most students a great service if we would devote the first half of the high school geometry course to informal activities and postpone formal proofs to the second semester. Then the students would move through the lower levels so proofs would not be an obstacle for them.

## ACTIVITIES

Here are some sample activities for your students. The way the students respond will tell you about their thought level.

A. Make a sheet of quadrilaterals and non-quadrilaterals in various positions. Ask the students to put an S on the squares, R on the rectangles, D on rhombi, P on parallelograms and T on trapezoids. Can they recognize them? Do they realize that a rectangle is a parallelogram?

B. Have several quadrilateral shapes cut out of cardboard. Ask the students to put together the figures that are "alike in some way." Ask the students why they chose that grouping. Repeat this several times to look for different ways of sorting.

C. Ask students to pretend they are telling a blind friend what certain figures are. How would they describe a square, rhombus, rectangle, parallelogram, etc. Then ask them to give the shortest descriptions that they can think of.

D. Ask students to list some properties of a parallelogram. Discuss the properties with them. What else can they observe? Then pick one of the properties and ask the student to explain why it is true. Formulate the converse of the statement and ask the students if that is also true, and why.

E. Show the students a picture of a kite and ask what type of a figure it is (some will say parallelogram). Ask about properties of the figure (many will say that the opposite angles are congruent). Can they explain why their claims are true?

You can try these and similar activities with students at all ages. You will be surprised at some of the responses. In fact, many students who have studied a year of high school geometry still seem to be thinking at level 1.

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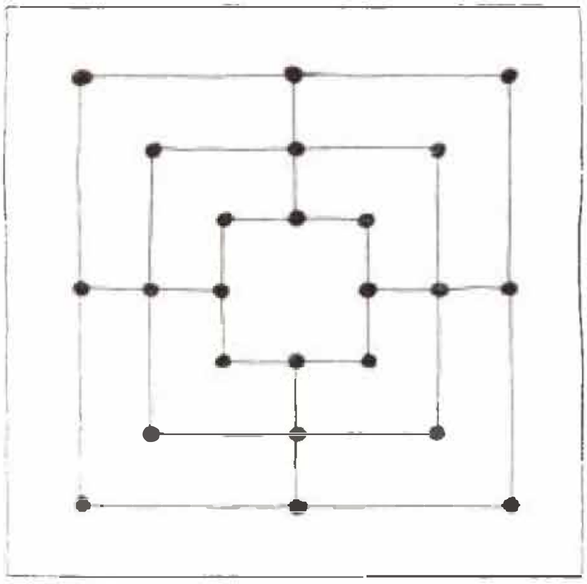


## Plan Ahead

(A Strategy game for two players)

by Janet Hewitt, Spokane Lutheran School

Equipment: above gameboard (20 cm square), 6 markers (3 of each color).

Rules: The object is to get your three pieces lined up along any one of the lines before your opponent can do so. In your first three turns, each of you places a piece on the board anywhere you desire. Thereafter, you can only move to an adjacent dot, and only along a line. You are not allowed to jump over a piece, nor can you move between dots not connected by a line.

I had my third and fourth graders make this game for their parents' Christmas gift, and they all seemed to have a good time playing the game before wrapping it up. It was a good exercise in measuring and using a ruler. We made the gameboard on tagboard, using colored pens to make it colorful. Then they printed the rules on the back and we covered both sides with contac paper. The children collected beer bottle caps and painted them two colors for markers (the twist off type are best and ones with a rebus inside are additional fun).

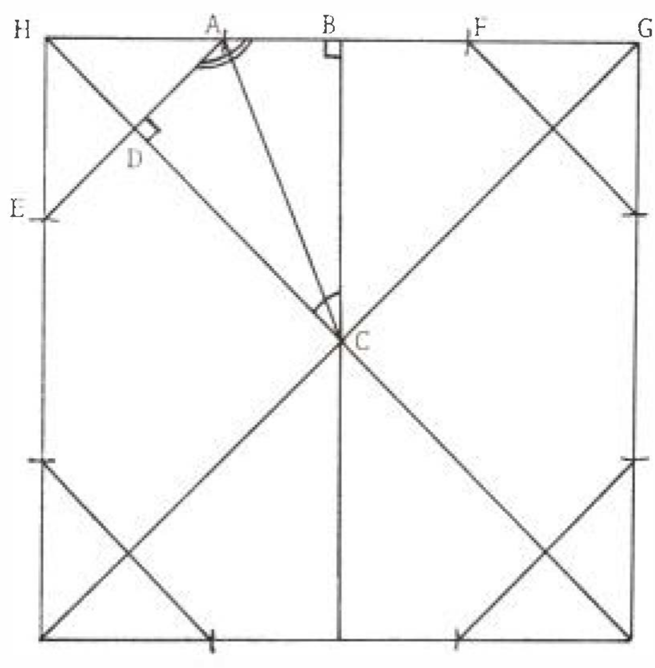
(Reprinted from *Washington Mathematician*, Vol. 25, No. 2, February 1981)

# How to Draw an Octagonal Figure Quickly

***William J. Bruce***

The following method, commonly used by a carpenter, is well within the understanding of students of elementary plane geometry.

Method: 1. Draw both diagonals of a square to intersect in the centre C as shown in the accompanying diagram.
2. With each corner of the square as centre and with radii equal to the length of a semi-diagonal, draw arcs to intersect all sides of the square.
3. The points of intersection in (2) are the corners of the required octagon. (See proof below.)




Proof: From the figure, B is a point of bisection of a side of the square. By construction, $\Delta ACG$ is isosceles with $AG \equiv CG$. $\angle AGC = 45^{\circ}$, so $\angle ACG = \angle CAG = 67\frac{1}{2}^{\circ}$. $\angle BCG = \angle HCB = 45^{\circ}$, so $\angle ACB = \angle ACD = 22\frac{1}{2}^{\circ}$. Thus $\angle CAD = 67\frac{1}{2}^{\circ}$ also. Therefore, by a.s.a., $\Delta$'s ABC and ADC are congruent. So $AB \equiv \angle AD$. But $AF = 2AB$ and $AE = 2AD$, by construction. Hence $AF \equiv AE$, as required for an octagon.

*Note:* If s units is the length of a side of the square and h units is the length of a side of the octagon, it is easy to obtain $h = s \tan 22\frac{1}{2}^{\circ}$ and, from the appropriate half-angle trigonometric identity, that $h = s(\sqrt{2} - 1)$.

# ? ? ? Problem Corner ? ? ?

***edited by William J. Bruce and Roy Sinclair***
*University of Alberta, Edmonton*

Problems suggested here are aimed at students of both the junior and senior high schools of Alberta. Solutions are solicited and a selection will be made for publication in the next issue of *delta-k*. Names of participants will be included. All solutions must be received (preferably in typewritten form) within 30 days of publication of the problem in *delta-k*.

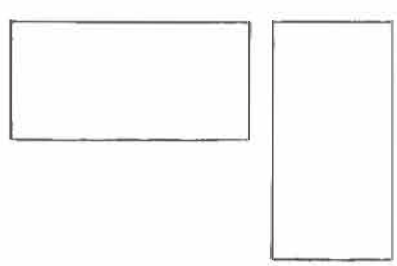
*Mail solutions to:* Dr. Roy Sinclair or Dr. Bill Bruce
Department of Mathematics
University of Alberta
Edmonton, Alberta T6G 2G1

**Problem 7:**

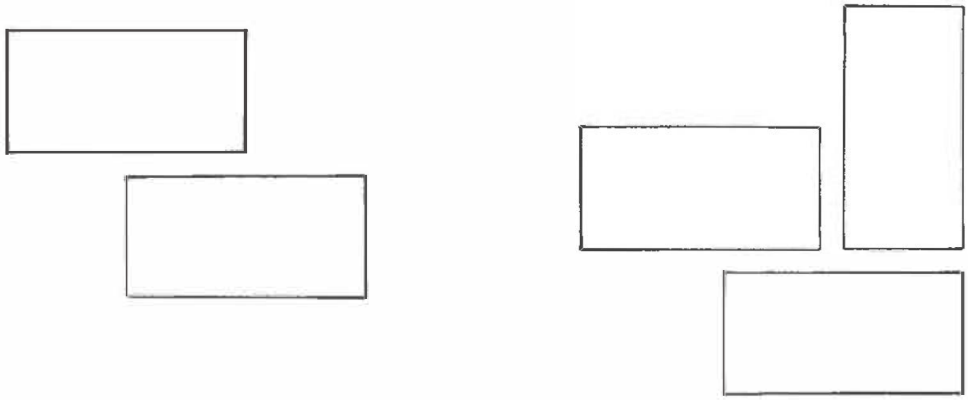
(submitted by William J. Bruce, University of Alberta)

The following problem is presented in two parts and both are suitable for trial and error approaches by students of all levels. Diagrams only are to be submitted for publication. However, proofs are welcome and will be acknowledged.

(a) Consider an 8 cm by 8 cm grid of 1 cm squares and a set of 1 cm by 2 cm dominos. Place the dominos on the checkerboard in such an arrangement so that any two dominos that touch each other do so only at right angles. Find the pattern that will leave the minimum number of spaces uncovered and state this number.



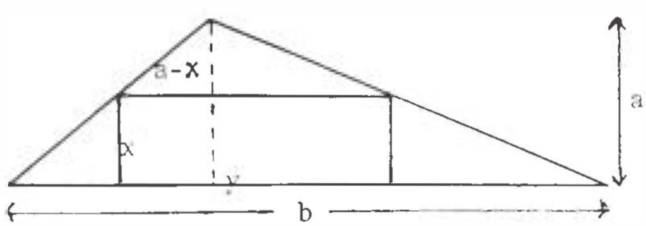
(b) As in (a) above except that, in addition to right angle butting, semi-adjacent parallelism is permitted also. By this we mean that the dominos may overlap half way along their sides. Again find the pattern that will leave the minimum number of spaces uncovered and state this number.



Note: The patterns developed in the above problems are a source for tiled table tops and floors or even for wall hangings done in needlepoint stitchery.

## Solution for Problem 5:

(suggested by Dr. William Bruce)




From the figure, $\frac{x}{b} = \frac{a-x}{a} \rightarrow x = \frac{b}{a}(a-x)$

and $\frac{a-x}{y} = \frac{a}{b} \rightarrow y = \frac{b}{a}(a-x)$

From the latter, $xy = \frac{b}{a}x(a-x)$ gives the area of the rectangle. We must maximize $xy$, i.e., $\frac{b}{a}x(a-x)$. Complete the square and get $\frac{-b}{a}[(x-\frac{a}{2})^2 - \frac{a^2}{4}]$. This will have a maximum when $x = \frac{a}{2} \rightarrow y = \frac{b}{2}$. So the maximum area is $\frac{ab}{4}$. But $\frac{ab}{2}$ is the area of the triangle no matter on which side the rectangle is drawn.

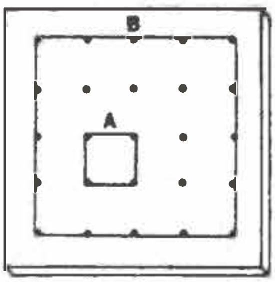
activities

# Geoboard Investigations

## Investigation 1

A is the smallest square you can form on the geoboard using one rubber band.

B is the largest square you can form on the geoboard using one rubber band.



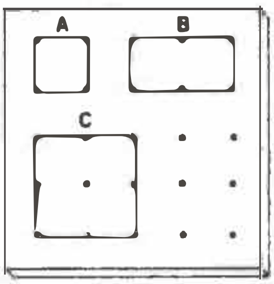

There are squares of 6 more different sizes that can be formed.

How many of them can you find and draw on dot paper?

## Investigation 2

Rubber band A encloses an area of 1 square.

Rubber band B encloses an area of 2 squares.

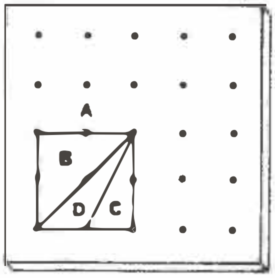



The area of C is 4 squares.

Can you show a square or a rectangle that has an area of 3? 5? 6? 7? 8? 9? 10? 11? 12? 13? 14? 15? 16?

## Investigation 3

What is the area of square A?
What is the area of B?
What is the area of C?




Using the information about the areas of A, B, and C, what is the area of triangle D?

There are 8 triangles of different shapes with area 1 that can be formed on the geoboard. How many of them can you find and draw on dot paper?

# Patterns in the Angles of Regular Polygons

1. Construct regular polygons of 3, 4, 5, 6, 7, and 8 sides. Choose any point P in each polygon. Join P to each vertex. How many triangles are formed in each polygon? The angle sum of each triangle is how many right angles?

2. Complete the following table, using right angles instead of degrees for angle sums.

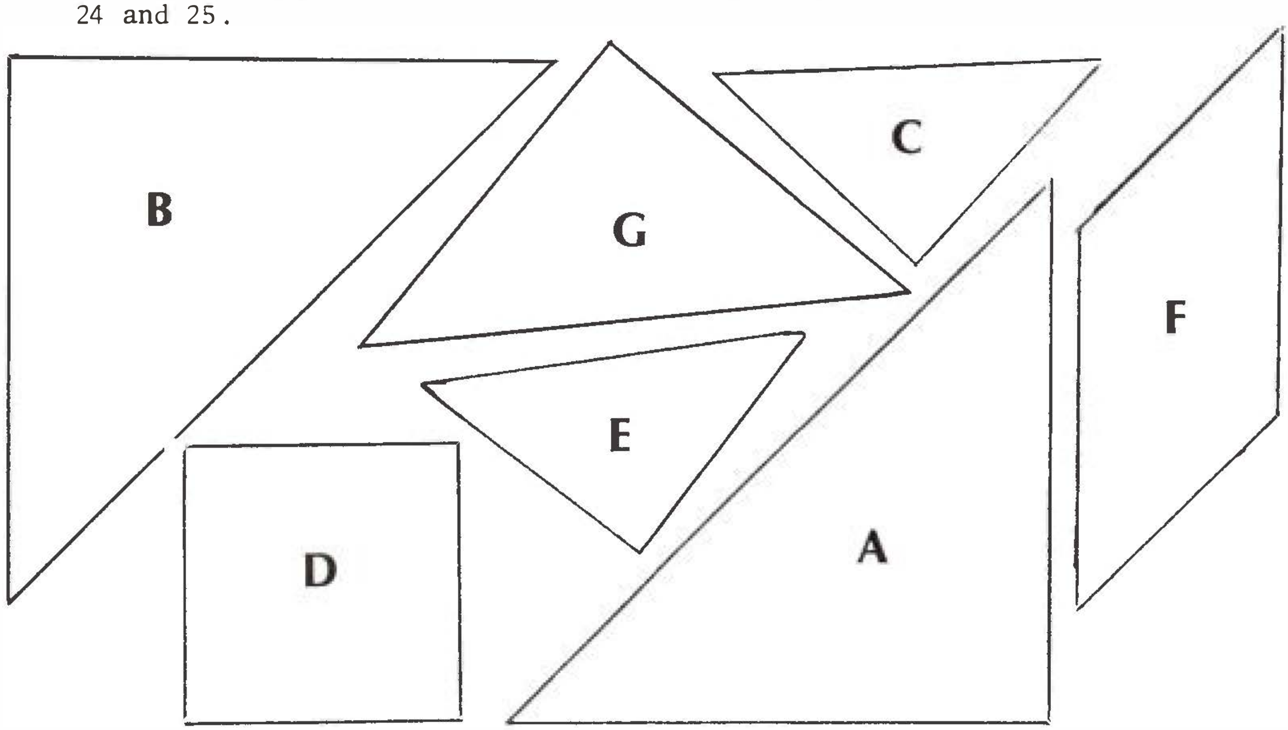
| No. of sides of polygon | No. of triangles | Angle sum of triangles | Angle sum at P | Angle sum of int. angles |
|---|---|---|---|---|
| 3 | 3 | 6 rt. angles | 4 rt. angles | 2 rt. angles |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |
| 7 | | | | |
| 8 | | | | |

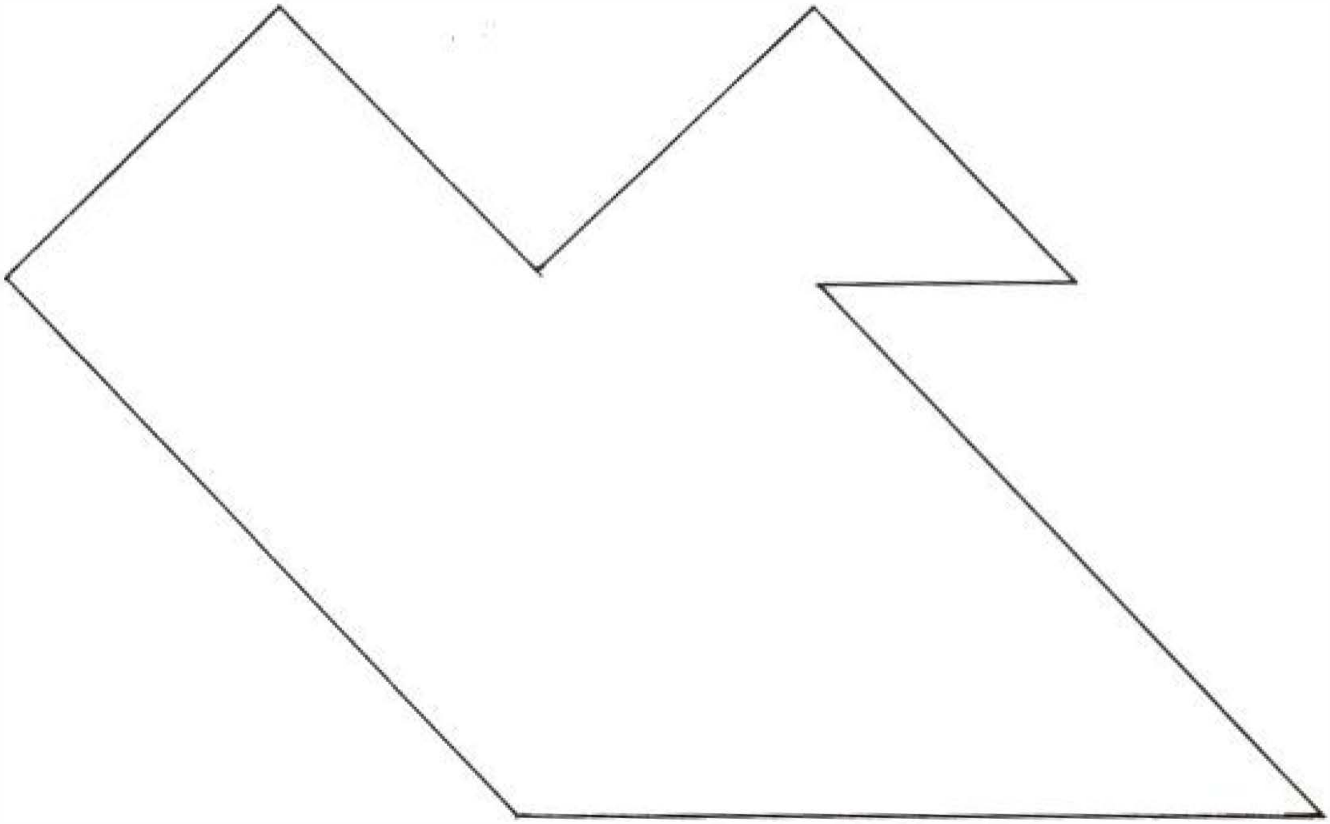
3. Find the pattern. What is the sum of the interior angles of polygons of 10, 20, 100 sides?

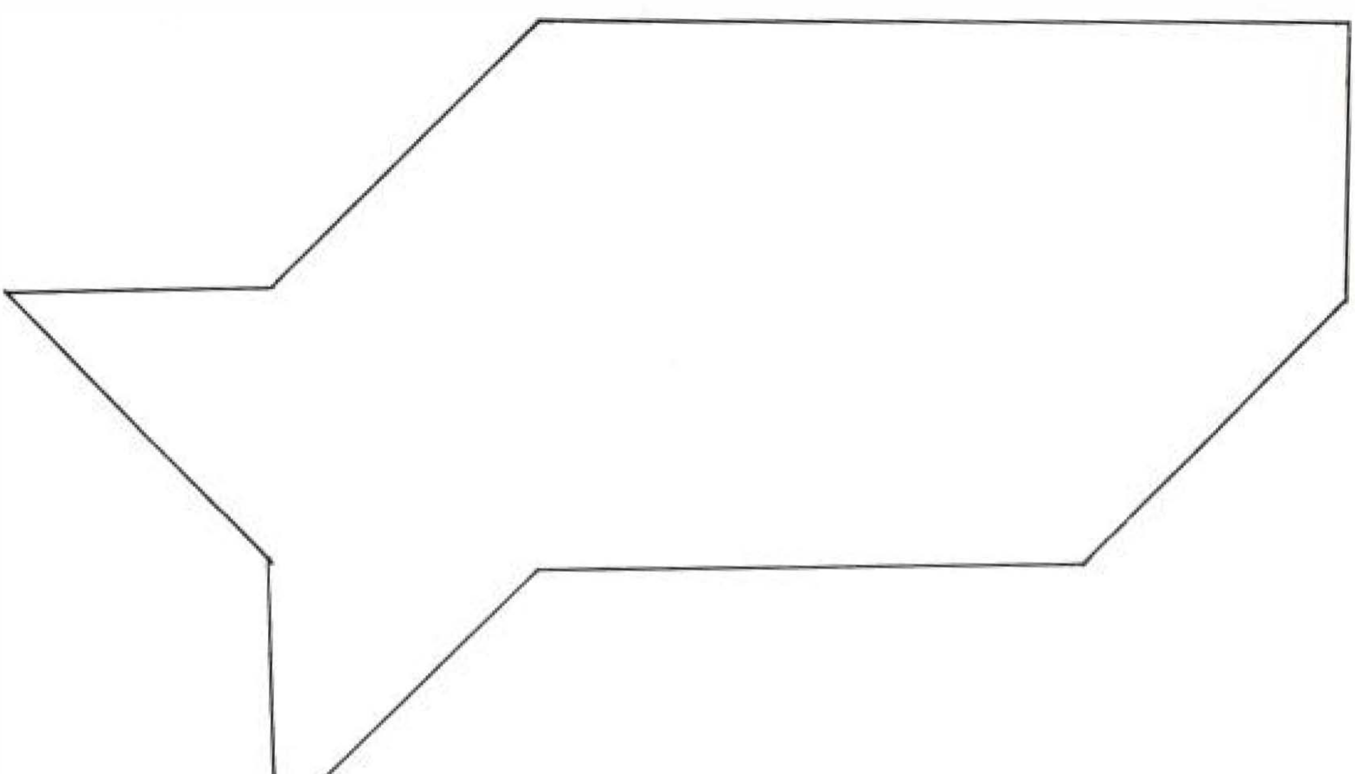
4. Graph the size of the interior angles against the number of sides. What conclusions do you draw?

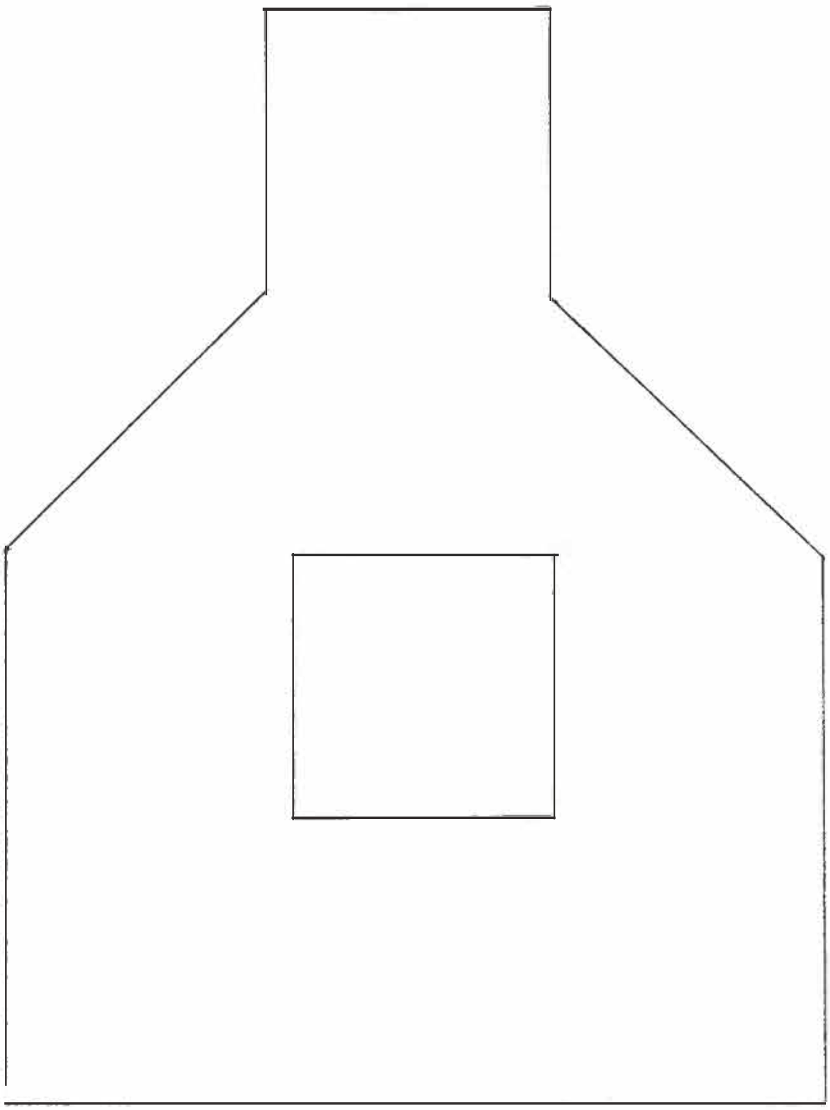
5. Can you find the sum of the interior angles of non-regular polygons?

Trace, cut out, and then use these seven pieces to make the figures on pages 24 and 25.

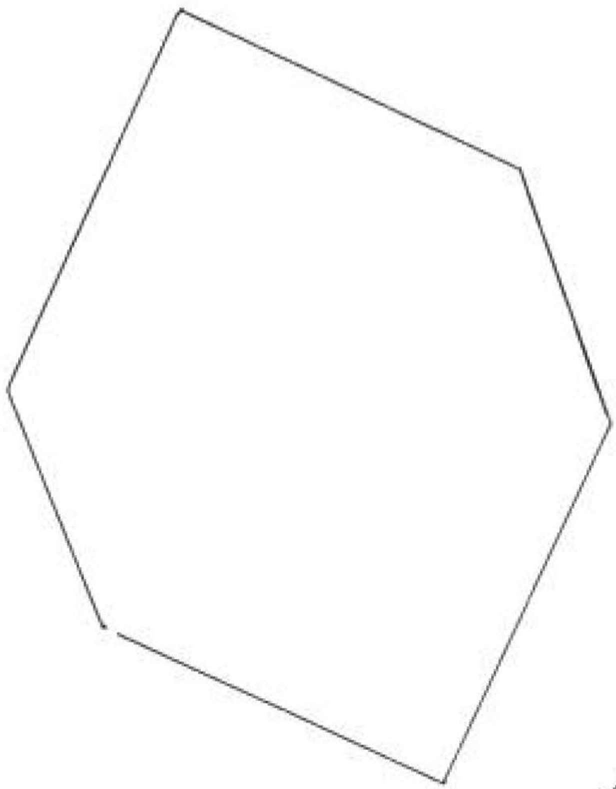


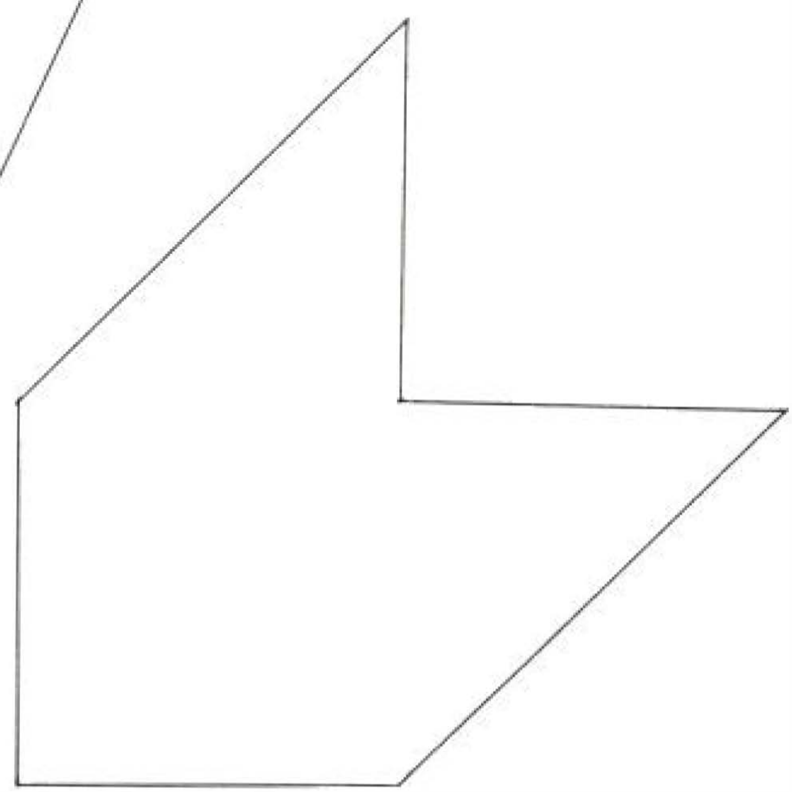




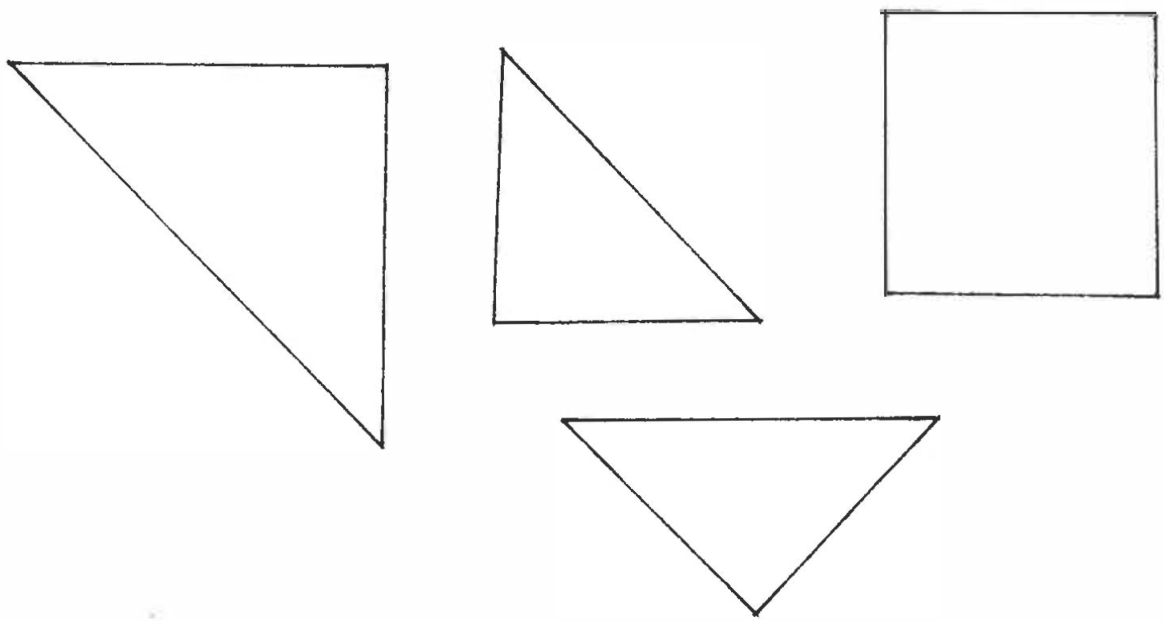


Use only the five smaller shapes (C, D, E, F, G) to make these figures.





## Use:



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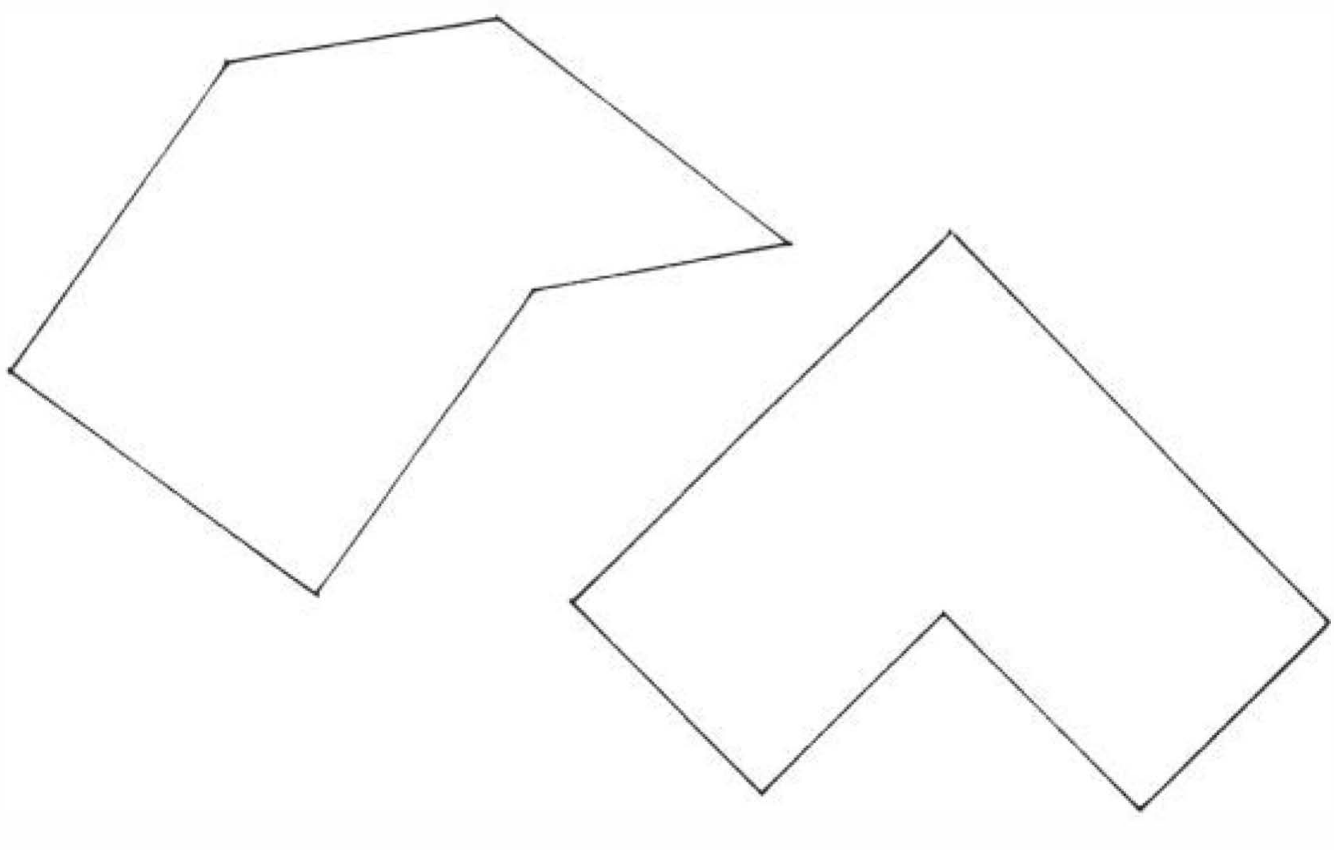
## To Make:



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