An Advisory Exam in Mathematics

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The Mathematics Department at the University of Alberta gave the following 50-minute examination to the 2,500 students in its introductory calculus courses at the beginning of the 1981 fall term. Those students whose background appeared weak were advised to attend a five-week remedial program along with their calculus course. In spite of the difficulties inherent in such a voluntary, no-credit program, the student response was really gratifying. Those students who had not taken Mathematics 30 recently were particularly appreciative of this opportunity to refresh the concepts involved. A lack of proficiency with the fundamentals is a major handicap in the calculus courses.

The examination is given below and is followed by some of its statistics. The mean score was 10.08. Students who had taken both Mathematics 30 and Mathematics 31 fared significantly better than those with only Mathematics 30 on the advisory exam and in their calculus course.

1. If $S = am + (m-1)d$, then $m =$

   (a) $\dfrac{S+d}{a+1}$ (b) $\dfrac{S+d}{a+d}$ (c) $\dfrac{S-d}{a+d}$ (d) $-\dfrac{S+d}{a+1}$ (e) none of these.

2. $\dfrac{x^2-y^2}{2xy} \div \dfrac{x^2+2xy+y^2}{y^2} =$

   (a) $\dfrac{y}{2x}$ (b) $-\dfrac{y}{2x}$ (c) $\dfrac{y(x-y)}{2x(x+y)}$ (d) $\dfrac{y}{2x}\cdot\left(\dfrac{x-y}{x+y}\right)^2$ (e) none of these.

3. $\dfrac{2x^4-x^3+x-2}{2x^2-x+2} =$

   (a) x^2-1 (b) x^2+1 (c) x^2+x-1 (d) x^2-x+1 (e) none of these.

4. $\dfrac{1}{\sqrt{x} + \sqrt{y}} =$

   (a) $\dfrac{1}{\sqrt{x+y}}$ (b) $x^{-\frac{1}{2}} + y^{-\frac{1}{2}}$ (c) $\sqrt{x} - \sqrt{y}$ (d) $\dfrac{\sqrt{x} - \sqrt{y}}{x-y}$ (e) none of these.

5. $x\sqrt{x} - 2\sqrt{x} + x^{-\frac{1}{2}} =$

   (a) $x-2$ (b) $\dfrac{x-1}{\sqrt{x}}$ (c) $\left(1-\dfrac{1}{x}\right)^2 \sqrt{x}$ (d) $\dfrac{(x-1)^2}{\sqrt{x}}$ (e) none of these.

6. $\dfrac{\sqrt[3]{p^2 q}}{\sqrt{pq^3}} =$

   (a) $\dfrac{\sqrt[6]{pq^5}}{q^2}$ (b) $\dfrac{(pq)^{\frac{1}{6}}}{q^2}$ (c) $\dfrac{(pq^5)^{\frac{1}{6}}}{q}$ (d) $\dfrac{p}{q} \sqrt[6]{pq^2}$ (e) none of these.

7. If $2x^2-4x+5$ is written as $a(x-h)^2+k$, then

   (a) $a=2$ and $k=\dfrac{3}{2}$ (b) $a=2$ and $k=3$ (c) $a=2$ and $k=5$ (d) $a=2$ and $k=9$

   (e) none of these.

8. The solution set of $x(x+2)=1$ is

   (a) $\{-1\}$ (b) $\{-1+\sqrt{2}\,i\,,-1-\sqrt{2}\,i\}$ (c) $\{-1+\sqrt{2}\,,\,-1-\sqrt{2}\}$ (d) $\{1\}$

   (e) none of these.

9. $x(9x-2)\le(3x+1)^2$ is equivalent to

   (a) $-\dfrac{1}{8} \le x$ (b) $-\dfrac{1}{5} \le x$ (c) $-\dfrac{1}{2} \le x$ (d) $-8\le x$ (e) none of these.

10. $|x-1|<\dfrac{1}{10}$ is equivalent to

   (a) $\dfrac{-11}{10} < x < \dfrac{11}{10}$ (b) $\dfrac{-9}{10} < x < \dfrac{9}{10}$ (c) $-\dfrac{9}{10} < x < \dfrac{11}{10}$ (d) $\dfrac{-11}{10} < x < \dfrac{9}{10}$

   (e) none of these.

11. If $x=-1$ and $y=3$, then $\dfrac{|x-y|}{|x|-|y|} =$

   (a) -1 (b) 1 (c) 2 (d) -2 (e) none of these.

12. The domain of $f(x) = \sqrt{x(1-x)}$ consists of all real numbers x such that

   (a) $0\le x$ (b) $x\le0$ or $x\ge1$ (c) $x\le1$ (d) $0\le x\le1$ (e) none of these.

13. If the graph of $f(x) = -x^3+2kx^2-\dfrac{3}{2}kx+1$ contains the point $(2,3)$, then $k =$

   (a) 0 (b) -1 (c) $-\dfrac{6}{5}$ (d) -2 (e) none of these.

14. If $f(x) = \dfrac{1}{x}$, then $\dfrac{f(1+h)-f(1)}{h} =$

   (a) $\dfrac{-1}{1+h}$ (b) $\dfrac{1}{h^2}$ (c) $\dfrac{1}{1+h}$ (d) $\dfrac{1}{h(1+h)}$ (e) none of these.

15. $\log_a\left(\dfrac{8x^3}{\sqrt{y}}\right) =$

   (a) $(\log_a 2x)^3 - (\log_a y)^{\frac{1}{2}}$ (b) $\log_a 8 + 3\log_a x - \dfrac{1}{2}\log_a y$

   (c) $24\log_a x - \dfrac{1}{2}\log_a y$ (d) $3\log_a(2x) + \dfrac{1}{2}\log_a y$ (e) none of these.

16. If $\log_2(x-1) = 3$, then $x =$

   (a) 8   (b) 10   (c) 7   (d) -7   (e) none of these.

17. Which of the following is true for the graph of $y = 10^x$?

   (a) It crosses every line $y = k$ exactly once where k is any positive constant.

   (b) It intersects the y-axis at $(0,10)$.

   (c) It is symmetric across the y-axis.

   (d) It crosses the x-axis exactly once.

   (e) none of these.

18. The equation of the line passing through $(1,2)$ and parallel to the line through $(3,-2)$ and $(4,-1)$ is

   (a) $y=2x$ (b) $y=-3x-1$ (c) $y=x+1$ (d) $y=3x-1$ (e) none of these.

19. The set of points $P(x,y)$ that are 5 units from $(5,-6)$ is given by

   (a) $x^2+y^2-10x+12y-16=0$ (b) $x^2+y^2-10x+12y+36=0$ (c) $x^2+y^2-10x+12y-36=0$

   (d) $x^2+y^2+10x-12y+36=0$ (e) none of these.

20. The graph of $y=16-9x^2$ has the following two properties

   (a) it opens downward and its vertex is at $(0,16)$.

   (b) it opens to the right and its vertex is at $(-\dfrac{4}{3}, 0)$.

   (c) it opens to the left and its vertex is at $(\dfrac{4}{3}, 0)$.

   (d) it opens upward and its vertex is at $(0,16)$.

   (e) none of these.

21. The degree measure equivalent to the radian measure $-\dfrac{7\pi}{4}$ is

   (a) $-315°$ (b) $45°$ (c) $-225°$ (d) $405°$ (e) none of these.

22. If $\tan\theta = \dfrac{\sqrt{5}}{2}$ and $\sec\theta = \dfrac{3}{2}$, then $\sin\theta =$

   (a) $-\dfrac{\sqrt{5}}{3}$ (b) $\dfrac{4}{3\sqrt{5}}$ (c) $\dfrac{\sqrt{5}}{3}$ (d) $\dfrac{3\sqrt{5}}{4}$ (e) none of these.

23. $\sin\pi\,\cos\dfrac{\pi}{2} - \cos\pi\,\sin\dfrac{\pi}{2} =$

   (a) $\sin\dfrac{\pi}{2}$ (b) $\sin\dfrac{3\pi}{2}$ (c) $\cos\dfrac{\pi}{2}$ (d) $\cos\dfrac{3\pi}{2}$ (e) none of these.

24. $\dfrac{1}{\sin^2\theta} - 1 =$

    (a) $\tan^2\theta$ (b) $\sec^2\theta$ (c) $\csc^2\theta$ (d) $\cot^2\theta$ (e) none of these.

25. For which of the following intervals is $f(\theta) = \sin\theta$ <u>both</u> positive and increasing?

    (a) $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$ (b) $0 < \theta < \dfrac{\pi}{2}$ (c) $\dfrac{\pi}{2} < \theta < \pi$ (d) $0 < \theta < \pi$

    (e) none of these.

Advisory Exam Statistics

Table I gives the percentage of students answering each of the 25 questions correctly.

Table II gives the relative frequency (R.F.) and cumulative frequency (C.F.) for each of the possible scores (that is, 0 to 25).

Table I	
Question	Percentage
1	54
2	60
3	46
4	31
5	28
6	14
7	28
8	43
9	64
10	31
11	72
12	40
13	64
14	19
15	38
16	20
17	27
18	57
19	34
20	60
21	41
22	52
23	25
24	19
25	40

Table II		
Score	R.F.	C.F.
0	0.6	0.6
1	0.9	1.4
2	1.4	2.8
3	2.8	5.6
4	3.7	9.3
5	5.8	15.2
6	7.0	22.1
7	8.2	30.3
8	9.4	39.7
9	10.4	50.1
10	8.2	58.3
11	8.8	67.1
12	6.3	73.4
13	5.8	79.1
14	4.4	83.5
15	3.2	86.8
16	3.3	90.1
17	2.7	92.7
18	2.0	94.7
19	1.3	96.0
20	0.9	96.9
21	1.2	98.1
22	0.8	98.9
23	0.5	99.4
24	0.5	99.9
25	0.1	100.0