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THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

Volume XXII, Number 1 — October 1982

Mathematics Council, ATA 22nd Annual Conference

October 22 and 23, 1982

Terrace Inn, Edmonton

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THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

Volume XXII, Number 1 **October 1982**

Contents

EDPRESS

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Editorial

A new school year has begun. What will 1982-83 hold for us as teachers of mathematics?

Technology has already made a significant impact upon us, but the impact will be even greater in 1982-83. While there are a few educators who view this as a sinister phenomenon, the majority see real possibilities, especially for microcomputer technology. Unfortunately, progress in the effective use of microcomputer technology will not be as great in 1982-83 as it could be, simply because there is no coordination of efforts. The government is moving at its proverbial snail's pace, and school boards seem unwilling to put a few extra dollars into efforts to coordinate. Consequently, it will be groups of teachers here and there throughout the province who make progress with using the computer effectively in 1982-83. New textbooks have been approved for elementary schools. The impact of the changeover will begin to be felt early in the 1982-83 school year.

According to the "Agenda for Action" (published by the National Council of Teachers of Mathematics), the focus for school mathematics in the 1980s was to be problem solving. Progress has certainly been made in this regard. Alberta Education has built a problem-solving focus into the revised elementary curriculum guide and published a pilot problem-solving booklet. However, its selection of new elementary texts suggests that it has not bought the problem-solving focus completely.

What are high school teachers doing with problem solving? One problem has been published in the "Problem Corner" of each of the last eight issues of *delta-K*. Teachers were asked to have their students solve the problems and send in their solutions, but not one solution has been received. Therefore, money - that great motivator - has been added as an enticement for students to send in solutions. See the "Problem Corner" of this issue for details.

Another event which could significantly affect your teaching in 1982-83 is the MCATA Annual Meeting in Edmonton, October 22 and 23. Plan to attend.

BEST WISHES FOR A PROFESSIONALLY REWARDING YEAR.

\- *George Cathcart*

Announcements

Dr. Tom Atkinson Retires

Dr. Tom Atkinson is retiring after a long and distinguished career as a mathematics teacher and educator.

Tom taught for five years before joining the air force during World War II. After the war, Tom returned to the University of Alberta for further training and then taught for 14 years with the Edmonton Public School Board, 11 of those 14 at Victoria Composite High School. Dr. Atkinson joined the Faculty of Education at the University of Alberta in 1963 and has taught mathematics education and related courses for the past 19 years.

MCATA has benefited from Tom's expertise and energy. Tom was the second president of MCATA and later did some editorial work on Council publications.

A retirement party for Tom Atkinson is planned to coincide with the MCATA Annual Meeting. It will be held in the Papachase Room of the Faculty Club (University Campus) on Saturday, October 23, 1982, at 1930 hours. Tickets are $25 each. This includes dinner, table wine, and a small contribution toward a gift for Tom and Dorothy.

Tickets may be obtained from Will Reese, Department of Elementary Education (432-5417) and will be limited to 90 participants. Make cheques payable to the Department of Elementary Education.

PRISM-Canada Report Available

The Canadian Priorities in School Mathematics (PRISM) study surveyed lay people as well as educators across Canada to determine priorities for school mathematics in the 1980s.

The PRISM-Canada report highlights the findings of this study and states some implications for action in the 1980s.

The study was supported by the Alberta Advisory Committee for Educational Studies, the Faculty of Education, the University of Alberta, the National Council of Teachers of Mathematics, and by the University of Alberta Humanities and Social Sciences Research Fund. Joan Worth, George Cathcart, Tom Kieren, Walter Worth, and Sharon Forth conducted the study.

Copies of the report are available for $8 each from the University of Alberta, Faculty of Education, Publications Office, Room 4-116 Education North, Edmonton, Alberta T6G 2G5.

MCATA 22nd Annual Conference
October 22-23, 1982
Terrace Inn, 4440 – Calgary Trail North, Edmonton

Theme:	"Mathematics: Beyond Four Walls"
Keynote Speaker:	Dr. Gerald A. Goldin Coordinator, Mathematics and Science Education Northern Illinois University
Accommodation:	Please make your own arrangements. Possible hotels and rates (subject to change): Terrace Inn (1-437-6010) $45 single, double, or family Relax Inn $28.95 single/$32.95 double Convention Inn $45 single/double

Conference Registration:

	Before Oct. 8	After Oct. 8
MCATA members	$30	$35
Non-members*	$40	$45
Student members	$10	$10
Student non-members*	$16	$16

*Includes membership.

Luncheon Ticket: $9

Registration forms will be sent to members and schools at the beginning of September.

For further information, contact:

Rod Anderson
3528 - 104 Street
Edmonton, Alberta
Phone (403)435-5580

Alberta Society for Computers in Education
Third Annual Conference
October 28, 29, and 30, 1982

Theme: "The Computer as Teacher's Helper"

Location: Chateau Lacombe Hotel, Edmonton

Preconference workshops will be held on October 28, with the conference taking place on October 29 and 30.

For further information, contact Dr. Gene Romaniuk, President, at 432-3802 or write to the Secretary, ASCE, 352 General Services Building, University of Alberta, Edmonton, Alberta T6G 2H1, or phone 432-4767.

ASCE Programming Contest

It used to be that extracurricular school activities were predictable: history clubs, science clubs, "Reach for the Top" clubs, and the like. But times change. As a result, there are groups of students who are attracted, some would even say addicted, to a new branch of technology - computers. As a result of this interest, schools are forming computer clubs. These clubs, through the efforts of the Alberta Society for Computers in Education, are now competing among themselves. The winners of area and regional contests are invited to compete in an annual invitational computer programming competition.

This year on May 29, at the University of Alberta, 27 students from senior high, junior high, and elementary schools gathered at the Education Building. At the end of the day, one of them returned home $4,000 richer, first prize being an APPLE computer with a disk drive, printer, and monitor. In addition to these prizes, a Commodore VIC20, a pocket computer, a programmable calculator, and a scientific calculator were also given away. A total of 20 different prizes and gifts were given out so that all students received at least four gifts, with 15 students each receiving prizes for his or her programming skills.

Prize winners were:

Senior High School

1st - D. Spitzer	St. Francis High (Calgary)
2nd - M. Tarrabain	M.E. Lazerte (Edmonton)
3rd - S. Bougerolle	Lethbridge Collegiate Institute
C. Mungan	Western Canada High (Calgary)
T. Tucker	Lethbridge Collegiate Institute
R. Sicclari	Lamont
D. Gilleland	Camrose Composite High
D. Hagglund	Ponoka Composite High
G. Teubert	M.E. Lazerte (Edmonton)
S. Slupsky	M.E. Lazerte (Edmonton)
P. Trevor	St. Francis High (Calgary)
G. McKenna	Western Canada High (Calgary)
D. Atkins	Western Canada High (Calgary)

Junior High School

1st - C. Anderson	Barnwell

Elementary School

1st - R. Ohlhauser	Lansdowne Elementary (Edmonton)

Others who competed were:

C. Anderson	Lansdowne Elementary
R. Boker	Barnwell
C. Cathcart	Greenfield Elementary
G. Chow	Fort McMurray Composite High
P. Hagemann	Ponoka Composite High
D. Jones	Ponoka Composite High
K. Mah	Lethbridge Collegiate Institute
S. Morgan	Camrose Composite High
D. Pullor	Western Canada High
L. Tanne	Lethbridge Collegiate Institute
K. McAlpine	Lansdowne Elementary

Prize donors were:

APPLE Canada	APPLE Computer and Diskdrive
Westworld	2 - $20 Gift Vouchers
Gage Publishing Company	1 Subscription
Altel Data (AGT)	1 Subscription
Department of Transport	50 Alberta Buttons
Department of Education	Certificates
Computerland	Centronics 739 Printer
Radio Shack (Mill Woods, Edmonton)	Book
Digital Equipment of Canada	5 Clocks
TJB Microcomputers Systems (Edmonton)	VIC 20 Computer
Data General Limited (Edmonton)	Plaque
University of Alberta	Lunches and Facilities
Galactica Computers (Edmonton)	Monitor
ASCE	Programmable Calculator, Scientific Calculator, TRS-80 Pocket Computer, T-Shirts, Plaques

The competition and registration began at 0800 when the judges started their briefing. After this time, the students checked out their equipment and became familiar with the surroundings. Between 0930 and 1000, rules and regulations of the contest were presented and any uncertainties were clarified. At 1000, the first of four half-hour problems were given out as well as the major problem. The major problem was to be planned and thought about, but no computer work could be carried out until 1400 hours. For the next 30 minutes, the students programmed the computer for the solution to the first problem. In this case, they had to write their names a specified number of times both vertically and horizontally, left to right diagonally, and then right to left diagonally. The next three half-hour problems consisted of: (1) removing all vowels from any word entered, (2) calculating what the money paid for Manhattan Island would be worth today, using various interest rates, and (3) evaluating the total of a series of positive and negative fractions.

The major and last problem of the day was to write a program that had a title page, and a menu. Also required was input key protection routines, internal documentation, and a flowchart of the program.

The day finished at 1730. Everyone was tired and had gained a great deal of appreciation for the programming skills of the students. A few participants were happy to be major prize winners.

Each of the problems was evaluated by two different judges, and the results of both scores were recorded as the student's mark. Scores with wide discrepancies were reviewed by other evaluators. The evaluators were guided by Dr. M. Petruk who did an excellent job of organizing the evaluators and preparing the evaluation forms.

The evaluators were:

Eugene Kozak (St. Rose School)
Hans Kruse (Wetaskiwin)
Dr. Milt Petruk (U of A)
Orson Gadowsky (Lansdowne)
Lyle Jubenville (Eastwood)
Alf Bilton (Dan Knott JHS)
R.E. Busse (NAIT)
Rick Roder (U of A)
Dave Rand (ALPHA ONE)
Judy Dobson (LTCHS)
Dr. George Cathcart (U of A)
Dr. Steve Hunka (U of A)

These people deserve our thanks, and all who worked with them appreciate their contribution to this exercise. They made the day a memorable occasion.

However, the people deserving the most thanks were those on the competition committee, as they spent weeks preparing for this event and spent many Saturday mornings huddled together working out the details. These people were:

Gloria Cathcart (Lansdowne Elementary)
Ray Lautt (M.E. Lazerte)
Eldred Stamp (Ponoka Composite)
Ed Wiecek (Ponoka Composite)

Each person was responsible for a specific task and each fulfilled his task aptly and diligently. The result was that the day's events ran smoothly and without a major problem.

In closing, I would like to say that this competition would not have been so satisfying without the participation of the students, many of whom came over 500 kilometres to attend, without the able assistance of the evaluators, and without the efforts of the organizing committee. So, until we meet again at next year's competition, good luck and good programming. □

Plus + +

The following material is reprinted from Plus ++ *(Volume 2, No. 2, Summer 1982), a short magazine informing mathematics educators across Canada about important events, research, curriculum development, and items of national interest.*

Canadian Mathematics Education Study Group 1982 Meeting

The Sixth Annual Conference of CMESG was held on June 3-7, 1982, at Queen's University, Kingston, Ontario. Major lectures were delivered by Doctors Philip Davis, Professor of Applied Mathematics, Brown University, and Gérard Vergnaud, Maître de Recherches au Centre National de La Recherche Scientifique, Paris. For membership in the study group and for copies of the proceedings of this conference, write to:

Dr. Joe Hillel
Secretary-Treasurer CMESG
Mathematics Department
Concordia University
7141 Sherbrooke Street West
Montreal, Quebec
H4B 1R6

The Second International Mathematics Study – Ontario

Ontario has joined with 24 other "nations" in the International Association for the Evaluation of Educational Achievement's (IEA) Second Study of Mathematics. The Study involves the assessment of students' attitudes toward and achievement in mathematics at two levels: Grades 8 and 12/13. Teachers are also queried on their teaching approaches as well as their attitudes toward mathematics and mathematics learning. Principals provide information on the size and structure of their schools.

Findings Expected from the Study

What will we know when all the data is collected and analyzed? First, we will have an excellent idea of what content is covered in Grade 8 and Grade 12/13 classes, how Ontario teachers convey this material, and how well Ontario students learn it.

Teachers' and students' attitudes toward mathematics and mathematics education will also be recorded, together with how much homework teachers assign and how much time students spend doing such homework. The same information is being collected in 24 other countries.

The linkage between teacher behaviors, student-teacher attitudes, and student achievement will then be explored. Are there certain ways of conveying information in the classroom that lead to greater student achievement? What attitudes do the students bring to the classroom, and to what extent are these attitudes related to teaching and learning in the classroom? Do certain visual or mechanical aids facilitate better attitudes and higher achievement in mathematics?

Student achievement will vary from country to country, of course, but what aspects of schooling are associated most often with high achievement?

It is rare that comparable data are collected around the world to address these questions, and Ontario educators will be poring over the results for the next few years. A conference of mathematics educators is being planned for November 1983 to consider the first results.

For further information, write to:

Dr. Les McLean, Head
Education Evaluation Centre, OISE
252 Bloor Street West
Toronto, Ontario
M5S 1V6

British Columbia Mathematics Assessment 1981

The 1981 Mathematics Assessment was designed to gather a broad range of information from the professional literature, review panels, interpretation panels, a major survey of teachers, and students enrolled in Grades 4, 8, 10 (a sample), and 12.

Five specific goals were established:

- to identify those curriculum models for mathematics which are prevalent in British Columbia and elsewhere;

- to evaluate and report on students' achievement in mathematics and their attitudes toward the subject;

- to assess the extent and direction of change in the pattern of students' achievement since the 1977 Assessment;

- to survey teachers of mathematics on a number of matters which affect the teaching and learning of mathematics;

- to coordinate B.C. participation in the Second International Study of Mathematics.

For complete details of the assessment, write to:

Learning Assessment Branch
Ministry of Education
7451 Elmbridge Way
Richmond, British Columbia
V6X 1B8
Telephone: (604) 278-3433

The Fourth 1982 Beatty Essay Contest

The assessors, Edward J. Barbeau, of the University of Toronto, and William P. Bisset, of A.Y. Jackson Secondary School in North York, are pleased to announce that four essays were deserving of a prize in the fourth annual Samuel Beatty Essay Contest.

It is hoped that the 1981 and 1982 winning essays will be published. The top essays from the first contest have already appeared in print and can be obtained at $4 a copy from Professor E.J. Barbeau, Department of Mathematics, University of Toronto, Toronto, Ontario M5S 1A1.

This is the last Beatty Essay Contest. However, we hope to keep on publishing good student essays in future. Teachers and organizers of other contests are invited to submit the best efforts of their students to E.J. Barbeau, and when we have a sufficient number of suitable essays, we will produce a third volume of Beatty Essays.

Nova Math – A New Journal for the Teachers of Mathematics

This is a new journal produced under the auspices of both the Department of Mathematics, Statistics, and

Computing Science, and the Department of Education at Dalhousie University. The intended audience is primarily the teachers of mathematics at the high school level and those people interested in mathematical education. Our aim is to produce a regional journal composed primarily of contributions from regional authors, paying particular attention to local issues. Other contributions are, of course, welcome. We hope to foster closer cooperation and dialogue between secondary and post-secondary teachers of mathematics. One topic we hope to emphasize is the understanding and use of (micro)computers and calculators, particularly in the classroom. For further information, write to:

The Editors, *Nova Math*
c/o Department of Mathematics
Statistics and Computing Science
Dalhousie University
Halifax, Nova Scotia
B3H 4H8

ICME 5 – August 24-30, 1984 – Adelaide, Australia

The ICME 5 Organizing Committee is pleased to announce that the Fifth International Congress on Mathematical Education will be held in Adelaide from August 24 to 30, 1984.

The program will cover all areas of education and the diverse needs and interests of the participants. Congress activities will include lectures, seminars, workshops, films, poster sessions, and exhibitions of current projects in mathematical education.

Special interest, working, and study groups are invited to meet and to contribute to the congress program. A large exhibition of aids and materials relevant to mathematical education and research is planned to be held in conjunction with the congress.

The congress venue is the University of Adelaide, whose compact campus is a few minutes' walk from the center of Adelaide, a city of 800,000 people.

The main language of communication of the congress is English. Simultaneous translation into several languages is anticipated for some sessions. Translated abstracts or summaries of presented papers are expected to be available.

A complimentary copy of the proceedings of ICME 5 will be sent to each registered full member of the congress.

THE SECOND ANNOUNCEMENT will be available in Fall 1983.

The firm of Travel Planners, Inc., located in San Antonio, Texas, has been appointed as the official North American coordinator for U.S. and Canadian delegates attending the ICME 5. For further information, write to:

ICME 5 Travel Planners
P.O. Box 32366
San Antonio, Texas 78216
Telephone: (512) 341-8131

In 1980, over 2,100 participants from more than 90 countries attended ICME 4 in Berkeley, California. □

Summing Sets of Odd Integers: Patterns in Powers

Bonnie H. Litwiller and David R. Duncan
Professors of Mathematics
University of Northern Iowa, Cedar Falls, Iowa

Teachers are constantly searching for activities that will aid in developing skills in computation and the use of calculators. It is motivational if number patterns can also be discovered.

Throughout this article we will use the set of positive odd integers (1, 3, 5, 7, ...). We will present several activities which are based upon sums of different sets of these integers.

Activity I

A rather well-known number pattern results from summing the first n odd integers for different values of n as shown in Table 1.

Table 1.

Indicated Sum	No. of Odd Integers in the Sum	Sum
1	1	$1 = 1^2$
1+3	2	$4 = 2^2$
1+3+5	3	$9 = 3^2$
1+3+5+7	4	$16 = 4^2$
1+3+5+7+9	5	$25 = 5^2$
.	.	.
.	.	.
.	.	.

In general the sum of the first n odd integers $[1+3+5+...+(2n-1)]$ yields n^2. The proof of this is found in chapters on mathematics induction in many mathematics texts.

Activity II

To generate cubes, again add consecutive odd integers. These sums are organized quite differently from those of Activity I. See Table 2.

Table 2.

Indicated Sum	No. of Odd Integers in the Sum	Sum
1	1	$1=1^3$
3+5	2	$8=2^3$
7+9+11	3	$27=3^3$
13+15+17+19	4	$64=4^3$
21+23+25+27+29	5	$125=5^3$
.	.	.
.	.	.
.	.	.

In general the sum of n appropriately chosen consecutive odd integers is n^3. In contrast to Activity I, these summed sets do not all begin at 1. In Activity II, each summed set begins immediately after the previous summed set concludes. No odd integer is used in more than one summed set. Because the summed sets partition the set of odd integers, call this the "partition sum method."

Let us verify the correctness of this summing procedure. We wish to sum n consecutive odd integers, so we need to determine the first odd integer to be summed. To find 1^3, 2^3, 3^3, ..., $(k-1)^3$ requires the summing of 1 odd integer, 2 odd integers, 3 odd integers, ..., (k-1) odd integers. Consequently, to find 1^3, 2^3, 3^3, ... , $(k-1)^3$ requires $1+2+3+...+(k-1)$ odd integers; that is, the first $\frac{(k-1)(k)}{2}$ odd integers have already been used in sums.

The last odd integer used in summing to find $(k-1)^3$ was $2(\frac{(k-1) \cdot k}{2}) -1$ or (k^2-k-1). The first odd integer to be summed to find k^3 is therefore $(k^2-k-1) + 2$ or (k^2-k+1). Since k odd integers are to be summed to generate k^3, the odd integers are:

$$\underset{\text{1st}}{\underline{k^2-k+1}}, \quad \underset{\text{2nd}}{\underline{k^2-k+3}}, \quad \underset{\text{3rd}}{\underline{k^2-k+5}}, \quad \ldots, \quad \underset{\text{4th}}{\underline{k^2-k+(2k-1)}}$$

This sum is thus:

$$\underbrace{(k^2-k+1) + (k^2-k+3) + (k^2-k+5) +\ldots+ (k^2-k+(2k-1))}_{\text{k terms}}$$

$$= k(k^2-k) + [1+3+5+\ldots+(2k-1)]$$

$$= k^3-k^2 + [1+3+5+\ldots+(2k-1)]$$

$$= k^3-k^2 + k^2 \quad \text{(Activity I)}$$

$$= k^3.$$

Activity III

To generate the fourth powers, sum the sets as shown in Table 3.

Table 3.

Indicated Sum	No. of Odd Integers in the Sum	Sum
1	1	$1 = 1^4$
1+(3+5+7)	4	$16 = 2^4$
1+(3+5+7) + (9+11+13+15+17)	9	$81 = 3^4$
1+(3+5+7) + (9+11+13+15+17)+ (19+21+23+25+27+29+31)	16	$256 = 4^4$
1+(3+5+7) + (9+11+13+15+17)+ (19+21+23+25+27+29+31)+ (33+35+37+39+41+43+45+47+49)	25	$625 = 5^4$
.	.	.
.	.	.
.	.	.

Activity III resembles Activity I in that all summed sets begin with 1. It differs from Activity I in the number of odd integers summed. In Activity I, n^2 was found by summing the first n odd integers; in this activity, n^4 is found by summing the first n^2 odd integers.

Algebraically, this pattern is easy to verify. The sum of the first n^2 odd integers is $1+3+5+...+(2n^2-1)$. By the results of Activity I, this sum is $(n^2)^2 = n^4$.

In general, this method enables us to generate all the even powers. To find n^{2k}, sum the first n^k odd integers, producing $1+3+5...+(2n^k-1) = (n^k)^2 = n^{2k}$.

For example, $3^8 = 1+3+5+...+(2 \cdot 3^4-1) = 1+3+5+...+ 161 = 6561$.

Activity IV

To generate the fifth powers, sum the disjoint sets of odd integers as shown in Table 4.

Table 4.

Indicated Sum	No. of Odds Used	Odd Integers Skipped	No. of Odds Skipped	Sum
1	1	None	0	$1 = 1^5$
5+7+9+11	4	3	1	$32 = 2^5$
19+21+23+25+...+35	9	13,15,17	3	$243 = 3^5$
49+51+53+...+79	16	37,39,...,47	6	$1024 = 4^5$
101+103+105+...+149	25	81,83,...,99	10	$3125 = 5^5$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

Activity IV resembles Activity II in that disjoint sums are used. It differs from Activity II in the following ways:

1. The number of odd integers summed to achieve n^5 is n^2 (to achieve n^3, add n odd integers).

2. Odd integers are skipped between summed sets. The number of odd integers skipped is 1, 3, 6, 10, ... ; this is the set of triangular numbers.

A more general pattern connecting Activities I and II and Activities III and IV may be noticed. In generating either n^2 or n^3, n consecutive odd integers are summed. The summed sets which generate n^2 all begin with 1, while the summed sets which generate n^3 are disjoint.

In generating either n^4 or n^5, n^2 consecutive odd integers are summed. The summed sets generating n^4 all begin with 1, while the summed sets which generate n^5 are disjoint.

Challenges for the Reader

1. We know that the pattern for generating n^6 is to sum the first n^3 odd integers where the summed sets all start with 1 (Activity III). From the relationship just noted among Activities I through IV, it might be conjectured that values of n^7 could be found by summing disjoint sets of n^3 odd integers.
   a) Is this conjecture true?
   b) If the conjecture is true, how many odd integers must be skipped between the summed sets?

2. Generate other odd powers, and conjecture a general pattern for these cases.

□

Product Differences of Consecutive Factors

David R. Duncan and Bonnie H. Litwiller
Professors of Mathematics
University of Northern Iowa, Cedar Falls, Iowa

Consider 3-factor products in the following form:

$n(n + a)(n + 2a)$
$(n + a)(n + 2a)(n + 3a)$
$(n + 2a)(n + 3a)(n + 4a)$
$(n + 3a)(n + 4a)(n + 5a)$
.
.
.

Table 1 reports the results of several 3-factor products. In each case, the third differences are constant.

Table 1.

Factors differ by 1

1 x 2 x 3 = 6			
	18		
2 x 3 x 4 = 24		18	
	36		6
3 x 4 x 5 = 60		24	
	60		6
4 x 5 x 6 = 120		30	
	90		6
5 x 6 x 7 = 210		36	
	126		6
6 x 7 x 8 = 336		42	
	168		
7 x 8 x 9 = 504			

Factors differ by 3

3 x 6 x 9 = 162			
	486		
6 x 9 x 12 = 648		486	
	972		162
9 x 12 x 15 = 1620		648	
	1620		162
12 x 15 x 18 = 3240		810	
	2430		162
15 x 18 x 21 = 5670		972	
	3402		162
18 x 21 x 24 = 9072		1134	
	4536		
21 x 24 x 27 = 13608			

Factors differ by 2

1 x 3 x 5 = 15			
	90		
3 x 5 x 7 = 105		120	
	210		48
5 x 7 x 9 = 315		168	
	378		48
7 x 9 x 11 = 693		216	
	594		48
9 x 11 x 13 = 1287		264	
	858		48
11 x 13 x 15 = 2145		312	
	1170		
13 x 15 x 17 = 3315			

Factors differ by 4

3 x 7 x 11 = 231			
	924		
7 x 11 x 15 = 1155		1056	
	1980		384
11 x 15 x 19 = 3135		1440	
	3420		384
15 x 19 x 23 = 6555		1824	
	5244		384
19 x 23 x 27 = 11799		2208	
	7452		384
23 x 27 x 31 = 19251		2592	
	10044		
27 x 31 x 35 = 29295			

Table 2 reports the factor differences and the third differences between products.

Table 2.

Factor Difference (a)	*Third Product Differences*
1	$6 = 6(1)^3$
2	$48 = 6(2)^3$
3	$162 = 6(3)^3$
4	$384 = 6(4)^3$

From the results of Table 2, Conjecture I is made. Consider the following 3-factor products:

$n(n + a)(n + 2a)$
$(n + a)(n + 2a)(n + 3a)$
$(n + 2a)(n + 3a)(n + 4a)$
$(n + 3a)(n + 4a)(n + 5a)$
.
.
.

Then, the third differences between consecutive products are $6a^3$.

PROOF — Select four consecutive products of the following form:

$$P_1 = (n + ai)(n + a(i+1))(n + a(i+2))$$
$$P_2 = (n + a(i+1))(n + a(i+2))(n + a(i+3))$$
$$P_3 = (n + a(i+2))(n + a(i+3))(n + a(i+4))$$
$$P_4 = (n + a(i+3))(n + a(i+4))(n + a(i+5))$$

Then,

$$\begin{aligned} D_1 &= P_2 - P_1 \\ &= (n + a(i+1))(n + a(i+2))(n + a(i+3)) - (n + ai)(n + a(i+1))(n + a(i+2)) \\ &= (n + a(i+1))(n + a(i+2))[(n+a(i+3)) - (n + ai)] \\ &= (n + a(i+1))(n + a(i+2))[3a] \end{aligned}$$

$$\begin{aligned} D_2 &= P_3 - P_2 \\ &= (n + a(i+2))(n + a(i+3))(n + a(i+4)) - (n + a(i+1))(n + a(i+2))(n + a(i+3)) \\ &= (n + a(i+2))(n + a(i+3))[3a] \end{aligned}$$

$$\begin{aligned} D_3 &= P_4 - P_3 \\ &= (n + a(i+3))(n + a(i+4))(n + a(i+5)) - (n + a(i+2))(n + a(i+3))(n + a(i+4)) \\ &= (n + a(i+3))(n + a(i+4))[3a] \end{aligned}$$

$$SD_1 = D_2 - D_1$$
$$= (n + a(i+2))(n + a(i+3))[3a] - (n + a(i+1))(n + a(i+2))[3a]$$
$$= (3a)(n + a(i+2))[2a]$$
$$= 6a^2(n + a(i+2))$$

$$SD_2 = D_3 - D_2$$
$$= (n + a(i+3))(n + a(i+4))[3a] - (n + a(i+2))(n + a(i+3))[3a]$$
$$= 6a^2(n+a(i+3))$$

$$TD = SD_2 - SD_1$$
$$= 6a^2(n + a(i+3)) - 6a^2(n + a(i+2))$$
$$= 6a^2(a)$$
$$= 6a^3$$

The third difference, $6a^3$, depends only upon the difference between consecutive factors and not upon the size of the factors themselves. Thus, Conjecture I is established.

Now consider 4-factor products of the form:

$n(n + a)(n + 2a)(n + 3a)$
$(n + a)(n + 2a)(n + 3a)(n + 4a)$
$(n + 2a)(n + 3a)(n + 4a)(n + 5a)$
$(n + 3a)(n + 4a)(n + 5a)(n + 6a)$
$(n + 4a)(n + 5a)(n + 6a)(n + 7a)$

.
.
.

Table 3 reports results of 4-factor products for a = 1, 2, 3, 4.

Table 3.

2 x 3 x 4 x 5 = 120				
	240			
3 x 4 x 5 x 6 = 360		240		
	480		120	
4 x 5 x 6 x 7 = 840		360		24
	840		144	
5 x 6 x 7 x 8 = 1680		504		24
	1344		168	
6 x 7 x 8 x 9 = 3024		672		24
	2016		192	
7 x 8 x 9 x 10 = 5040		864		
	2880			
8 x 9 x 10 x 11 = 7920				

3 x 5 x 7 x 9 = 945				
	2,520			
5 x 7 x 9 x 11 = 3,465		3,024		
	5,544		1,728	
7 x 9 x 11 x 13 = 9,009		4,752		384
	10,296		2,112	
9 x 11 x 13 x 15 = 19,305		6,864		384
	17,160		2,496	
11 x 13 x 15 x 17 = 36,465		9,360		384
	26,520		2,880	
13 x 15 x 17 x 19 = 62,985		12,240		
	38,760			
15 x 17 x 19 x 21 = 101,745				

1 x 4 x 7 x 10 = 280				
	3,360			
4 x 7 x 10 x 13 = 3,640		7,560		
	10.920		6,480	
7 x 10 x 13 x 16 = 14,560		14,040		1,944
	24,960		8,424	
10 x 13 x 16 x 19 = 39,520		22,464		1,944
	47,424		10,368	
13 x 16 x 19 x 22 = 86,944		32,832		1,944
	80,256		12,312	
16 x 19 x 22 x 25 = 167,200		45,144		
	125,400			
19 x 22 x 25 x 28 = 292,600				

2 x 6 x 10 x 14 = 1,680				
	13,440			
6 x 10 x 14 x 18 = 15,120		26,880		
	40,320		21,504	
10 x 14 x 18 x 22 = 55,440		48,384		6,144
	88,704		27,648	
14 x 18 x 22 x 26 = 144,144		76,032		6,144
	164,736		33,792	
18 x 22 x 26 x 30 = 308,880		109,824		6,144
	274,560		39,936	
22 x 26 x 30 x 34 = 583,440		149,760		
	424,320			
26 x 30 x 34 x 38 = 1,007,760				

The fourth differences are constant in each case. Table 4 shows the factor differences and the fourth differences of the products for each case.

Table 4.

Factor Difference (a)	*Fourth Product Difference*
1	$24 = 24(1)^4$
2	$384 = 24(2)^4$
3	$1944 = 24(3)^4$
4	$6144 = 24(4)^4$

An examination of Table 4 yields Conjecture II. Consider the following 4-factor products:

$n(n + a)(n + 2a)(n + 3a)$
$(n + a)(n + 2a)(n + 3a)(n + 4a)$
$(n + 2a)(n + 3a)(n + 4a)(n + 5a)$
$(n + 3a)(n + 4a)(n + 5a)(n + 6a)$
$(n + 4a)(n + 5a)(n + 6a)(n + 7a)$
.
.
.

Then, the fourth differences between consecutive products are $24a^4$.

PROOF — Select 4-factor products of the following form:

$$P_1 = (n + ai)(n + a(i+1))(n + a(i+2))(n + a(i+3))$$
$$P_2 = (n + a(i+1))(n + a(i+2))(n + a(i+3))(n + a(i+4))$$
$$P_3 = (n + a(i+2))(n + a(i+3))(n + a(i+4))(n + a(i+5))$$
$$P_4 = (n + a(i+3))(n + a(i+4))(n + a(i+5))(n + a(i+6))$$
$$P_5 = (n + a(i+4))(n + a(i+5))(n + a(i+6))(n + a(i+7))$$

Then, $D_1 = P_2 - P_1$
$= (n + a(i+1))(n + a(i+2))(n + a(i+3))[4a]$

$D_2 = P_3 - P_2$
$= (n + a(i+2))(n + a(i+3))(n + a(i+4))[4a]$

$D_3 = P_4 - P_3$
$= (n + a(i+3))(n + a(i+4))(n + a(i+5))[4a]$

$D_4 = P_5 - P_4$
$= (n + a(i+4))(n + a(i+5))(n + a(i+6))[4a]$

$SD_1 = D_2 - D_1$
$= (4a)[(n + a(i+2))(n + a(i+3))](3a)$
$= 12a^2[(n + a(i+2))(n + a(i+3))]$

$SD_2 = D_3 - D_2$
$= 12a^2[(n + a(i+3))(n + a(i+4))]$

$SD_3 = D_4 - D_3$
$= 12a^2[(n + a(i+4))(n + a(i+5))]$

$TD_1 = SD_2 - SD_1$
$= 12a^2(n + a(i+3))[2a]$
$= 24a^3(n + a(i+3))$

$$TD_2 = SD_3 - SD_2$$
$$= 24a^3(n + a(i+4))$$

$$FD = TD_2 - TD_1$$
$$= 24a^3(a)$$
$$= 24a^4$$

Thus, Conjecture II is established.

Challenges for the Reader

1. We have investigated products resulting from 3-factor and 4-factor combinations in which the factors differed by a fixed amount, and the second factor of any given product became the first factor in the next product. Extend the patterns to include 5-factor products, 6-factor products, . . . Table 5 should result from your computations, where a is the difference between factors.

Table 5.

Number of Factors (j)	j^{th} *Product Differences*
3	$6a^3 = 3!a^3$
4	$24a^4 = 4!a^4$
5	$120a^5 = 5!a^5$
6	$720a^6 = 6!a^6$
.	.
.	.
.	.

2. The results of Table 5 may be generalized by Conjecture III. Consider the following j-factor products:

$$n(n + a)(n + 2a)(n + 3a) \;.\;.\;.\; (n + (j-1)a)$$
$$(n + a)(n + 2a)(n + 3a)(n + 4a) \;.\;.\;.\; (n + ja)$$
$$(n + 2a)(n + 3a)(n + 4a)(n + 5a) \;.\;.\;.\; (n + (j+1)a)$$
$$\vdots \qquad\qquad \vdots$$

Then, the j^{th} product differences are all $j!(a)^j$. Prove this conjecture.

3. In each of the examples of this article, the second factor of any given product became the first factor of the next product. What happens if the third (fourth, fifth, . . .) factor of a given product becomes the first factor of the next product and then consecutive differences (third, fourth, fifth, . . .) are found? □

An Advisory Exam in Mathematics

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The Mathematics Department at the University of Alberta gave the following 50-minute examination to the 2,500 students in its introductory calculus courses at the beginning of the 1981 fall term. Those students whose background appeared weak were advised to attend a five-week remedial program along with their calculus course. In spite of the difficulties inherent in such a voluntary, no-credit program, the student response was really gratifying. Those students who had not taken Mathematics 30 recently were particularly appreciative of this opportunity to refresh the concepts involved. A lack of proficiency with the fundamentals is a major handicap in the calculus courses.

The examination is given below and is followed by some of its statistics. The mean score was 10.08. Students who had taken both Mathematics 30 and Mathematics 31 fared significantly better than those with only Mathematics 30 on the advisory exam and in their calculus course.

---

1. If $S = am + (m-1)d$, then $m =$

   (a) $\frac{S+d}{a+1}$ (b) $\frac{S+d}{a+d}$ (c) $\frac{S-d}{a+d}$ (d) $-\frac{S+d}{a+1}$ (e) none of these.

2. $\frac{x^2-y^2}{2xy} \div \frac{x^2+2xy+y^2}{y^2} =$

   (a) $\frac{y}{2x}$ (b) $-\frac{y}{2x}$ (c) $\frac{y(x-y)}{2x(x+y)}$ (d) $\frac{y}{2x}\cdot\left(\frac{x-y}{x+y}\right)^2$ (e) none of these.

3. $\frac{2x^4-x^3+x-2}{2x^2-x+2} =$

   (a) x^2-1 (b) x^2+1 (c) x^2+x-1 (d) x^2-x+1 (e) none of these.

4. $\frac{1}{\sqrt{x}+\sqrt{y}} =$

   (a) $\frac{1}{\sqrt{x+y}}$ (b) $x^{-\frac{1}{2}} + y^{-\frac{1}{2}}$ (c) $\sqrt{x}-\sqrt{y}$ (d) $\frac{\sqrt{x}-\sqrt{y}}{x-y}$ (e) none of these.

5. $x\sqrt{x} - 2\sqrt{x} + x^{-\frac{1}{2}} =$

(a) $x-2$ (b) $\frac{x-1}{\sqrt{x}}$ (c) $\left(1-\frac{1}{x}\right)^2\sqrt{x}$ (d) $\frac{(x-1)^2}{\sqrt{x}}$ (e) none of these.

6. $\frac{\sqrt[3]{p^2q}}{\sqrt{pq^3}} =$

(a) $\frac{\sqrt[6]{pq^5}}{q^2}$ (b) $\frac{(pq)^{\frac{1}{6}}}{q^2}$ (c) $\frac{(pq^5)^{\frac{1}{6}}}{q}$ (d) $\frac{p}{q}\sqrt[6]{pq^2}$ (e) none of these.

7. If $2x^2-4x+5$ is written as $a(x-h)^2+k$, then

(a) $a=2$ and $k=\frac{3}{2}$ (b) $a=2$ and $k=3$ (c) $a=2$ and $k=5$ (d) $a=2$ and $k=9$

(e) none of these.

8. The solution set of $x(x+2)=1$ is

(a) $\{-1\}$ (b) $\{-1+\sqrt{2}\,i, -1-\sqrt{2}\,i\}$ (c) $\{-1+\sqrt{2}, -1-\sqrt{2}\}$ (d) $\{1\}$

(e) none of these.

9. $x(9x-2) \leq (3x+1)^2$ is equivalent to

(a) $-\frac{1}{8} \leq x$ (b) $-\frac{1}{5} \leq x$ (c) $-\frac{1}{2} \leq x$ (d) $-8 \leq x$ (e) none of these.

10. $|x-1| < \frac{1}{10}$ is equivalent to

(a) $\frac{-11}{10} < x < \frac{11}{10}$ (b) $\frac{-9}{10} < x < \frac{9}{10}$ (c) $-\frac{9}{10} < x < \frac{11}{10}$ (d) $\frac{-11}{10} < x < \frac{9}{10}$

(e) none of these.

11. If $x=-1$ and $y=3$, then $\frac{|x-y|}{|x|-|y|} =$

(a) -1 (b) 1 (c) 2 (d) -2 (e) none of these.

12. The domain of $f(x) = \sqrt{x(1-x)}$ consists of all real numbers x such that

(a) $0 \leq x$ (b) $x \leq 0$ or $x \geq 1$ (c) $x \leq 1$ (d) $0 \leq x \leq 1$ (e) none of these.

13. If the graph of $f(x) = -x^3+2kx^2-\frac{3}{2}kx+1$ contains the point $(2,3)$, then **k** =

(a) 0 (b) -1 (c) $-\frac{6}{5}$ (d) -2 (e) none of these.

14. If $f(x) = \frac{1}{x}$, then $\frac{f(1+h)-f(1)}{h} =$

(a) $\frac{-1}{1+h}$ (b) $\frac{1}{h^2}$ (c) $\frac{1}{1+h}$ (d) $\frac{1}{h(1+h)}$ (e) none of these.

15. $\log_a\left(\frac{8x^3}{\sqrt{y}}\right) =$

(a) $(\log_a 2x)^3 - (\log_a y)^{\frac{1}{2}}$ (b) $\log_a 8 + 3\log_a x - \frac{1}{2}\log_a y$

(c) $24\log_a x - \frac{1}{2}\log_a y$ (d) $3\log_a(2x) + \frac{1}{2}\log_a y$ (e) none of these.

16. If $\log_2(x-1) = 3$, then $x =$

(a) 8 (b) 10 (c) 7 (d) -7 (e) none of these.

17. Which of the following is true for the graph of $y = 10^x$?

(a) It crosses every line $y = k$ exactly once where k is any positive constant.
(b) It intersects the y-axis at (0,10).
(c) It is symmetric across the y-axis.
(d) It crosses the x-axis exactly once.
(e) none of these.

18. The equation of the line passing through (1,2) and parallel to the line through (3,-2) and (4,-1) is

(a) $y=2x$ (b) $y=-3x-1$ (c) $y=x+1$ (d) $y=3x-1$ (e) none of these.

19. The set of points P(x,y) that are 5 units from (5,-6) is given by

(a) $x^2+y^2-10x+12y-16=0$ (b) $x^2+y^2-10x+12y+36=0$ (c) $x^2+y^2-10x+12y-36=0$

(d) $x^2+y^2+10x-12y+36=0$ (e) none of these.

20. The graph of $y=16-9x^2$ has the following two properties

(a) it opens downward and its vertex is at (0,16) .
(b) it opens to the right and its vertex is at $(-\frac{4}{3}, 0)$.
(c) it opens to the left and its vertex is at $(\frac{4}{3}, 0)$.
(d) it opens upward and its vertex is at (0,16) .
(e) none of these.

21. The degree measure equivalent to the radian measure $-\frac{7\pi}{4}$ is

(a) -315° (b) 45° (c) -225° (d) 405° (e) none of these.

22. If $\tan\theta = \frac{\sqrt{5}}{2}$ and $\sec\theta = \frac{3}{2}$, then $\sin\theta =$

(a) $-\frac{\sqrt{5}}{3}$ (b) $\frac{4}{3\sqrt{5}}$ (c) $\frac{\sqrt{5}}{3}$ (d) $\frac{3\sqrt{5}}{4}$ (e) none of these.

23. $\sin\pi\cos\frac{\pi}{2} - \cos\pi\sin\frac{\pi}{2} =$

(a) $\sin\frac{\pi}{2}$ (b) $\sin\frac{3\pi}{2}$ (c) $\cos\frac{\pi}{2}$ (d) $\cos\frac{3\pi}{2}$ (e) none of these.

24. $\frac{1}{\sin^2\theta} - 1 =$

(a) $\tan^2\theta$ (b) $\sec^2\theta$ (c) $\csc^2\theta$ (d) $\cot^2\theta$ (e) none of these.

25. For which of the following intervals is $f(\theta) = \sin\theta$ both positive and increasing?

(a) $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ (b) $0 < \theta < \frac{\pi}{2}$ (c) $\frac{\pi}{2} < \theta < \pi$ (d) $0 < \theta < \pi$

(e) none of these.

Advisory Exam Statistics

Table I gives the percentage of students answering each of the 25 questions correctly.

Table II gives the relative frequency (R.F.) and cumulative frequency (C.F.) for each of the possible scores (that is, 0 to 25).

Table I

Question	Percentage
1	54
2	60
3	46
4	31
5	28
6	14
7	28
8	43
9	64
10	31
11	72
12	40
13	64
14	19
15	38
16	20
17	27
18	57
19	34
20	60
21	41
22	52
23	25
24	19
25	40

Table II

Score	R.F.	C.F.
0	0.6	0.6
1	0.9	1.4
2	1.4	2.8
3	2.8	5.6
4	3.7	9.3
5	5.8	15.2
6	7.0	22.1
7	8.2	30.3
8	9.4	39.7
9	10.4	50.1
10	8.2	58.3
11	8.8	67.1
12	6.3	73.4
13	5.8	79.1
14	4.4	83.5
15	3.2	86.8
16	3.3	90.1
17	2.7	92.7
18	2.0	94.7
19	1.3	96.0
20	0.9	96.9
21	1.2	98.1
22	0.8	98.9
23	0.5	99.4
24	0.5	99.9
25	0.1	100.0

□

Dissection-Transformation Activities: Rectangulations in the Plane

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This article is dedicated to the Canadian Mathematics Education Study Group/ Geometry Subgroup.

"Geometry is the study of space and spatial relationships."

Geometry is a subject rich in history and broad in scope. It was developed from practical activities and from the problems of daily life. The properties of geometric concepts as well as the concepts themselves have been abstracted from the world around us. It was necessary for people to draw many straight lines before they developed the axiom that a straight line can be drawn through any two arbitrary distinct points. They had to move various plane regions about and apply them to one another on many occasions before they could come to generalize their experiences to the notion of superposition of plane regions and employ this notion for the proof of theorems. This was done in the famous theorems about the congruence of traingular regions.

The following are some hand-mind investigations which might offer students straightforward physical experiences vital to formal conceptions of some goemetric ideas.

Objectives

1. To develop a general pattern in dissecting a triangular region into a rectangular region of equal area.

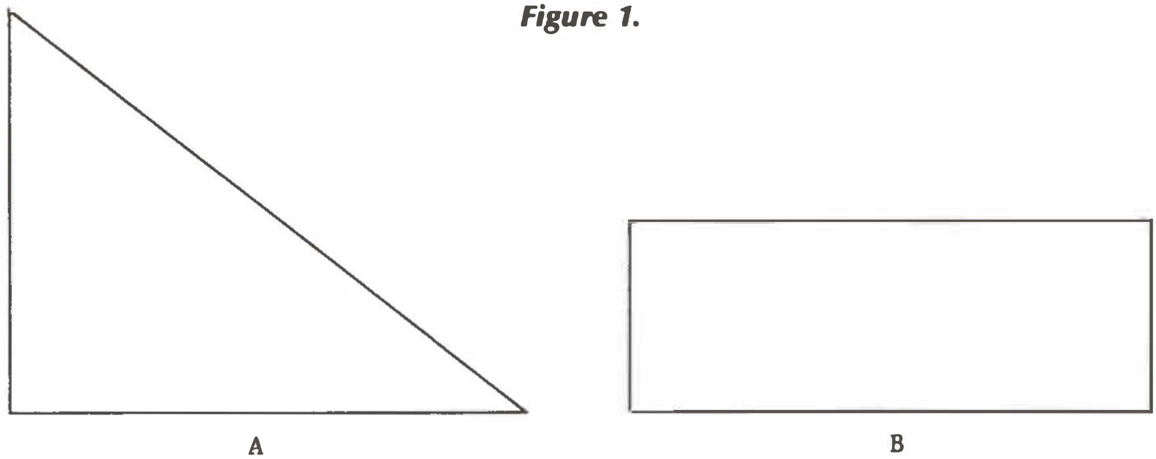
2. To develop the ability to distinguish between the perimeter and area of a given triangular region and to correct the common misconception that area is measured by the perimeter.

3. To expose interrelationships that exist among various polygnal regions.

Investigation I

A is a right triangular region and B is a rectangular region.

Figure 1.



There are certain relations between A and B related to their perimeters, areas, and corresponding sides. To investigate and identify them:

1. Trace and cut out a copy for each region. Dissect region A into pieces (two, three, or more; use the fewest pieces possible) and try to cover region B completely.

   Use = , < , or > to complete the following statement:

   Area of A _____ Area of B

2. Measure the sides of each region and calculate their perimeters. Use = , < , or > to complete the following statement:

   Perimeter of A ____ Perimeter of B.

3. Complete the following statement:

   A right triangular region and a rectangular region of ____ areas can be of ____ perimeters.

Instructional Suggestions:

1. Clues and leading questions may be given based on the following facts:

   a) There is a linear congruency between the bases (this is crucial to recognize).
   b) The altitude of A equals twice the altitude of B, and hence the midpoint concept is vital.
   c) In particular, the midpoints of the altitude and the hypotenuse are to be identified, and cutting through the line joining them is the key to a successful dissection.
   d) A half-turn motion is involved.

2. The dissection of A into B below can be looked at as:

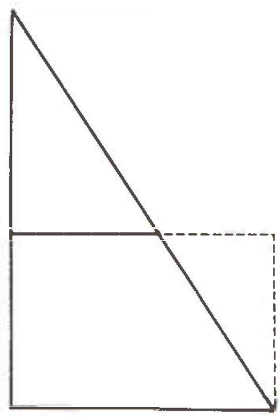


Figure 2.

A dissection of one-half of an isosceles triangular region and a dissection of an isosceles triangular region into an area-equivalent rectangular region would follow where the third side is taken as a base.

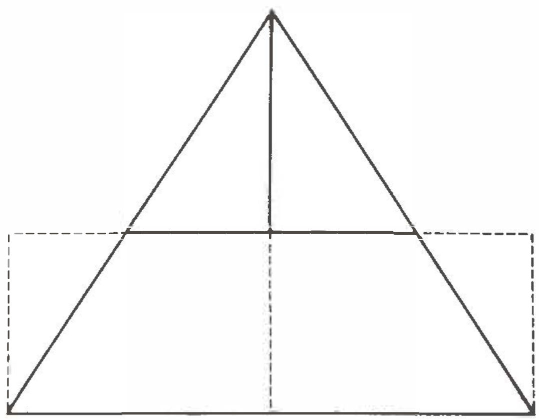


Figure 3.

3. The dissection of A into B can be used further to dissect any triangular region into an area-equivalent rectangular region, provided that in the obtuse case, the longest side is to be taken as a base.

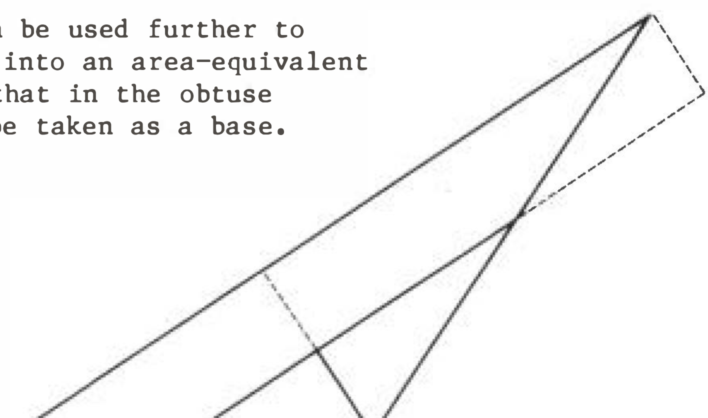


Figure 4.

Investigation II

Below are three copies of a right triangular region with the lengths of the sides as indicated.

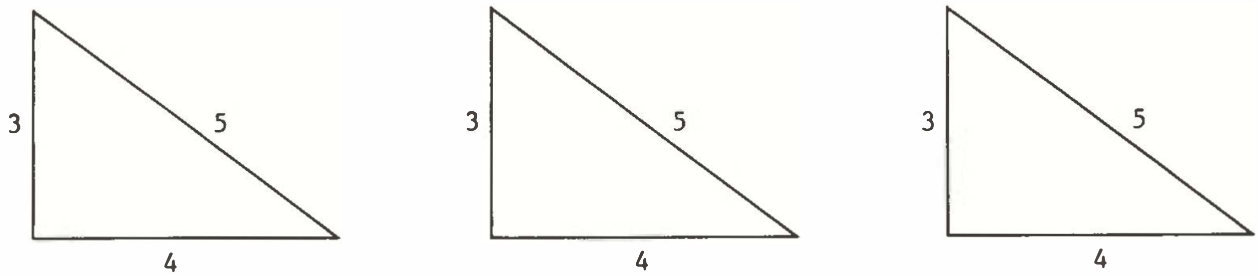


Figure 5.

1. Dissect each region into a rectangular region such that a different side at a time is taken as a base for both the proposed rectangular region and the triangular region.

2. Complete the following table:

	First Copy	Second Copy	Third Copy
Base of □ Altitude of □	4	3	5
Base of ◺ Altitude of ◺			
Perimeter of □ Perimeter of ◺			
Area of □ Area of ◺			

3. Using the data in the table, choose a word from those in the brackets to make a valid statement out of the following statement:

   While the areas for the two regions are (equal/not equal), the perimeters inscribed by them are (varied/not varied).

4. Justify your statement by numerical evidences from the table.

5. Accordingly, make a valid conclusion out of the following:

   The perimeter of a region (can/cannot) be a measure for the inscribed area.

Instructional Suggestions:

1. For the case where the hypotenuse is taken as a base for the proposed rectangular region, it is quicker to calculate the altitude of the resultant rectangular region by:

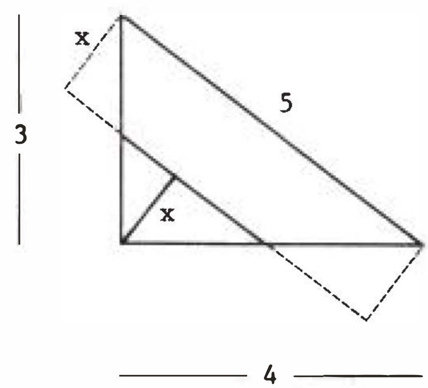
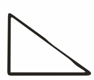


Figure 6.

Area of  = Area of  (by dissection)

$$5(x) = \frac{1}{2}(4)(3)$$

$$x = 1.2$$

rather than by the Pythagorean Theorem:

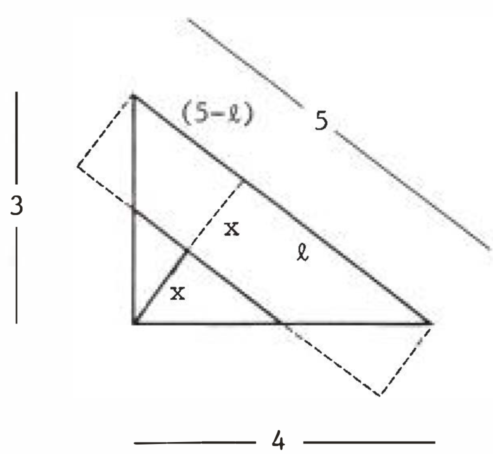


Figure 7.

$$(2x)^2 + \ell^2 = 4^2$$

$$(2x)^2 + (5 - \ell)^2 = 3^2 \qquad \text{and so on.}$$

2. The table above indicates that while the areas for the two regions in all cases are equal (6 area units), the perimeters inscribed by them vary (taking the values 10, 11, 12, or 12.4 length units). *Therefore, the perimeter cannot be a measure for the inscribed area.*

3. Clues and leading questions related to such key concepts as midpoints and the type of motion are to be offered throughout the instructional period so that continuous work will be maintained. In the third copy, for example, and depending on the level of the class, the corresponding altitude of the triangular region (2.4 length units) may be given.

Investigation III

A, B, C, and D are triangular regions. Trace, cut out copies, and then use for the following:

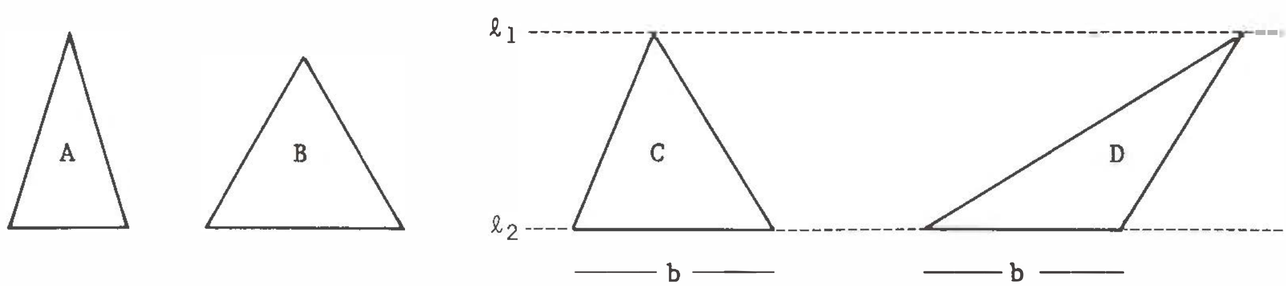


Figure 8.

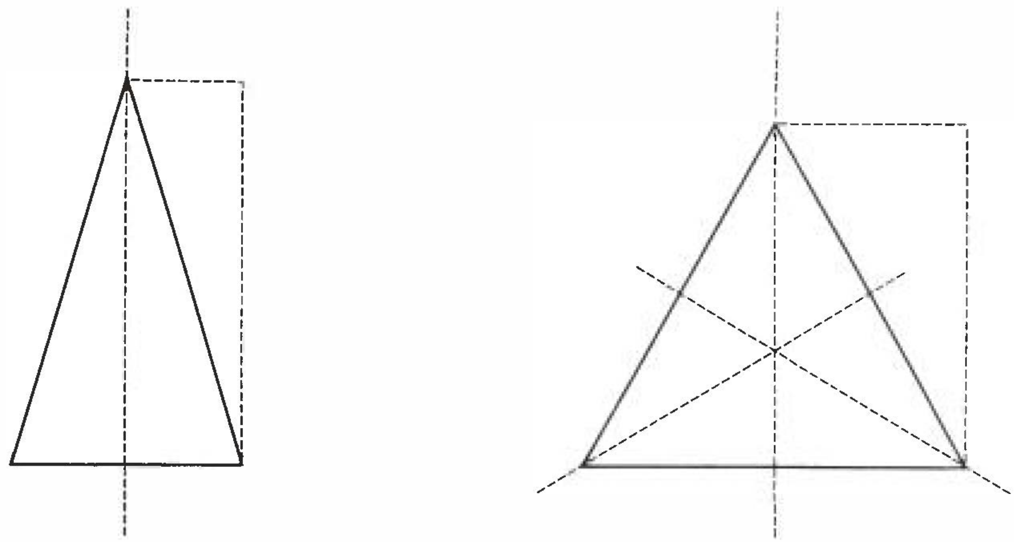
1. Dissect each of them into a rectangular region of equal area. (Use the fewest pieces possible.)

2. Dissect each of A and B into an area-equivalent rectangular region using only one cut. How many lines of symmetry does A have? Does B have? You may use these lines.

3. Dissect C and D into area-equivalent rectangular regions with base = b length units. Note that $\ell_1 \parallel \ell_2$. Are the resulting rectangular regions congruent? What can you tell, then, about the areas of the triangular regions? How many lines of symmetry does C have? Does D have?

Instructional Suggestions:

1. The concept of symmetry may be introduced throughout the dissection procedures of A and B where the lines of symmetry would give simple cuts for A and B in terms of the number of pieces used.



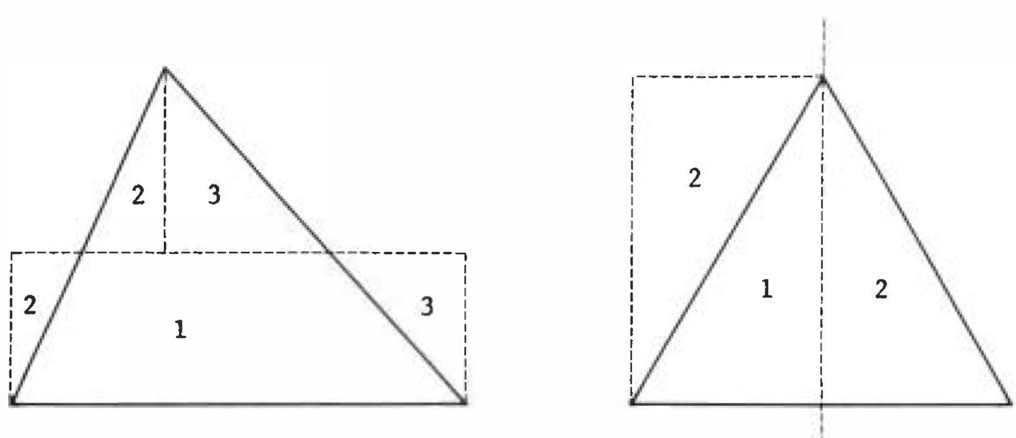
One line of symmetry indicates one option for the cutting.

The lines of symmetry indicate three options for the cutting.

Figure 9.

2. Clues and leading questions on some involved concepts such as symmetry, midpoints, and the motions of slide, turn, and flip are to be offered.

3. A comparison may be made on the role of the line of symmetry, when it exists, similar to the following:



In a triangular region, whenever a line of symmetry exists, the number of pieces will be less, and the dissection will be simpler.

Figure 10.

□

Time Out for Traveling

Ron Cammaert
Barnwell School, Taber

This is a modification of an article by Richard Trimarco which was originally published in the May 1981 issue of the Mathematics Teacher. *The original article applied to the Boston-Philadelphia area and used imperial units.*

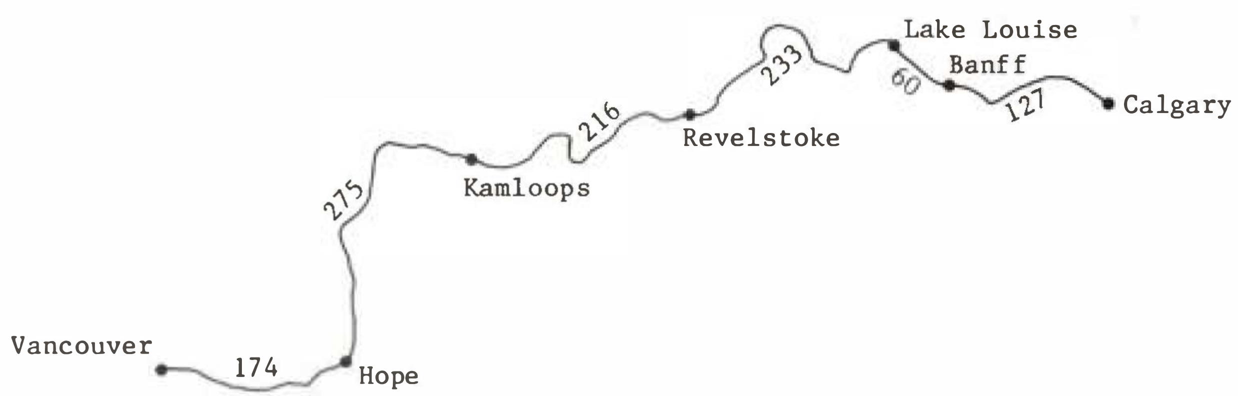
Activity 1.

If you travel by bus from Calgary to Vancouver, departure is at 1830 and arrival is at 1000 the next morning. (Don't forget the time zone change.) A one-way ticket costs $39.10.

1. How far is Vancouver from Calgary by bus? (Distance between towns is given in kilometres.)

2. What is the average speed of the bus, assuming only a five-minute stop at each center shown on the map?

3. What does it cost per kilometre to travel by bus?



Activity 2.

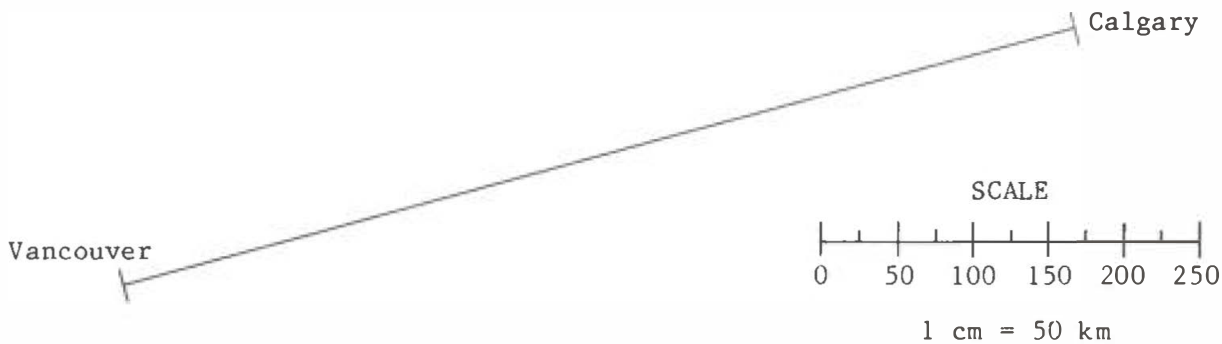
If you fly from Calgary to Vancouver, you can leave Calgary at 1720 and arrive in Vancouver at 1740 on a DC-9 airplane.

1. Use your ruler and the scale given to find the distance by air from Calgary to Vancouver.

2. What is the flying time? (Don't forget the time zone change.)

3. Find the average flight speed for the DC-9 in kilometres per hour.

4. If the one-way economy fare is $87, what is the cost per kilometre?



Activity 3.

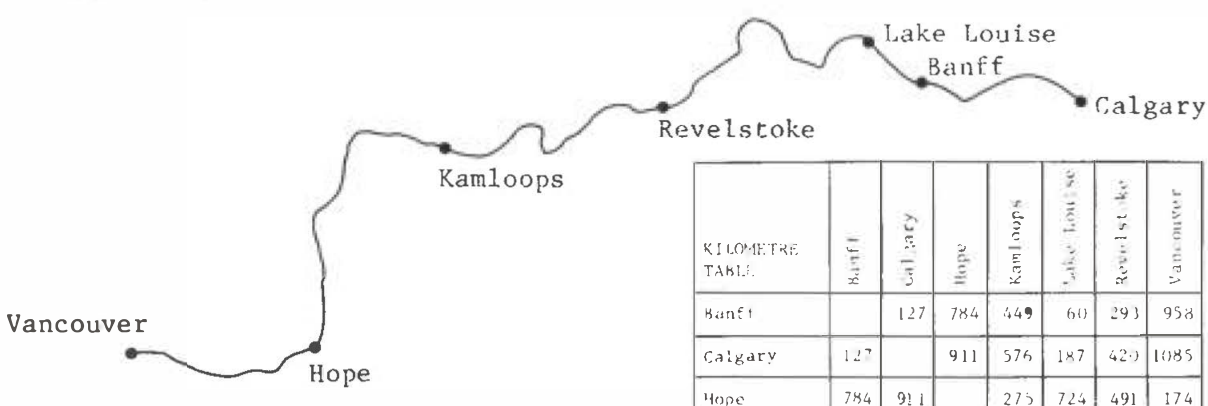
Suppose you travel by automobile from Calgary to Vancouver on the Trans-Canada Highway. Gasoline costs an average of 38 cents per litre.

1. How far is Vancouver from Calgary?

2. If you average 80 km/hour and drive nonstop, how long would the trip take in hours and minutes?

3. Find the cost of gasoline for the trip if your car averages 11 litres/km.

4. Estimate the cost per kilometre of the automobile trip. What cost factors other than those given should be considered in order to obtain a valid estimate of the actual cost per kilometre?



KILOMETRE TABLE	Banff	Calgary	Hope	Kamloops	Lake Louise	Revelstoke	Vancouver
Banff		127	784	449	60	293	958
Calgary	127		911	576	187	420	1085
Hope	784	911		275	724	491	174
Kamloops	449	576	275		449	216	449
Lake Louise	60	187	724	449		233	958
Revelstoke	293	420	491	216	233		898
Vancouver	958	1085	174	449	958	898	

□

? ? ? Problem Corner ? ? ?

edited by William J. Bruce and Roy Sinclair
University of Alberta, Edmonton

Problems suggested here are aimed at students in both the junior and senior high schools of Alberta. Solutions are solicited, and a selection will be made for publication in the next issue of *delta-K*. Names of participants will be included. All solutions must be received (preferably in typewritten form) within 60 days of publication of the problem in *delta-K*.

The Department of Mathematics, University of Alberta, has made prize money available for solutions: First Prize - $15; Second Prize - $10. Decision of the editors is final.

Mail solutions to: Dr. Roy Sinclair or Dr. Bill Bruce
Department of Mathematics
University of Alberta
Edmonton, Alberta T6G 2G1

Problem 8:

(submitted by Roy Sinclair, University of Alberta)
Reprinted from the March 1982 issue of delta-K.

A fly is located 1 m from the ceiling and in the middle of one end of a room. A hungry spider is located in the middle of the other end of the room and 1 m from the floor. Find the shortest path that the spider can take along the surface of the room to get to the fly if the room is 20 m long, 10 m high, and either (a) 10 m wide or (b) 15 m wide.

HINT: Unfold the room surface in each case to lie flat on a plane and solve both problems.

Problem 9:

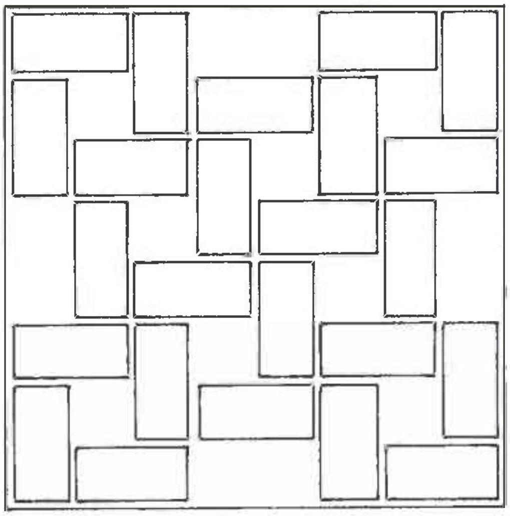
(submitted by Roy Sinclair, University of Alberta)

Use your hand-held calculator to solve the equation $\theta_n = \cos \theta_{n-1}$, $n \geq 1$ *either* in degrees *or* in radians. Indicate the program that you used and obtain the answer correct to eight figures. Include a sketch of the portion of the graph, which is involved, so as to show how to zero in on the point of intersection of the line and the curve.

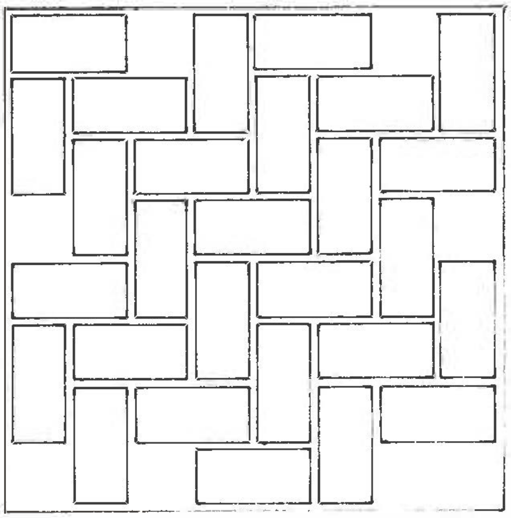
NOTE: This problem can be thought of as a calculator-assisted treasure hunt in which the hidden treasure is located at the point of intersection.

Solution to Problem 7:

(by William J. Bruce, University of Alberta)



(a) Right angle butting only. Minimum space unused - 16 squares.



(b) Right angle butting and semi-adjacent parallelism. Minimum space unused - 8 squares.

Note: It has been shown that these are the minima.

□

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New Computer Software Reference

Sales of computers to schools have created a need for professional instructional software. McKilligan Supply Corporation, a nationwide school supplier, has published a source book for computer software and equipment used in the classroom. It was published to guide teachers and administrators in making the best computer decisions for their students.

The 48-page reference is a compilation of the software produced by hundreds of programers and covers the best of the instructional software that the McKilligan firm has found to exist. Software is referenced to all the popular computers used in schools.

Among the software programs available are ones for math, science, reading, languages, guidance, art, vocational education, word processing, exceptional children, music, and accounting. In addition, there are programs for programming computers, as well as instructions on college-board tests for students and graduate record exams for their teachers.

Regular revisions of the publication are planned which will incorporate such new issues as software books, literature, teaching aids, and equipment that is available to assist teachers. The McKilligan firm believes this to be the first and most complete publication of its kind to serve the school in computer-assisted instruction. The publication will be sent, without charge, to a school address. For your copy, write to:

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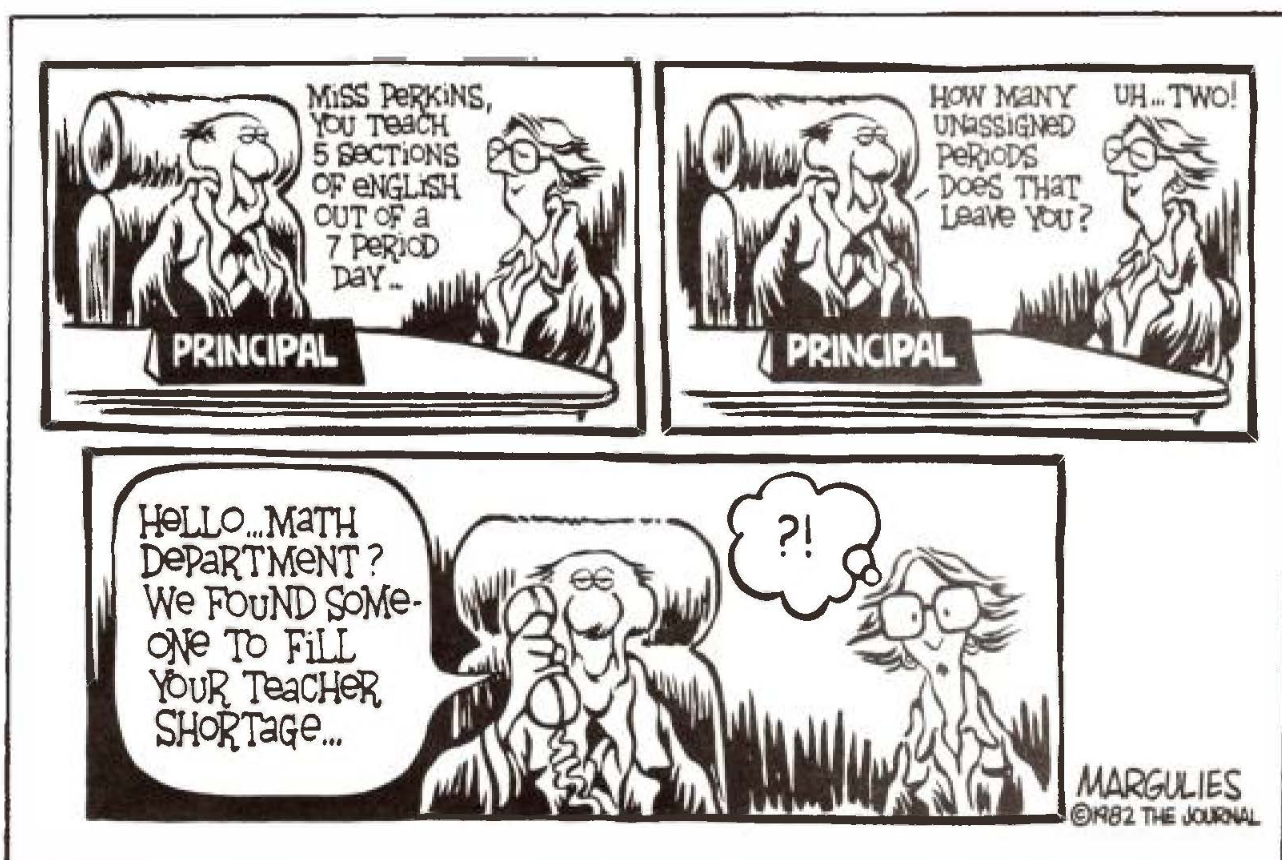
Johnson City, New York 13790

Phone: (607) 729-6511

Did You Miss the June Issue?

The *Canadian Mathematics Journal* which was mailed with the March issue of *delta-K* replaces the June issue. This was a trial run only for the *Canadian Mathematics Journal*.



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